Chapter 18: A Hybrid Model for the Routing and Wavelength Assignment Problem

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ECLiPSe ELearning
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Outline

1. Problem
2. Model
3. Worked Example
4. Results
What We Want to Introduce

- Hybridisation by decomposition
- Combination of MIP and FD solver
- Best current solution to routing and wavelength assignment problem
Outline

1. Problem
2. Model
3. Worked Example
4. Results
Routing and Wavelength Assignment (Demand Acceptance)

In an optical network, traffic demands between nodes are assigned to a route through the network and a specific wavelength. The route (called *lightpath*) must be a simple path from source to destination. Demands which are routed over the same link must be allocated to different wavelengths, but wavelengths may be reused for demands which do not meet. The objective is to find a combined routing and wavelength assignment which maximizes the number of accepted demands.
Example Network (NSF, 5 wavelengths)
Lightpath from node 5 to node 13 (5 ⇒ 13)
Conflict with demand 1 $\Rightarrow$ 12: Use different frequencies
Conflict with demand 1 ⇒ 12: Use different path
Conflict with demand 1 $\Rightarrow$ 12: Reject demand
Outline

1. Problem
2. Model
3. Worked Example
4. Results
Solution Approaches

- Greedy heuristic
- Optimization algorithm for complete problem
- Decomposition into two problems
  - Route maximal number of demands
  - Assign wavelengths
Solution Approaches

- Greedy heuristic
- Optimization algorithm for complete problem
- Decomposition into two problems
  - Route maximal number of demands
  - Assign wavelengths
Step 1: Route Maximal Number of Demands

- Ignore wavelengths
- Capacity constraints on all links
- Solve as MIP problem
- Source aggregation
- Find DAG to supply (all) demands with shared source
- Maximize number of accepted demands
Notation

- \( y_{sd} \), integer number of accepted demands from \( s \) to \( d \)
- \( z_{se} \), integer capacity used on edge \( e \) to satisfy demands sourced in \( s \)
- \( C \), number of available wavelengths, edge capacity
- \( P_{sd} \), requested number of demands from \( s \) to \( d \)
- \( T_s \), total number of requested demands sourced from \( s \)
- \( D_s \), nodes which have a requested demand sourced in \( s \)
Model (Step 1)

\[
\max \sum_{s \in N} \sum_{d \in D_s} y_{sd}
\]

s.t.

\[
y_{sd} \in \{0, 1 \ldots P_{sd}\}, \quad z_{se} \in \{0, 1 \ldots T_s\}
\]

\[
\forall e \in E : \quad \sum_{s \in \mathcal{N}} z_{se} \leq C
\]

\[
\forall s \in \mathcal{N} : \quad \sum_{e \in \text{In}(s)} z_{se} = 0
\]

\[
\forall s \in \mathcal{N}, \forall d \in D_s : \quad \sum_{e \in \text{In}(d)} z_{se} = \sum_{e \in \text{Out}(d)} z_{se} + y_{sd}
\]

\[
\forall s \in \mathcal{N}, \forall n \neq s, n \notin D_s : \quad \sum_{e \in \text{In}(n)} z_{se} = \sum_{e \in \text{Out}(n)} z_{se}
\]
Observation

- Optimal cost is upper bound for full problem
- LP Relaxation is also upper bound for full problem
- No 0/1 variables in model
- Source aggregation has massive impact on efficiency
  - Much better than treating each demand on its own
  - Reason 1: Reduced number of variables
  - Reason 2: Avoids symmetries due to multiple demands between nodes
Finding Accepted Demands

- Solution to MIP does not tell how demands are routed
- Program required to convert source “tree” into sets of paths
- Conversion not deterministic, may allow different solutions
- Solution may contain loops, these need to be removed
Step 2: Assign Wavelengths

- For each accepted demand, find frequency
- All demands routed over a link compete for frequencies
- Graph coloring problem
- Graph given as sets of cliques
- Solve with finite domains
- If solution found, then optimal for complete problem
Model (Step 2)

- $X_d$ finite domain variable 1..C for each accepted demand
- One \texttt{alldifferent} constraint for each edge
- Many \texttt{alldifferent} constraints are at capacity
- Possible to improve model
What Happens If No Solution Found

- **Problem infeasible**
  - Remove some demand and try again until solution found
  - Possibly sub-optimal solution of high quality
  - Different solution to MIP problem may lead to optimal solution

- **No solution found within time limit**
  - Try harder!
  - Improve reasoning and/or search technique
  - Special techniques to show infeasibility
Solution Approach

1. MIP Resource Model
2. Extract Accepted Demands
3. FD Graph Coloring
   - Infeasible
     - Yes: Provide Explanation
     - No: Solution
   - Remove Demand
     - Yes: Provide Explanation

Hybrid Model for RWA
Outline

1. Problem
2. Model
3. Worked Example
4. Results
Demand Matrix (100 Demands)

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Color Distance

- Color: Blue = 1, Green = 2, Red = 3, Yellow = 4, Orange = ≥ 5
Source Model Solution

Source Node 1

Source Model Solution

Source Node 1

Source Model Solution

Source Node 1

Source Model Solution

Source Node 1

Source Model Solution

Source Node 1
Source Model Solution

Source Node 2
Source Model Solution

Source Node 4
Source Model Solution

Source Node 5

Source Model Solution

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Hybrid Model for RWA
Source Model Solution

Source Node 6

Source Model Solution

Source Node 6

Source Model Solution

Source Node 6

Source Model Solution

Source Node 6

Source Model Solution

Source Node 6
Source Model Solution

Source Node 7

[Network diagram showing connections and colors labeled as Source, Sink, Unreached, Chosen Link]
Source Model Solution

Source Node 8

- **Source Node 8**: Represents the starting point of the network graph.

- **Color and Type Legend**:
  - Source
  - Sink
  - Unreached
  - Chosen Link

- **Graph Structure**:
  - Nodes are connected by edges with assigned colors and types.
  - The graph shows the flow of traffic from the source node to the sink node through chosen links.

- **Node Details**:
  - **Source Node**: Identified by a green circle labeled 'S'.
  - **Sink Node**: Identified by a red circle labeled '1'.

- **Edge Details**:
  - Edges are labeled with numbers indicating the flow or capacity, e.g., '1' and '2'.

- **Graph Complexity**:
  - The graph is hybrid, involving multiple nodes and edges, illustrating the network's complexity and connectivity.

- **Network Analysis**:
  - The chosen link is highlighted, indicating the path from the source to the sink.
Source Model Solution

Source Node 9
Source Model Solution

Source Node 10
Source Model Solution

Source Node 11

Graph with nodes and links representing the network topology and color-coded for different types: Source, Sink, Unreached, and Chosen Links.
Source Model Solution

Source Node 12

Hybrid Model for RWA
Source Model Solution

Source Node 13

Color
Type
Source
Sink
Unreached
Chosen Link

Hybrid Model for RWA
Source Model Solution

Source Node 14

Source Model Solution
Accepted Demands (86 Demands)
Accepted demands do not always use shortest path
Tendency to reject demands with larger minimal distance
These use more resources
Not compensated in objective function
Not fair
Graph Coloring Problem
Graph Coloring Solution

Color Wavelength
1
2
3
4
5
Observation

- All demands could be assigned to frequencies
- Optimal solution to complete problem
Explaining Infeasibility
Ad-hoc: Find pattern which show infeasibility
- Find large cliques
- If clique is larger than number of colors, problem is infeasible
- This is simple for graphs given

General explanation techniques
- Active research area
Explanation Method Used: QuickXPlain

- Find minimal subset of constraints which is infeasible
- Conflict set
- Works when overall problem fails without search
- Requires some trick to be applied here
Assigned Wavelengths

Wavelength 1

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Hybrid Model for RWA
Assigned Wavelengths

Wavelength 2

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Hybrid Model for RWA 51
Assigned Wavelengths

Wavelength 3

Color
1
2
3
4
≥ 5
unused
Assigned Wavelengths

Wavelength 4
Assigned Wavelengths

Wavelength 5

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Hybrid Model for RWA
Accepted Demands (86 Demands)

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Problem Model Worked Example Results

Hybrid Model for RWA

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Outline

1. Problem
2. Model
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4. Results
Benchmarks

- **Fixed network structure**
  - *nsf* 14 nodes, 42 edges
  - *eon* 20 nodes, 78 edges
  - *mci* 19 nodes, 64 edges
  - *brezil* 27 nodes, 140 edges

- **Random network structure**
  - Sizes from 30 to 100 nodes
  - Edge density 0.25
  - 500 demands, 30 wavelengths
### Overall Distribution of Solutions

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## Selected Examples (100 Runs Each)

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## Random Networks (Edge Density 0.25, 100 Runs Each)

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MIP and LP relaxation of phase 1 are very good bounds
Solved to optimality in most cases
Simple decomposition quite effective
Good solution even if initial graph coloring infeasible
Special structure of graph coloring helps FD model
Conclusions

- Combination of MIP and FD solver in problem decomposition
- Each doing what they do best
  - MIP: optimal solution, select items to include
  - FD: find feasible solution, explain infeasibility
- Hybrid model produces very high quality results
- Proven optimality in over 99.85% of problems tested
- Near optimal solutions by relaxation
- Much faster than monolithic MIP solution

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CSCLP 2009, Barcelona, Spain, June 2009.