# Chapter 17: Using Mixed Integer Linear Programming (Routing and Wavelength Assignment)

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ECLiPSe ELearning Overview

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<th>Helmut Simonis</th>
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What we want to introduce

- Mixed Integer Linear Programming in ECLiPSe
- eplex Library
- Alternative Models for Routing and Wavelength Assignment in Optical Networks
Problem Definition

Routing and Wavelength Assignment (Demand Acceptance)

In an optical network, traffic demands between nodes are assigned to a route through the network and a specific wavelength. The route (called *lightpath*) must be a simple path from source to destination. Demands which are routed over the same link must be allocated to different wavelengths, but wavelengths may be reused for demands which do not meet. The objective is to find a combined routing and wavelength assignment which maximizes the number of accepted demands.

Difference to Previous Problem

- **Static problem**
  - Accept all demands
  - Minimize number of wavelengths used
  - Design problem, minimize cost of network

- **Demand acceptance problem**
  - Number of wavelengths fixed
  - Maximize number of accepted demands
  - Operational problem, maximize use of network
Example Network (NSF, 5 wavelengths)

Lightpath from node 5 to node 13 (5 ⇒ 13)
Conflict with demand 1 ⇒ 12: Use different frequencies

Conflict with demand 1 ⇒ 12: Use different path
Conflict with demand 1 \(\Rightarrow\) 12: Reject demand

**Solution Approaches**

- Greedy heuristic
- Optimization algorithm for complete problem
- Decomposition into two problems
  - Find routing
  - Assign wavelengths
Optimization Solutions

- Link Based Model
  - Individual demands
  - Source aggregation
- Path Based Model

Link Based Model - Individual Demands

- Decide for each demand whether it is accepted and which wavelength is used
- Zero/One decision variable $y_d^\lambda$
- Atmost one wavelength may be used for demand
- Decide for each link and wavelength if it is used for demand
- Zero/One decision variables $x_{de}^\lambda$
- Different demands can not use the same wavelength on the same link
- Maximize the total number of demands accepted
Notation

- Directed graph $G = (N, E)$
- Nodes $n \in N$
- Edges $e \in E$
- Given Wavelengths $\lambda \in \Lambda$
- Demands $d \in D$ from source $s(d)$ to sink $t(d)$
- Edges Out($n$) leaving node $n$
- Edges In($n$) pointing to node $n$

Model Variables

- All Variables 0/1 Integer
- $x_{de}^\lambda$ wavelength $\lambda$ on link $e$ are used for demand $d$
- $y_d^\lambda$ wavelength $\lambda$ is used for demand $d$
Demand Acceptance Model 1

\[
\begin{align*}
\max \sum_{d \in D} \sum_{\lambda \in \Lambda} y^\lambda_d \\
\text{s.t.} \\
y^\lambda_d \in \{0, 1\}, x^\lambda_{de} \in \{0, 1\} \\
\forall d \in D : \sum_{\lambda \in \Lambda} y^\lambda_d \leq 1 \\
\forall e \in E, \forall \lambda \in \Lambda : \sum_{d \in D} x^\lambda_{de} \leq 1 \\
\forall d \in D, \forall \lambda \in \Lambda : \sum_{e \in \text{In}(s(d))} x^\lambda_{de} = 0, \sum_{e \in \text{Out}(s(d))} x^\lambda_{de} = y^\lambda_d \\
\forall d \in D, \forall \lambda \in \Lambda : \sum_{e \in \text{Out}(t(d))} x^\lambda_{de} = 0, \sum_{e \in \text{In}(t(d))} x^\lambda_{de} = y^\lambda_d \\
\forall d \in D, \forall \lambda \in \Lambda, \forall n \in N \setminus \{s(d), t(d)\} : \sum_{e \in \text{In}(n)} x^\lambda_{de} = \sum_{e \in \text{Out}(n)} x^\lambda_{de}
\end{align*}
\]

- Minimize/maximize some linear objective
- While satisfying linear equality/inequality constraints
- 0/1, integer or continuous variables
Solving the problem

- This is a standard MILP problem
- MILP = Mixed Integer Linear Programming
- ECLiPSe provides an interface to such solvers
- eplex library
- Works with commercial or open-source MIP/LP solvers

Main eplex Features used

- Variable definition: \( X :: 0.0 .. 1.0 \)
- Linear constraints \( X + Y \$= 1 \)
- Integrality constraints \( \text{integers}([X,Y]) \)
- Solver setup \( \text{eplex_solver_setup}(\min(M)) \)
- Optimization call \( \text{eplex_solve}(\text{Cost}) \)
- We can solve multiple MIP problems at the same time.
- We therefore need to state which problem we want to affect.
- This is done with `eplex` instances.
- Works like a module: `route: (X+Y $=1)` adds constraint to instance `route`.
- Create, use, cleanup.

Why not use finite domain solver?

- For this type of problem, finite domain reasoning is very weak.
- Each constraint is treated independently.
- Interaction through 0/1 variables is limited.
- No concept of minimizing objective function.
MILP solver basics

- Considers all constraints together
- In form of continuous relaxation
- Use Simplex algorithm to find optimal solution for relaxation
- Integer solutions found by forcing values to be integral
- By branching and/or by adding constraints (cutting planes)

Benchmarks

- Fixed network structure
  - nsf 14 nodes, 42 edges
  - eon 20 nodes, 78 edges
  - mci 19 nodes, 64 edges
  - brezil 27 nodes, 140 edges
### Selected Results (100 runs)

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**Idea**

- Combine all demands starting same node
- Build distribution tree (graph) rooted in source $s$
- Decide whether link/frequency is used for this distribution graph
- Graphs for different source nodes compete for resources
- Enforce sufficient conditions to extract routes for individual demands
Model Notation

- **Constants**
  - $P_{sd}$ integer, total number of requested demands from $s$ to $d$
  - $D_s$, set of all destination nodes for demands sourced in $s$

- **Variables**
  - $y_{sd}$ integer variable, how many demands from $s$ to $d$ are accepted (domain 0 to $P_{sd}$)
  - $x_{se}^\lambda$ 0/1 integer variable, frequency $\lambda$ on link $e$ is used to transport demands starting in $s$

Source Aggregation Model

$$\text{max} \sum_{s \in N} \sum_{d \in D_s} y_{sd}$$

s.t.

- $y_{sd} \in \{0, 1, \ldots, P_{sd}\}$, $x_{se}^\lambda \in \{0, 1\}$
- $\forall e \in E, \forall \lambda \in \Lambda : \sum_{s \in N} x_{se}^\lambda \leq 1$
- $\forall s \in N, \forall \lambda \in \Lambda : \sum_{e \in \text{In}(s)} x_{se}^\lambda = 0$
- $\forall s \in N, \forall d \in D_s, \forall \lambda \in \Lambda : \sum_{e \in \text{In}(d)} x_{se}^\lambda \geq \sum_{e \in \text{Out}(d)} x_{se}^\lambda$
- $\forall s \in N, \forall d \in D_s : \sum_{e \in \text{In}(d)} x_{se}^\lambda = \sum_{e \in \text{Out}(d)} x_{se}^\lambda + y_{sd}$
- $\forall s \in N, \forall n \neq s, n \notin D_s, \forall \lambda \in \Lambda : \sum_{e \in \text{In}(n)} x_{se}^\lambda = \sum_{e \in \text{Out}(n)} x_{se}^\lambda$
• MIP Solution does not say which demands are accepted
• ...nor how they are routed through the network
• Needs solutions extraction, for each source
• At the same time remove loops from routes

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### More Information

**Rajiv Ramaswami and Kumar N. Sivarajan.**

**Dhritiman Banerjee and Biswanath Mukherjee.**

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**Brigitte Jaumard, Christophe Meyer, and Babacar Thiongane.**

**Brigitte Jaumard, Christophe Meyer, and Babacar Thiongane.**


Helmut Simonis. Solving the static design routing and wavelength assignment problem. CSCLP 2009, Barcelona, Spain, June 2009.