Chapter 16: More Global Constraints (Car Sequencing)

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ECLiPSe ELEarning

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What we want to introduce

- Car sequencing problem
- \texttt{gcc} global cardinality constraint
- \texttt{sequence} constraint
- Search does not always have to be based on original problem variables
- Can be useful to consider additional variables which allow more clever search
Car Sequencing

We have to schedule a number of cars for production on an assembly line. Each car is of a certain type, and we know how many cars of each type we have to produce. Car types differ in the options they require, i.e. sun-roof, air conditioning. For each option, we have capacity limits on the assembly line, expressed as $k$ cars out of $n$ consecutive cars on the line may have some option. Find an assignment which produces the correct number of cars of each type, while satisfying the capacity constraints.

Example (DSV88)

- 100 cars
- 18 types
- 5 options
  - Option 1: 1 out of 2
  - Option 2: 2 out of 3
  - Option 3: 1 out of 3
  - Option 4: 2 out of 5
  - Option 5: 1 out of 5
## Car Types

<table>
<thead>
<tr>
<th>Type</th>
<th>Cars Required</th>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
<th>Option 4</th>
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Modelling Alternatives

- Assign start time (sequence number) to each car
  - 100 variables, each with 100 values
  - Handling of car types implicit
  - Symmetry breaking for cars of same type (inequalities)?
  - Capacity constraints?
- Assign car type to each slot on assembly line
  - 100 variables, 18 values
  - How to control number of cars of each type?
  - How to express capacity constraints?

Model

- 100 variables ranging over car types
- gcc constraint to control number of items with same type
- $5 \times 100$ 0/1 variables indicating use of option for each slot
- element constraints to map car types to options used
- sequence constraints to enforce limits on each option
Reminder: \textit{gcc}(\textit{Pattern}, \textit{Variables})

- \textit{gcc} \textit{global cardinality constraint}
- \textit{Pattern} is list of terms \textit{gcc}(\textit{Low}, \textit{High}, \textit{Value})
- The overall number of variables taking value \textit{Value} is between \textit{Low} and \textit{High}
- Generalization of \textit{alldifferent}
- Domain consistent version in ECLiPSe

\textbf{gcc Example}

\begin{verbatim}
X1 :: [2,4], X2 :: [1,3,4], X3 :: [1,2,3,4],
X4 :: [3,4,5], X5 :: [3,4,5],
gcc([gcc(1,1,1),gcc(2,3,2),gcc(1,3,3),
     gcc(0,4,4),gcc(1,3,5)],
    [X1,X2,X3,X4,X5]),
X1 = ?, X2 = ?, X3 = ?, X4 = ?, X5 = ?
\end{verbatim}
X1 :: [2,4], X2 :: [1,3,4], X3 :: [1,2,3,4],
X4 :: [3,4,5], X5 :: [3,4,5],
gcc([gcc(1,1,1), gcc(2,3,2), gcc(1,3,3),
    gcc(0,4,4), gcc(1,3,5)],
    [X1, X2, X3, X4, X5]),

X1 = ?2, X2 = ?, X3 = ?2, X4 = ?, X5 = ?

X1 :: [2,4], X2 :: [1,3,4], X3 :: [1,2,3,4],
X4 :: [3,4,5], X5 :: [3,4,5],
gcc([gcc(1,1,1), gcc(2,3,2), gcc(1,3,3),
    gcc(0,4,4), gcc(1,3,5)],
    [X1, X2, X3, X4, X5]),

X1 = 2, X2 = ?1, X3 = 2, X4 = ?, X5 = ?
Problem
Program
Search
Improved Search Strategy

gcc Continued

\[
\begin{align*}
X_1 & : [2, 4], X_2 : [1, 3, 4], X_3 : [\_, 2, 3, 4], \\
X_4 & : [3, 4, 5], X_5 : [3, 4, 5],
\end{align*}
\]

\[
gcc([gcc(1, 1, 1), gcc(2, 3, 2), gcc(1, 3, 3),
\quad gcc(0, 4, 4), gcc(1, 3, 5)],
\quad [X_1, X_2, X_3, X_4, X_5]),
\]

\[
X_1 = 2, X_2 = 1, X_3 = 2, X_4 = ?, X_5 = ?
\]

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gcc Made Domain Consistent

\[
\begin{align*}
X_1 & : [2, 4], X_2 : [1, 3, 4], X_3 : [\_, 2, 3, 4], \\
X_4 & : [3, 4, 5], X_5 : [3, 4, 5],
\end{align*}
\]

\[
gcc([gcc(1, 1, 1), gcc(2, 3, 2), gcc(1, 3, 3),
\quad gcc(0, 4, 4), gcc(1, 3, 5)],
\quad [X_1, X_2, X_3, X_4, X_5]),
\]

\[
X_1 = 2, X_2 = 1, X_3 = 2, X_4 \in \{3, 5\}, X_5 \in \{3, 5\}
\]
How does the constraint solver do that?

Explain in optional material at end

Reminder: element (X, List, Y)

- List is a list of integers
- The $X^{th}$ element of List is Y
- The index starts from 1
- Typical uses:
  - Projection
  - Cost
Prime is 1 iff $X \in 1..10$ is a prime number

$X :: 1..10,$
element(X,[1,1,1,0,1,0,1,0,0,0],Prime),

Cost is the cost corresponding to the assignment of $Y$

$Y :: 1..10,$
element(Y,[5,3,34,0,3,1,12,12,1,3],Cost)

sequence_total(Min,Max,Low,High,K,Vars)

- Variables $Vars$ have 0/1 domain
- Between $Min$ and $Max$ variables have value 1
- For every sub-sequence of length $K$, between $Low$ and $High$ variables have value 1
sequence_total Example

\[ [X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}] :: 0..1, \]
\[ \text{sequence_total}(2,3,1,2,3,} \]
\[ [X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}], \]
\[ X_1 = 0, X_4 = 0, X_7 = 0, X_{10} = 0 \]
Mathematical Equivalent

\[ \text{Vars} = [x_1, x_2, \ldots x_N] \]

\[ \text{Min} \leq \sum_{1 \leq i \leq N} x_i \leq \text{Max} \]

\[ 1 \leq s \leq N - k + 1 : \quad \text{Low} \leq \sum_{s \leq j \leq s + k - 1} x_j \leq \text{High} \]

- Pruning very different when using finite domain inequalities
- Currently no domain consistent implementation of \text{sequence_total}
- Weaker version \text{sequence} (no global counters) domain consistent
- Currently using decomposition:
  - \text{sequence_total} = \text{sequence} + \text{gcc} + \text{more}
Main Program

:-module(car).
:-export(top/0).
:-lib(ic).
:-lib(ic_global_gac).

top:-
    problem(Problem),
    model(Problem,L),
    writeln(L).

Structure Definitions

:-local struct(problem(cars,
                        models,
                        required,
                        using_options,
                        value_order)).

:-local struct(option(k,
                       n,
                       index_set,
                       total_use)).
Model (Part 1)

```
model(problem{cars:NrCars,
           models:NrModels,
           required:Required,
           using_options:List,
           value_order:Ordered},L):-
    length(L,NrCars),
    L :: 1..NrModels,
    (foreach(Cnt,Required),
     count(J,1,_),
     foreach(gcc(Cnt,Cnt,J),Card) do
      true
    ),
    gcc(Card,L),
    ...
```

Model (Part 2)

```
... (foreach(option{k:K,
                  n:N,
                  index_set:IndexSet,
                  total_use:Total},List),
    param(L,NrCars) do
    (foreach(X,L),
     foreach(B,Binary),
     param(IndexSet) do
      element(X,IndexSet,B)
    ),
    sequence_total(Total,Total,0,K,N,Binary)
  ),
  search(L,0,input_order,ordered(Ordered),
```
Data

```
problem(100, 18,
    [5, 3, 7, 1, 10, 2, 11, 5, 4, 6, 12, 1, 1, 5, 9, 5, 12, 1],
    [option(1, 2, [1, 2, 3, 5, 6, 7, 8, 14],
      [1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0], 48),
     option(2, 3, [1, 2, 3, 4, 5, 9, 10, 11, 15],
      [1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0], 57),
     option(1, 3, [3, 4, 8, 11, 12, 13, 18],
      [0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1], 28),
     option(2, 5, [2, 4, 7, 10, 13, 17],
      [0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1], 34),
     option(1, 5, [1, 6, 9, 12, 16],
      [1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0], 17)]
    [1, 3, 2, 4, 6, 8, 7, 12, 13, 5, 9, 11, 10, 14, 16, 18, 17, 15]).
```

Data Generation

- Data not really stored as facts
- Generated from text data files in different format
- Benchmark set from CSPLIB (http://www.csplib.org)
Another Example (PR97)

- 100 cars
- 22 types
- 5 options
  - Option 1: 1 out of 2
  - Option 2: 2 out of 3
  - Option 3: 1 out of 3
  - Option 4: 2 out of 5
  - Option 5: 1 out of 5

Second Example: Car Types

<table>
<thead>
<tr>
<th>Type</th>
<th>Cars Required</th>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
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</table>
Search (Stopped After 1000 Nodes)

- This does not look good
- Typical thrashing behaviour
- We made a wrong choice at some point
- ... but did not detect it
- Many additional choices are made before failure is detected
- We have to explore the complete tree under the wrong choice
- This is far too expensive
Change of Search Strategy

- Do not label car slot variables
- Decide instead if slot should use an option or not
- This restricts the car models which can be placed in this slot
- Start with the most restricted option
- When all options are assigned, the car type is fixed
- Potential problem: We now have 500 instead of 100 decision variables
- Naive searchspace $2^{500} = 3.2 \times 10^{150}$ instead of $2^{100} = 1.7 \times 10^{34}$

Second Modification

- Instead of assigning values left to right
- Start assigning in middle of board
- And alternate around middle until you reach edges
- Idea: Slots at edges are less constrained, i.e. easier to assign
- Save those slots until the end
- We already encountered this idea for the N-Queens problem
Modified Search

Assignment Step 2
Observations

- Important to start in middle
- Making hard choices first
- Concentrate on difficult to satisfy sub-problem
- Number of choices is much smaller than number of variables
- Some assignments lead to a lot of propagation

Conclusions

- Introduced global constraint *sequence*
- Reuse *gcc* and *element*
- Search on auxiliary variables can work well
- Raw search space measures are unreliable
- Modelling idea
  - Decide what to make in a given time slot
  - ... and not when to schedule some given activity
Making gcc Domain Consistent

Method: Max Flow Model

- Express constraint as max-flow problem
- Any flow solution corresponds to a valid assignment
- Only work with one flow solution
- Obtain all others by considering
  - residual graph and
  - strongly connected components
- Classical method, faster methods exist
- Use of max flow based propagators for many constraints
Making gcc Domain Consistent

Start with Value Graph

\[ \begin{align*}
X_1 &\rightarrow 1 & X_2 &\rightarrow 2 \\
X_2 &\rightarrow 3 & X_3 &\rightarrow 4 \\
X_3 &\rightarrow 5 & X_4 &\rightarrow 2 \\
X_4 &\rightarrow 4 & X_5 &\rightarrow 5 \\
\end{align*} \]

More Global Constraints

Convert to Flow Problem

\[ \begin{align*}
\text{s} &\rightarrow X_1 & 1 \\
\text{s} &\rightarrow X_2 & 2 \\
\text{s} &\rightarrow X_3 & 3 \\
\text{s} &\rightarrow X_4 & 4 \\
\text{s} &\rightarrow X_5 & 5 \\
X_1 &\rightarrow X_2 & 1 \\
X_1 &\rightarrow X_3 & 1 \\
X_1 &\rightarrow X_4 & 1 \\
X_1 &\rightarrow X_5 & 1 \\
X_2 &\rightarrow X_3 & 1 \\
X_2 &\rightarrow X_4 & 1 \\
X_2 &\rightarrow X_5 & 2 \\
X_3 &\rightarrow X_4 & 1 \\
X_3 &\rightarrow X_5 & 0 \\
X_4 &\rightarrow X_5 & 1 \\
\text{t} &\rightarrow X_1 & 1 \\
\text{t} &\rightarrow X_2 & 2 \\
\text{t} &\rightarrow X_3 & 3 \\
\text{t} &\rightarrow X_4 & 4 \\
\text{t} &\rightarrow X_5 & 5 \\
\end{align*} \]
Find Maximal Flow

Mark Value Edges in Flow
Residual Graph

Find Strongly Connected Components
Mark Edges

Remove Unmarked Edges

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Constraint is Domain Consistent

More Information

Mehmet Dincbas, Helmut Simonis, and Pascal Van Hentenryck.
Solving the car-sequencing problem in constraint logic programming.

Jean-Charles Regin and Jean-Francois Puget.
A filtering algorithm for global sequencing constraints.
Making gcc Domain Consistent

More Information

Christine Solnon, Van Dat Cung, Alain Nguyen, and Christian Artigues.

Willem Jan van Hoeve, Gilles Pesant, Louis-Martin Rousseau, and Ashish Sabharwal.
Revisiting the sequence constraint.

Michael J. Maher, Nina Narodytska, Claude-Guy Quimper, and Toby Walsh.
Flow-based propagators for the sequence and related global constraints.