

Chapter 11: Limits of Propagation (Costas Array)

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ECLiPSe ELearning [Overview](#)



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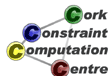


Outline

- 1 Problem
- 2 Program
- 3 Search
- 4 Improvements

What we want to introduce

- Improving propagation does not always pay
- For some problems, simple backtracking is best
- CP may not always be the best method
- CP should always be fastest way to model problem
- Consider time to target
 - Time required to *run* program
 - Time required to *write* program
- Problem: Costas Array (Antenna design, sonar systems)



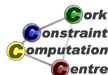
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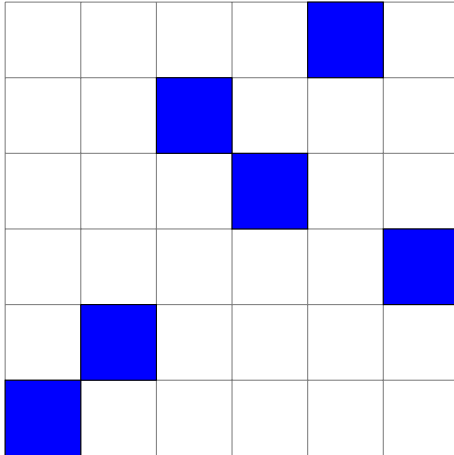
Problem Definition

Costas Array (Wikipedia)

A Costas array (named after John P. Costas) can be regarded geometrically as a set of N points lying on the squares of a $N \times N$ checkerboard, such that each row or column contains only one point, and that all of the $N(N - 1)/2$ vectors between each pair of dots are distinct.

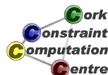


Example (Size 6)

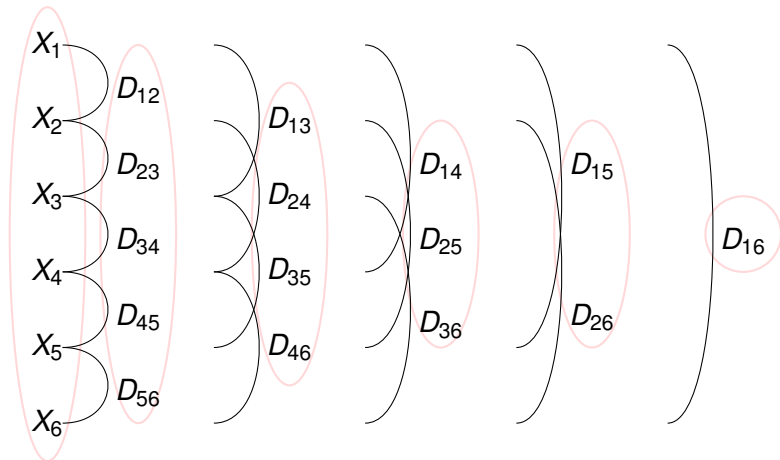


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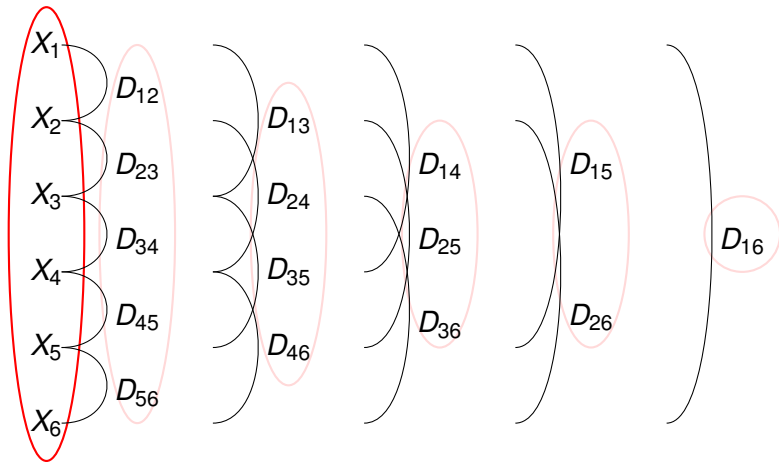
- A variable for each column, ranging from 1 to N
- A list of N variables for the columns
- A difference variable between each ordered pair of variables
- `alldifferent` constraint between variables
- `alldifferent` constraints for all differences



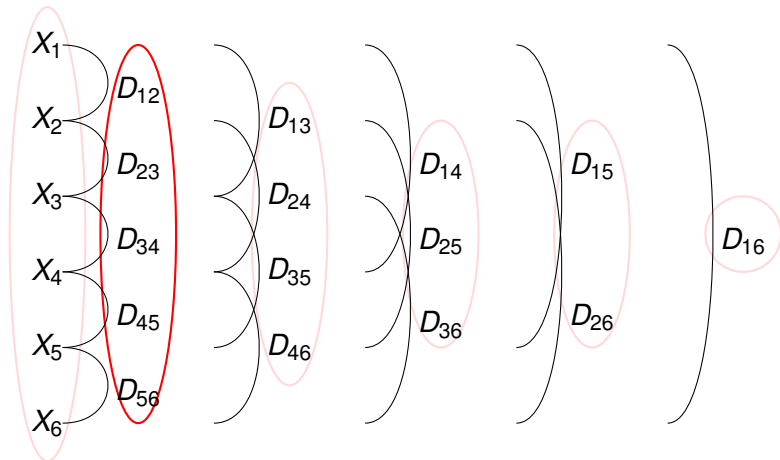
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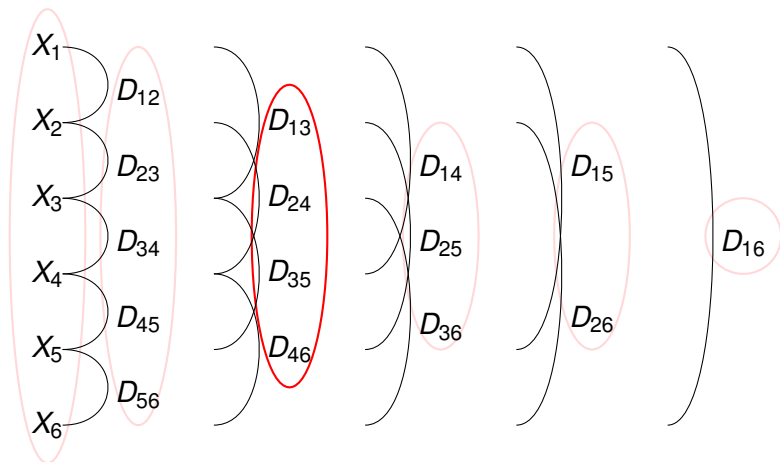
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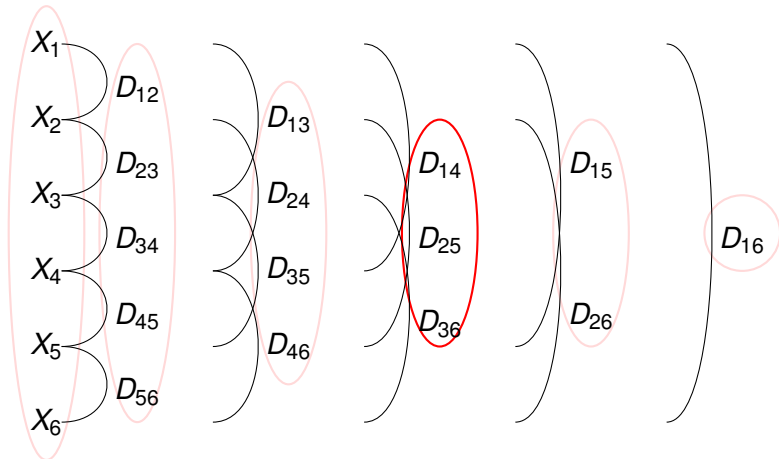
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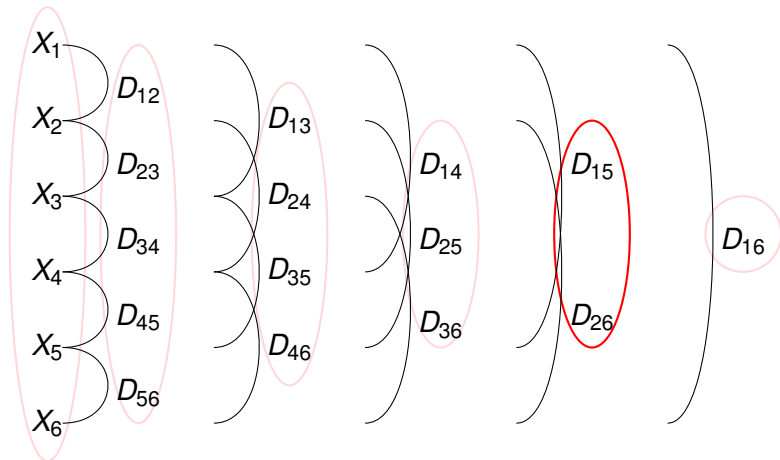
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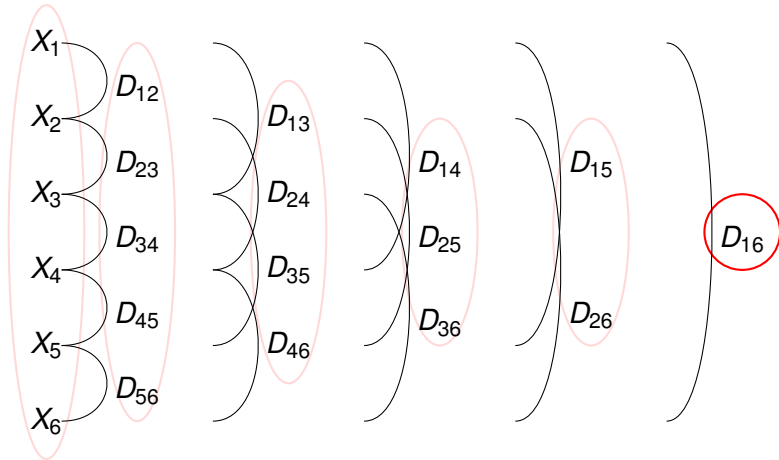
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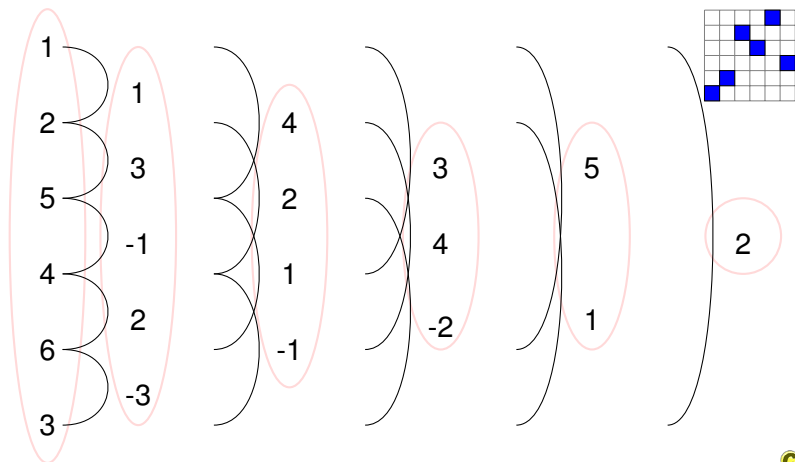
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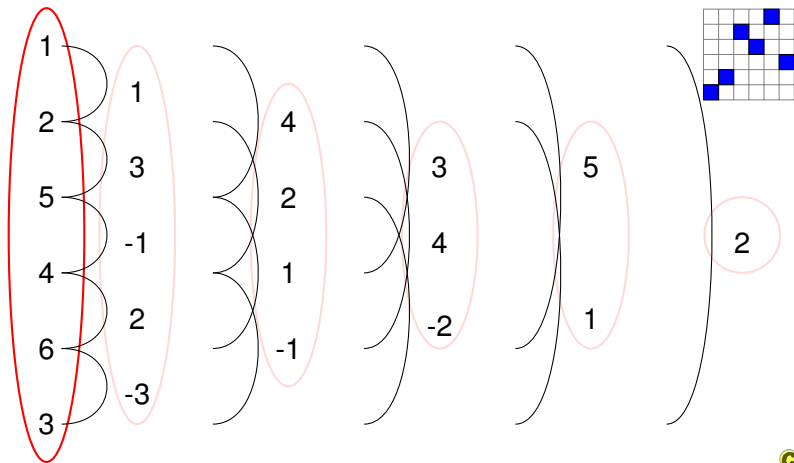
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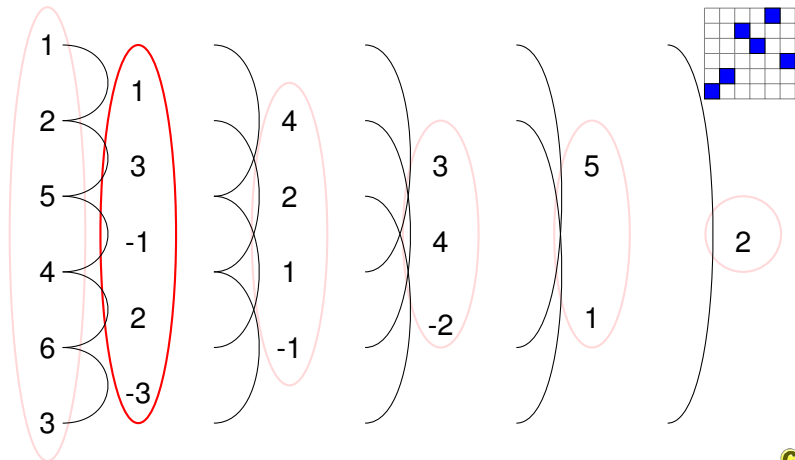
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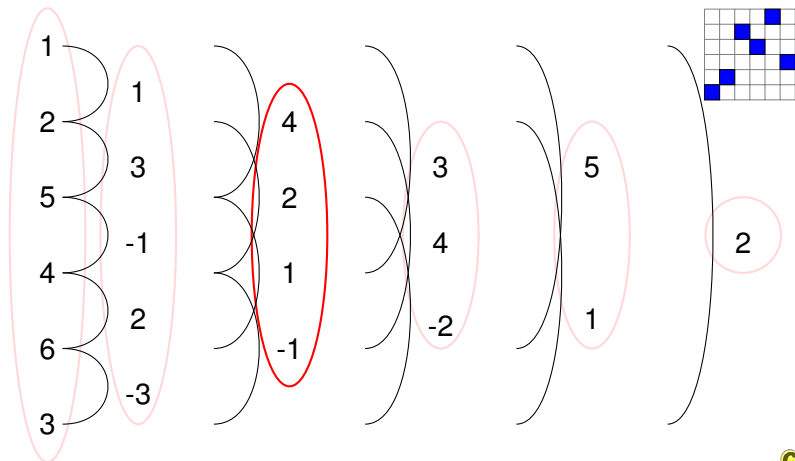
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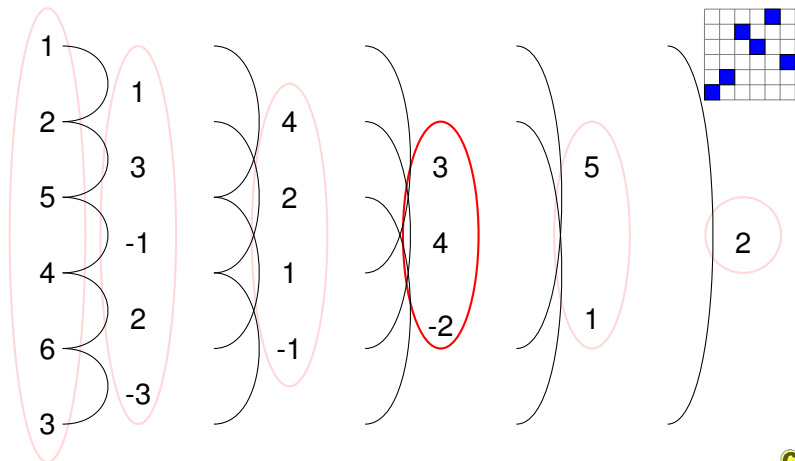
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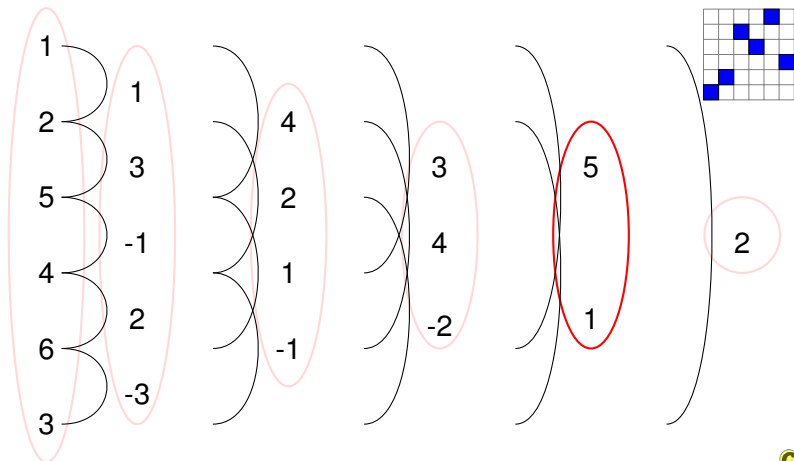
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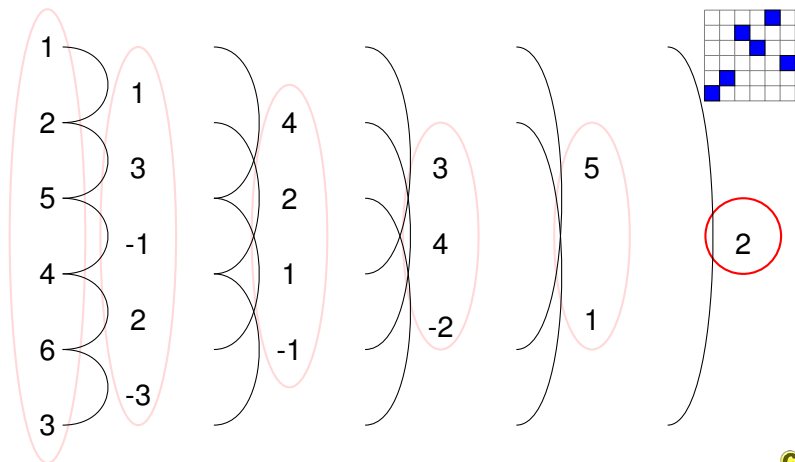
Example



Example



Example



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Declarations

```
:-module(costas).
```

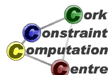
```
:-export(top/0).
```

```
:-lib(ic).
```



Main Program

```
top:-  
    (for(N,3,20) do  
        costas(N,_)  
    ).  
  
costas(N,L):-  
    length(L,N),  
    L :: 1..N,  
    alldifferent(L),  
    L = [_|L1],  
    diffs(L,L1),  
    search(L,0,first_fail,indomain,  
        complete,[]).
```



Differences

```

diffs(_, []).
diffs(L, [H|T]) :-
    diff_pairs(L, [H|T], Diffs),
    alldifferent(Diffs),
    diffs(L, T).

diff_pairs(_, [], []).
diff_pairs([X|X1], [Y|Y1], [D|D1]) :-
    X #= Y+D,
    diff_pairs(X1, Y1, D1).
  
```

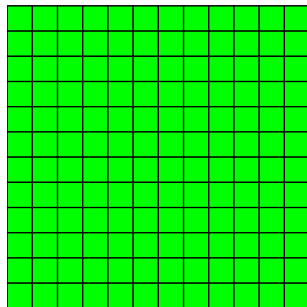


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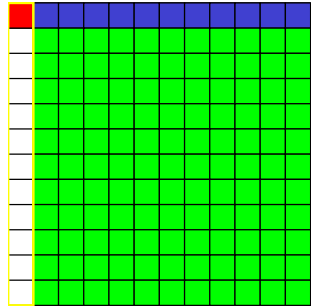
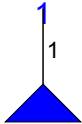
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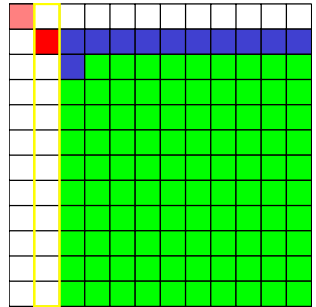
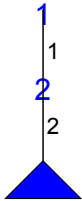
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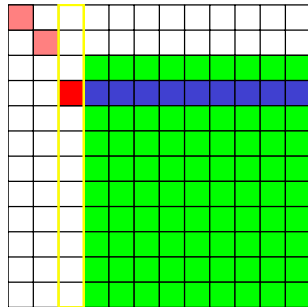
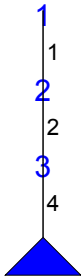
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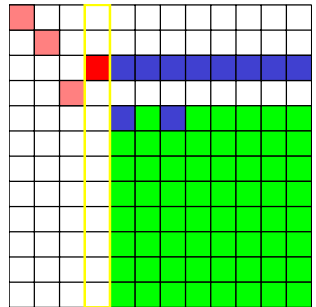
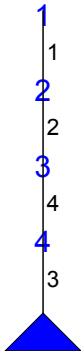
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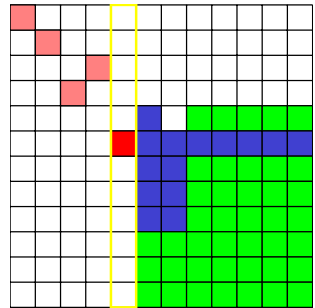
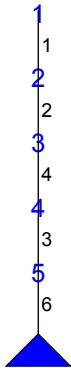
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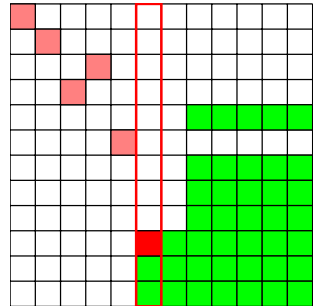
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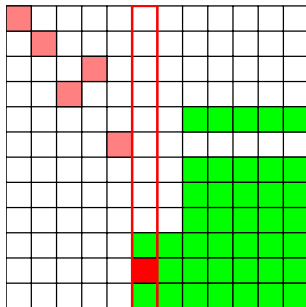
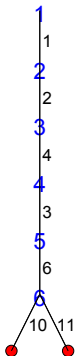
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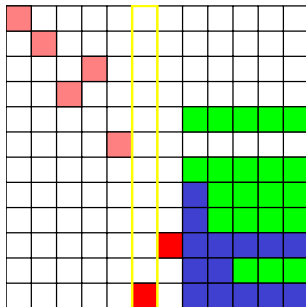
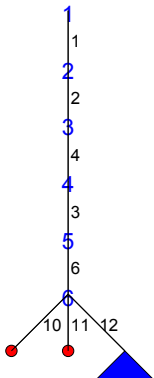
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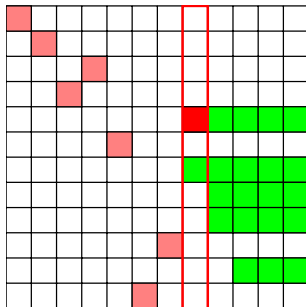
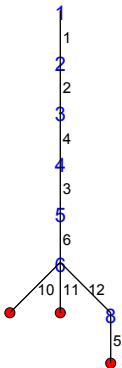


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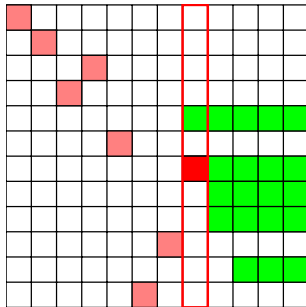
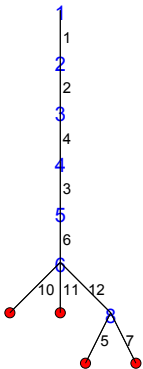
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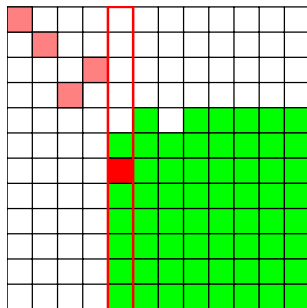
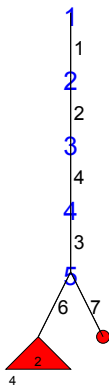
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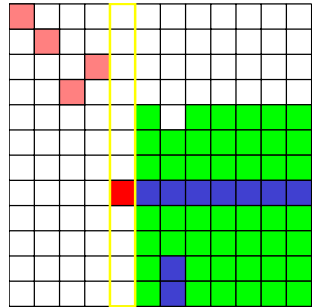
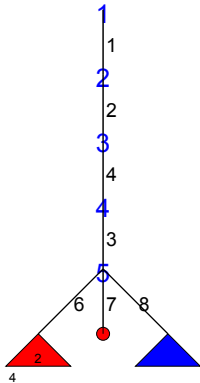
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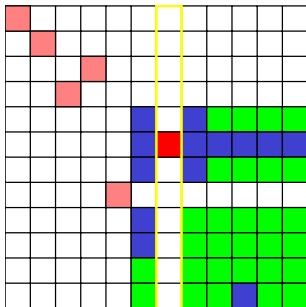
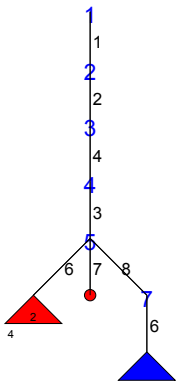
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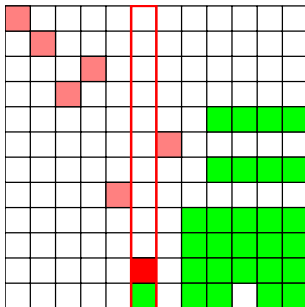
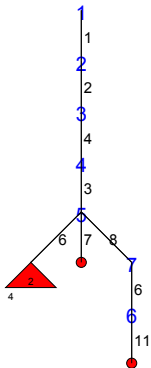
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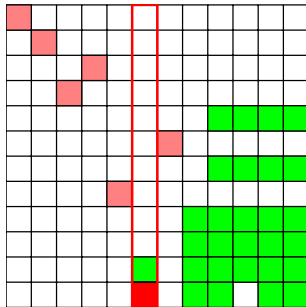
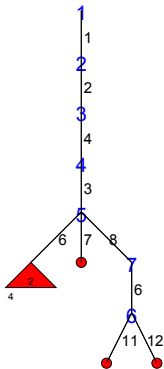
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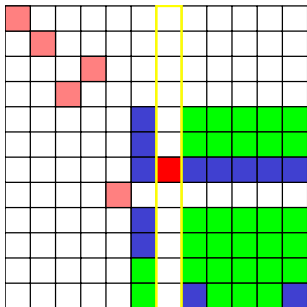
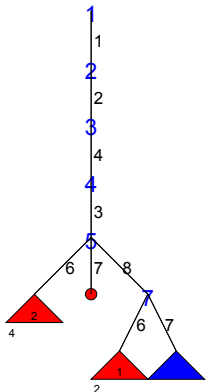
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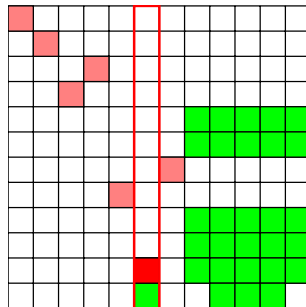
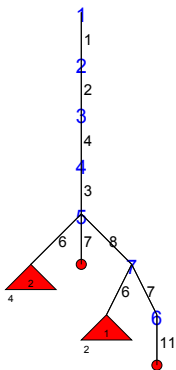


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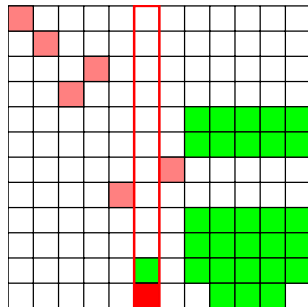
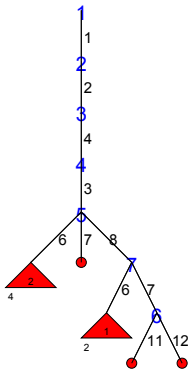


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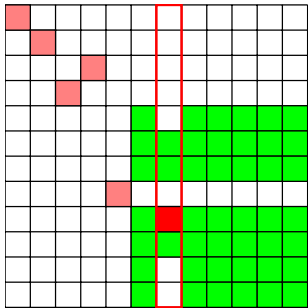
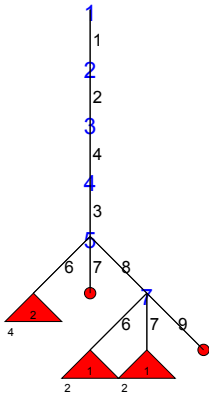
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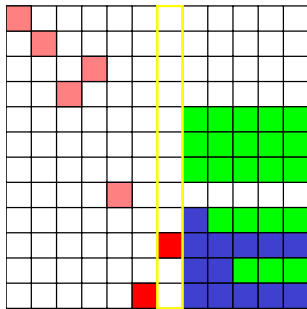
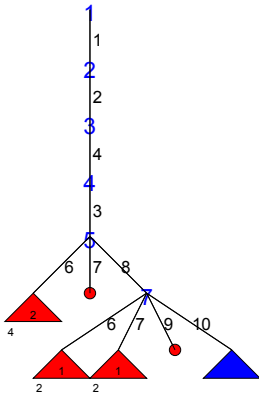
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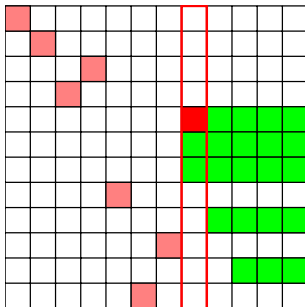
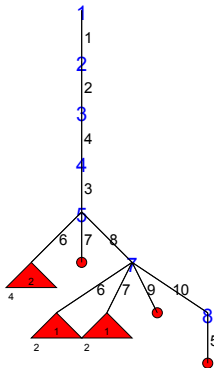
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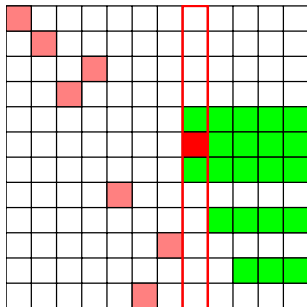
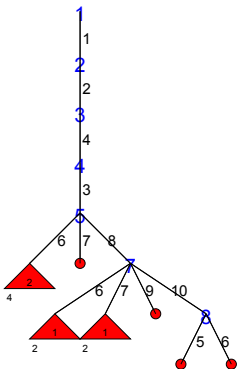


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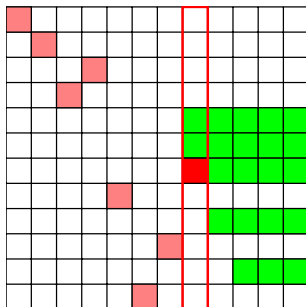
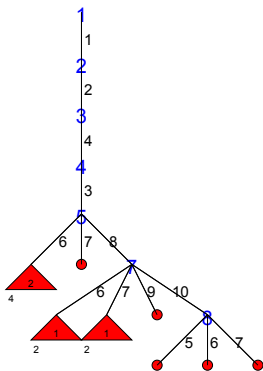
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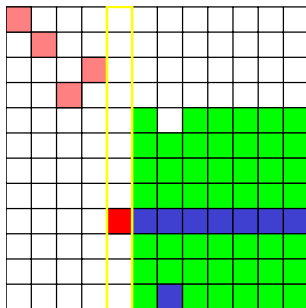
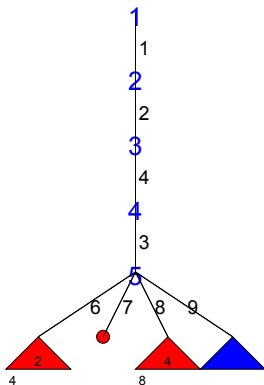
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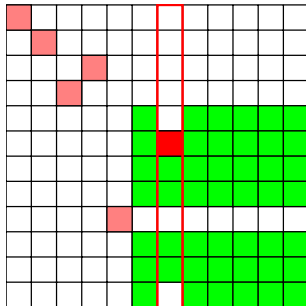
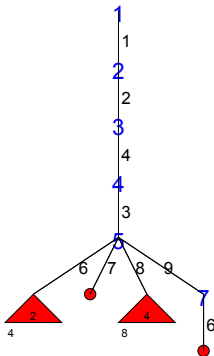


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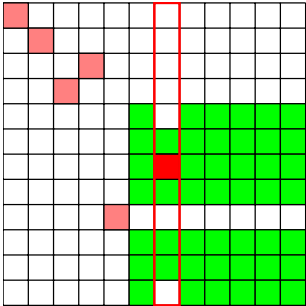
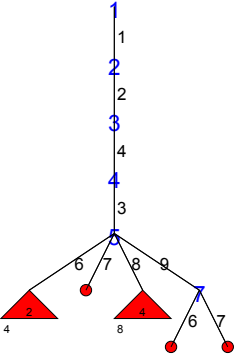
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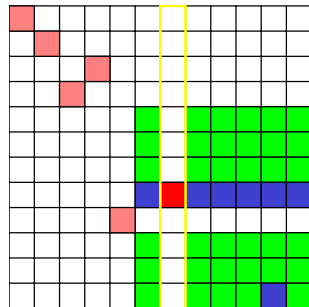
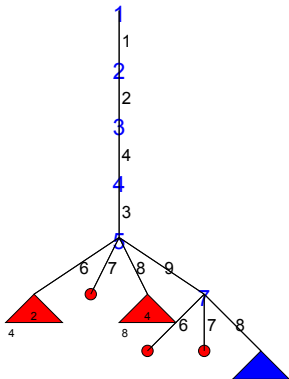
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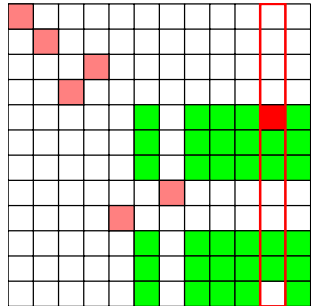
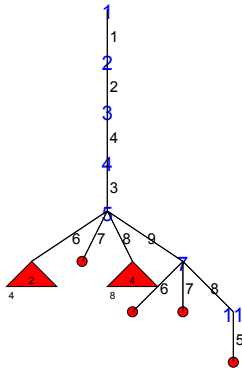


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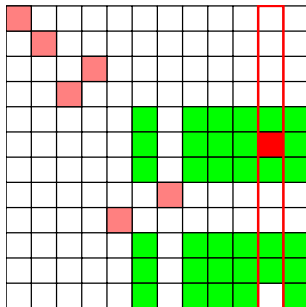
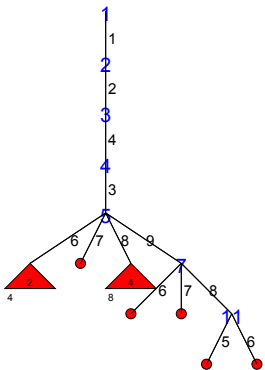
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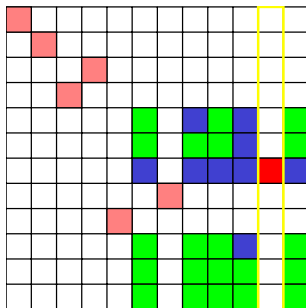
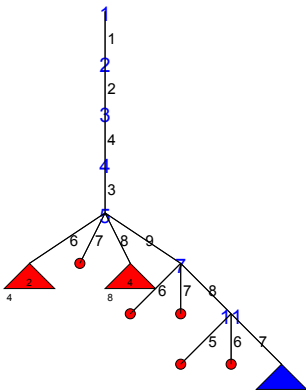
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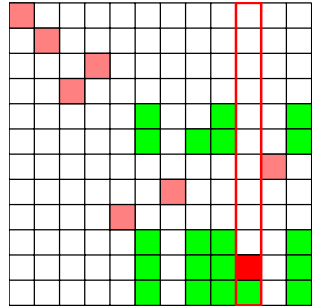
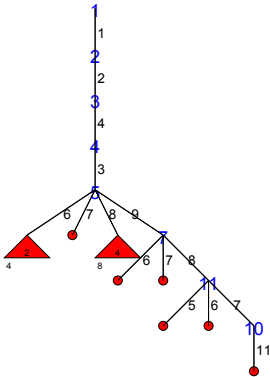
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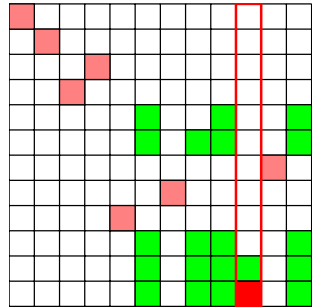
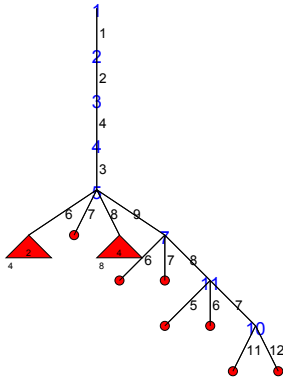
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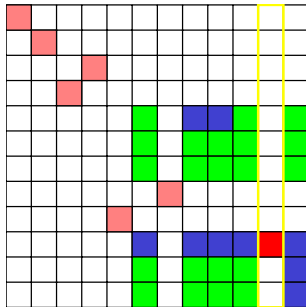
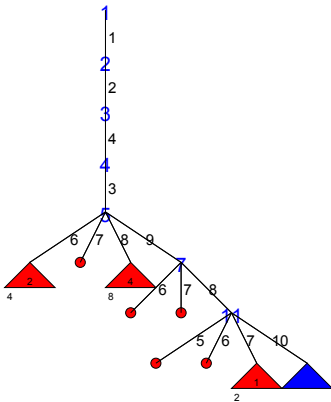
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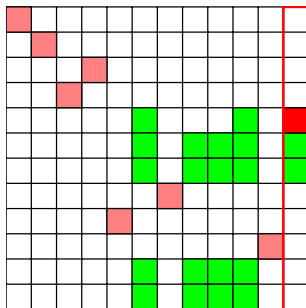
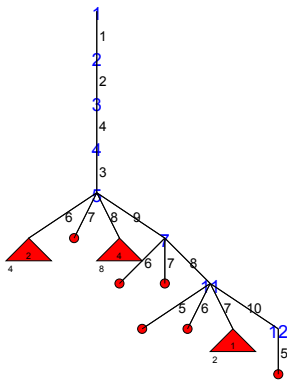
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▶▶ Skip Animation

Basic Model

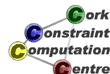


Basic Model

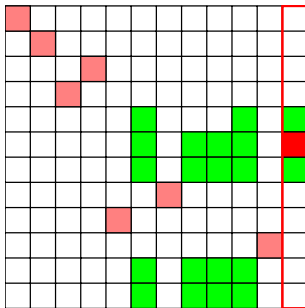
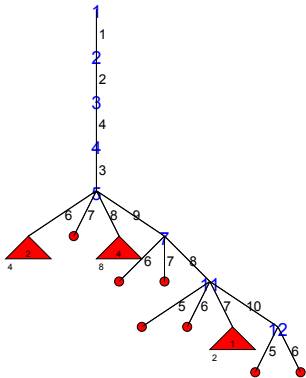


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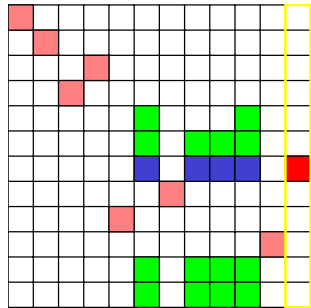
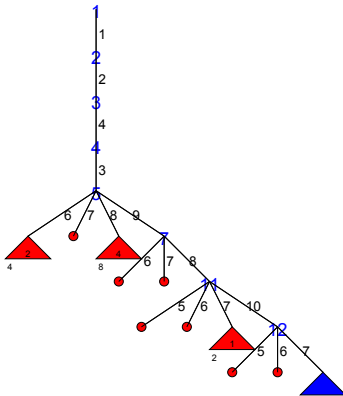
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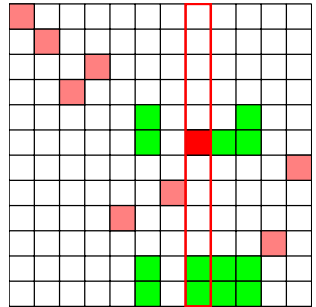
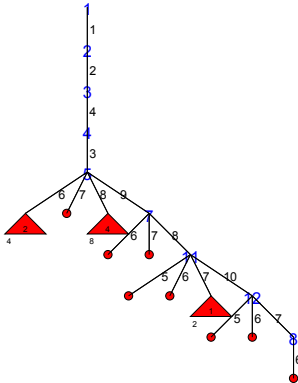
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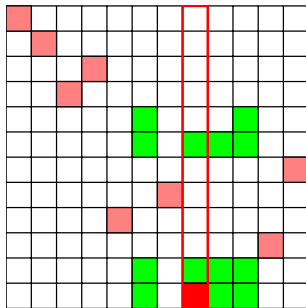
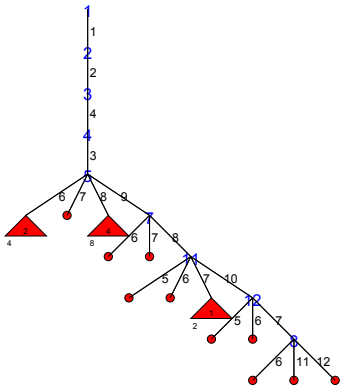
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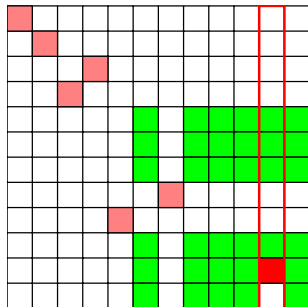
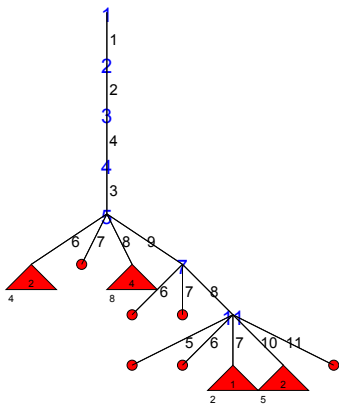
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Basic Model



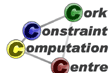
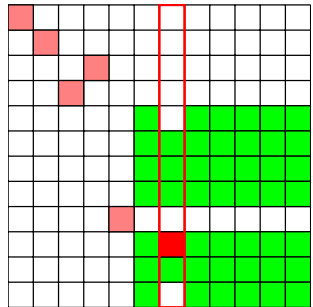
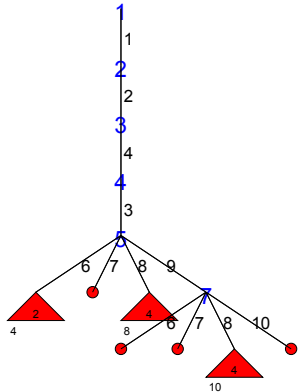
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◀ Back to Start

▶ Skip Animation

Basic Model

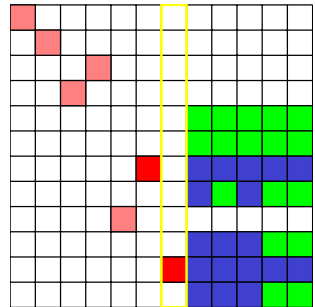
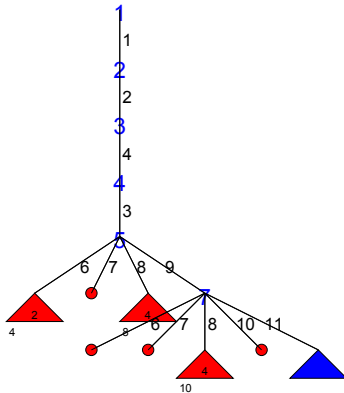


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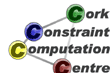
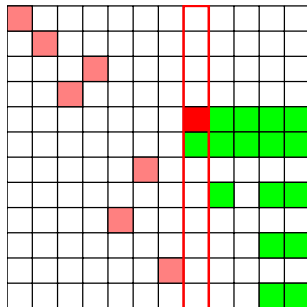
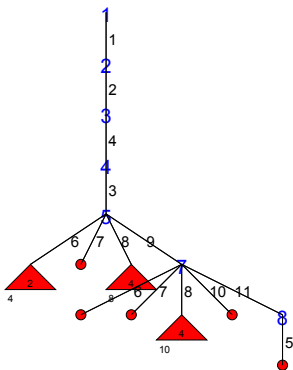
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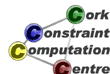
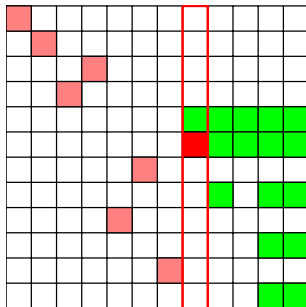
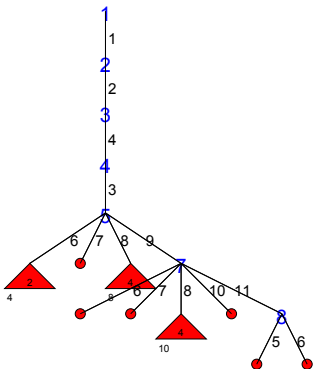
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Basic Model



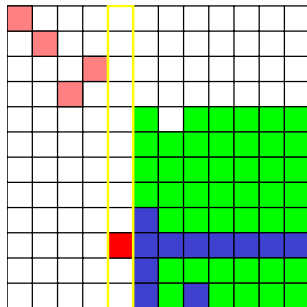
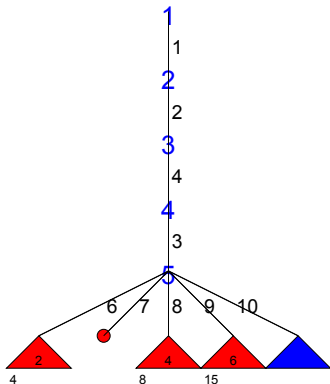
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◀ Back to Start ▶ Skip Animation



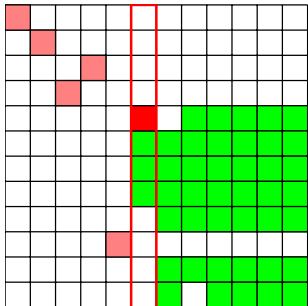
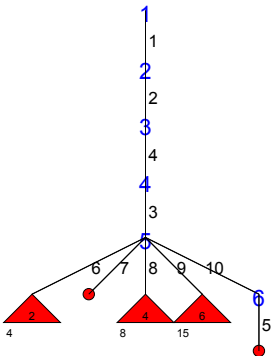
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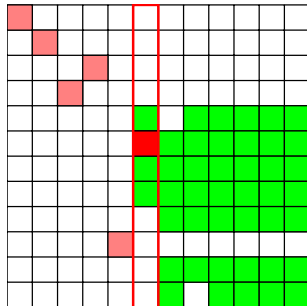
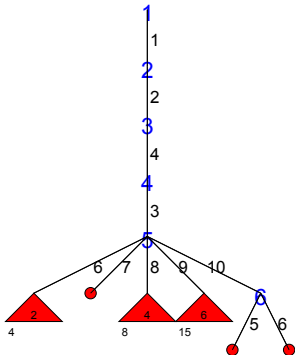
◀ Back to Start

▶▶ Skip Animation

Basic Model

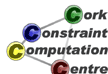


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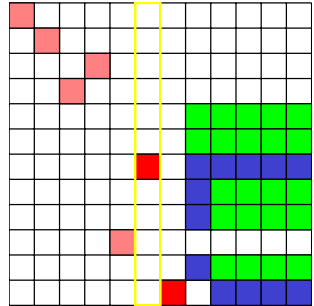
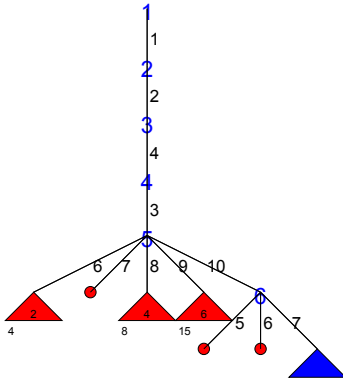


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Basic Model

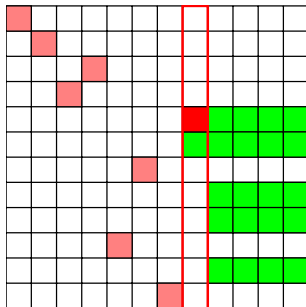
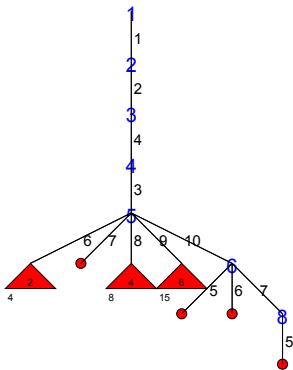


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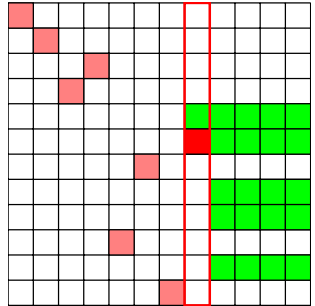
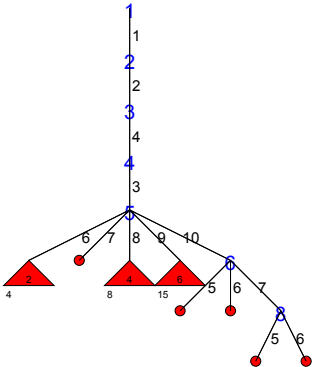
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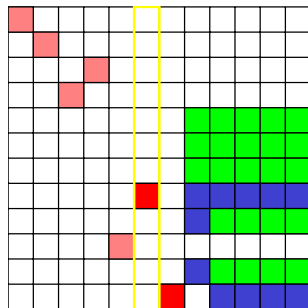
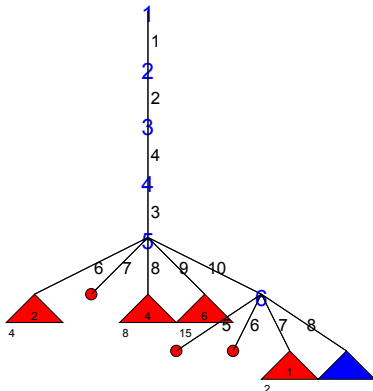
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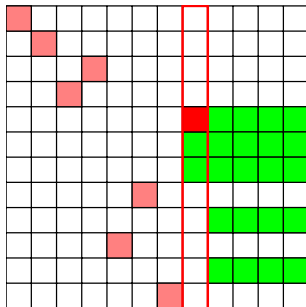
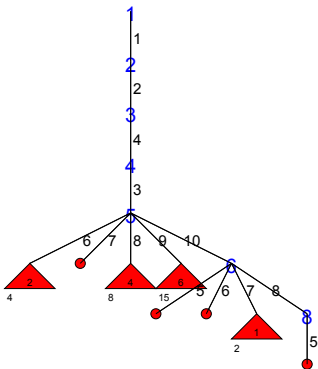
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Basic Model



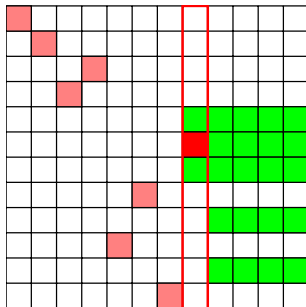
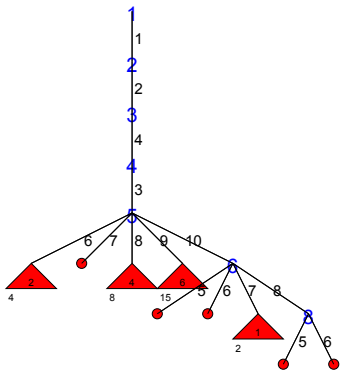
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Basic Model

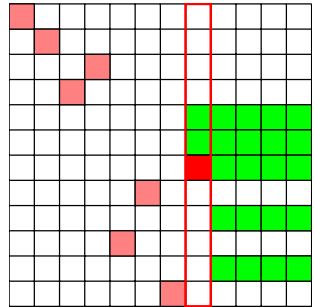
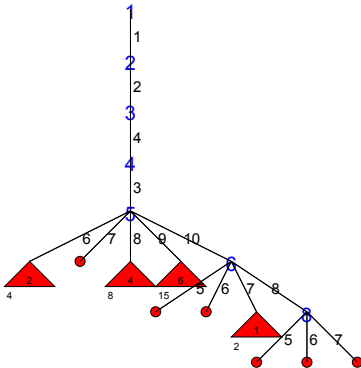


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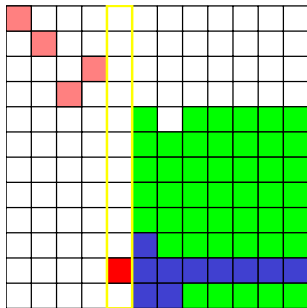
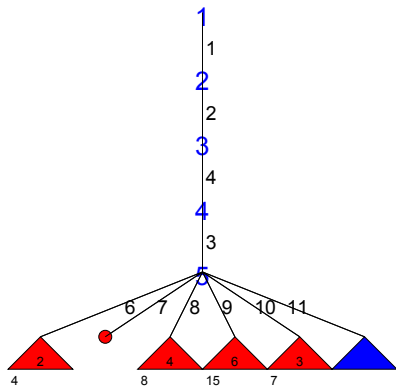
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Basic Model



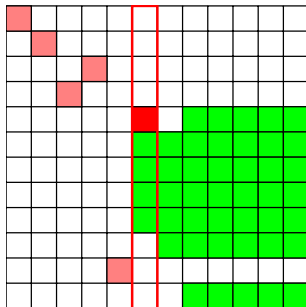
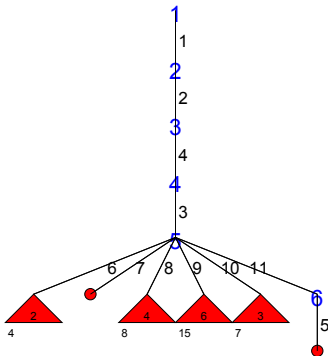
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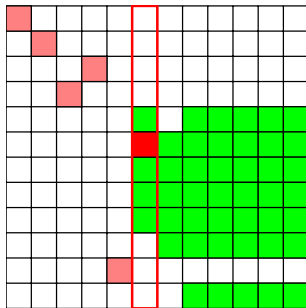
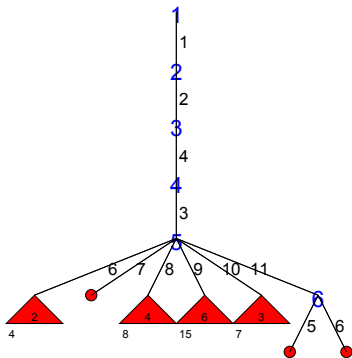
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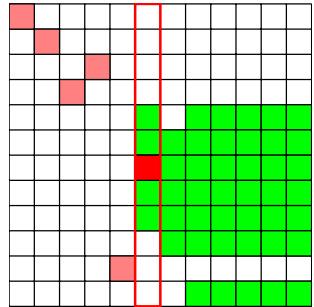
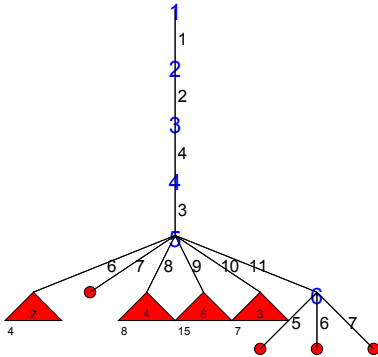
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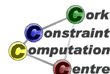
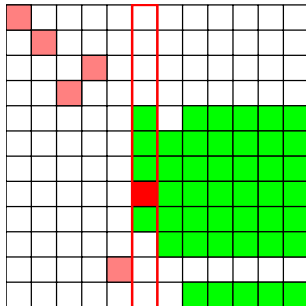
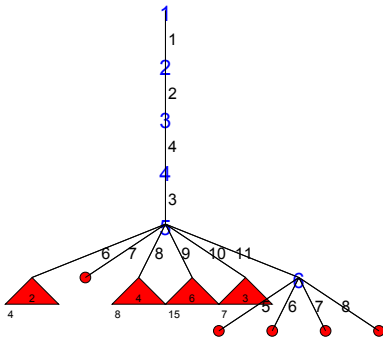
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Basic Model



Basic Model

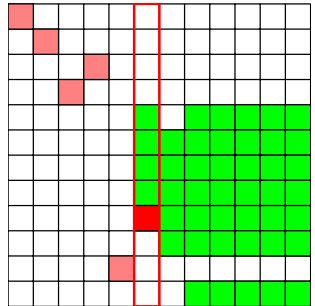
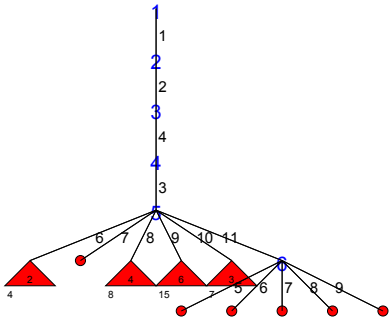


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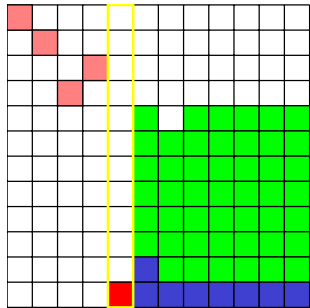
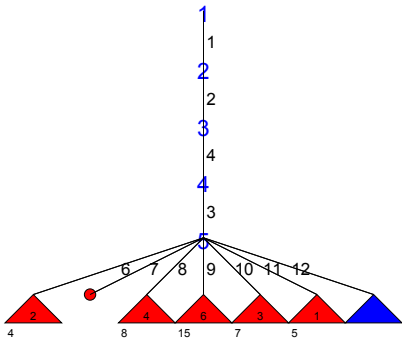
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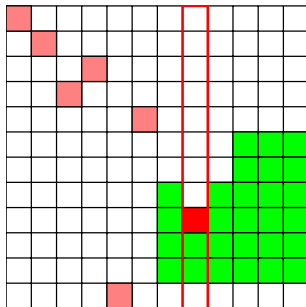
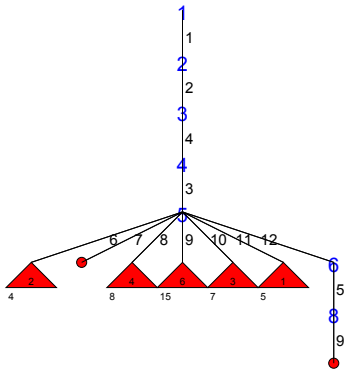
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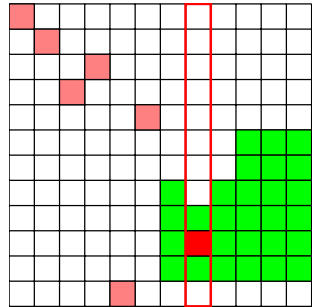
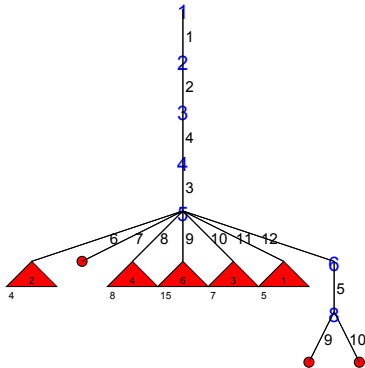
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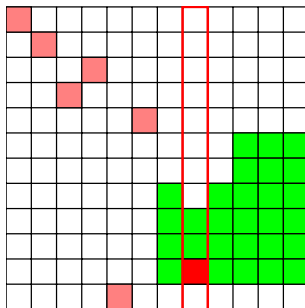
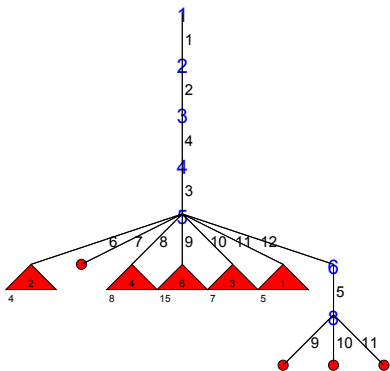
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Basic Model

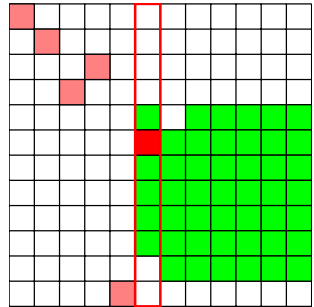
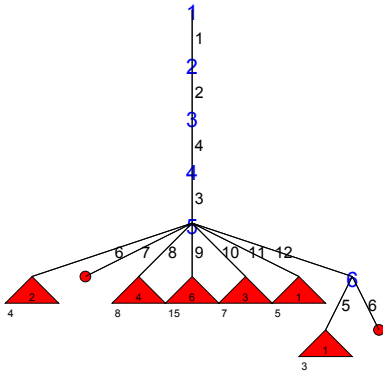


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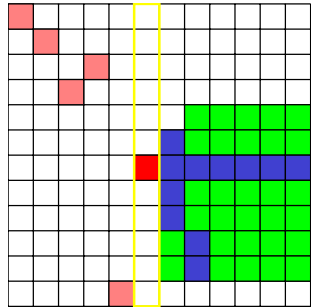
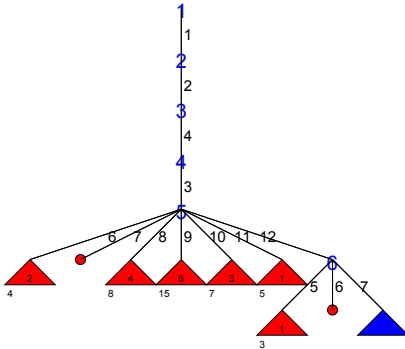
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Basic Model



Basic Model

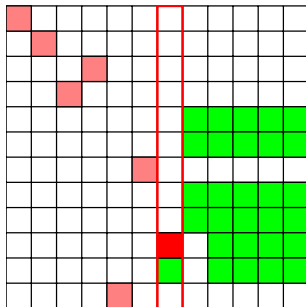
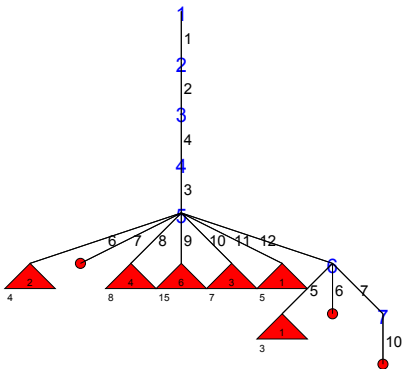


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Basic Model

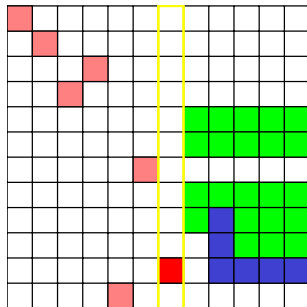
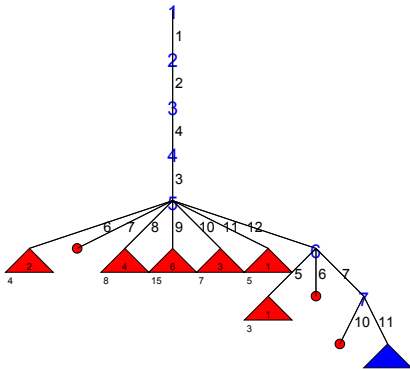


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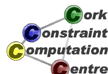
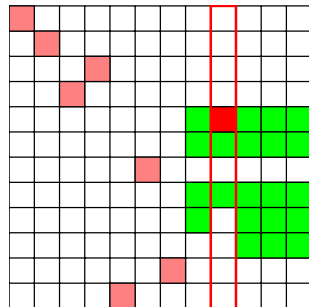
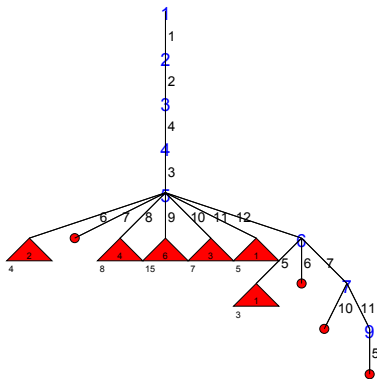
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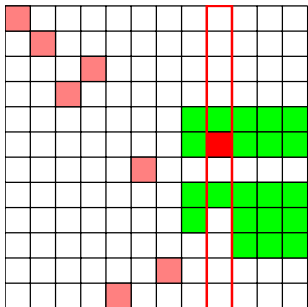
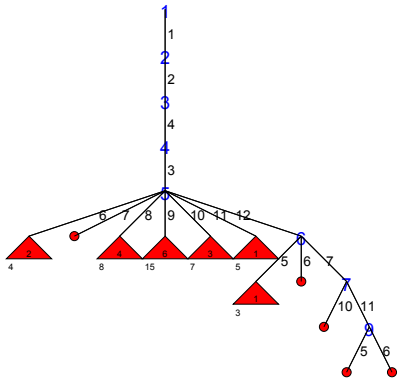
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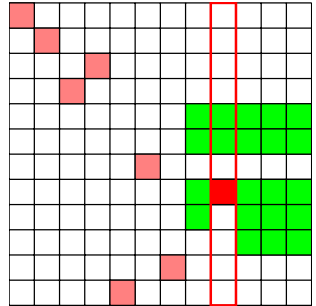
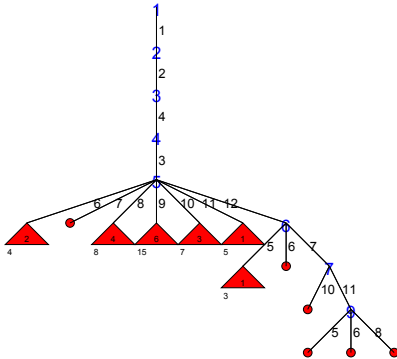
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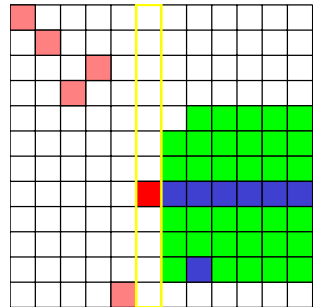
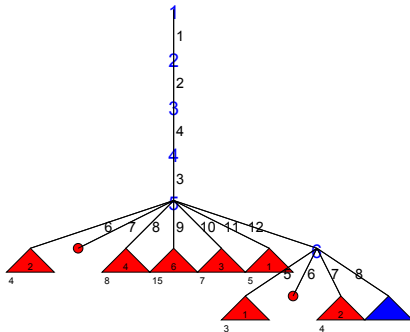
◀ Back to Start

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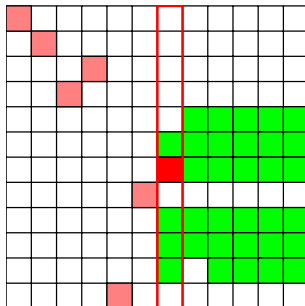
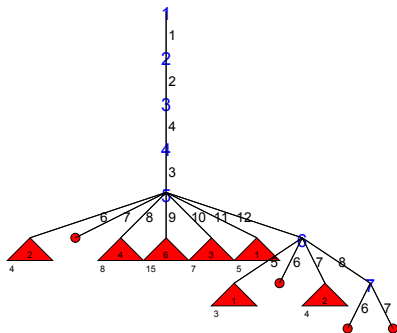
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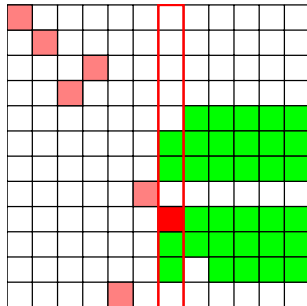
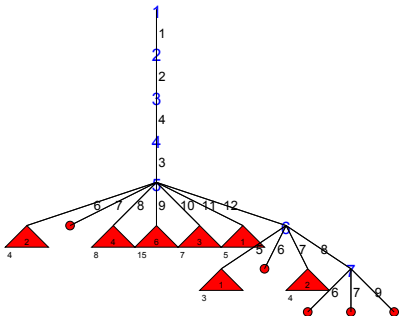
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▶▶ Skip Animation

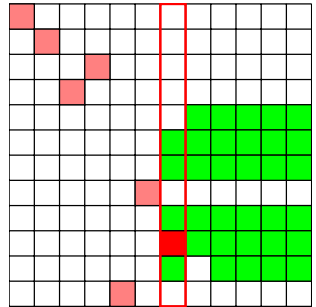
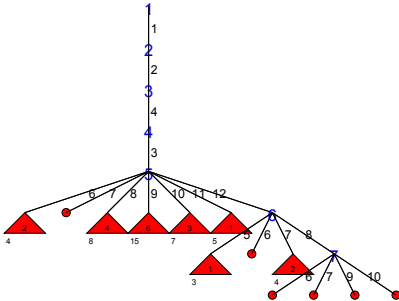
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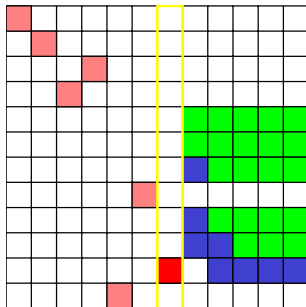
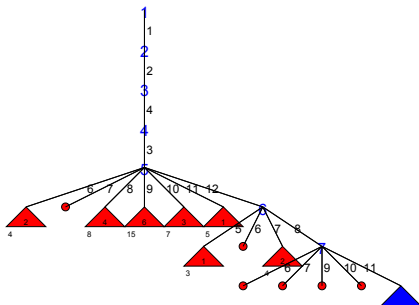
◀ Back to Start

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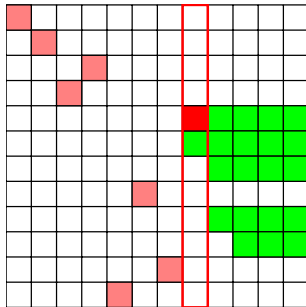
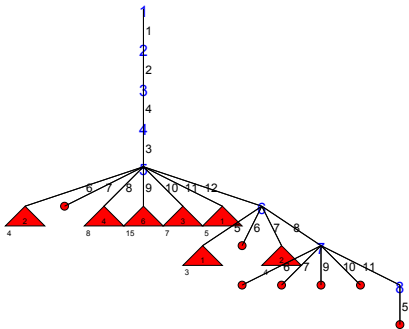
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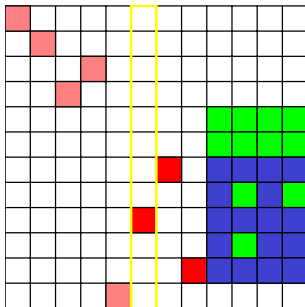
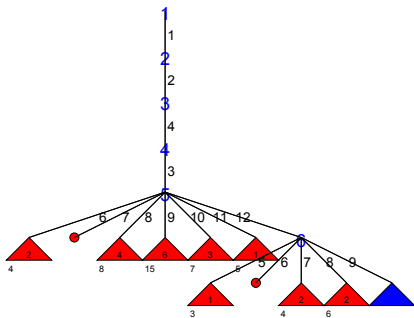
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Basic Model



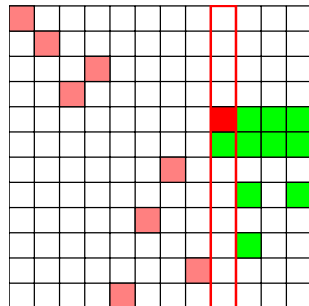
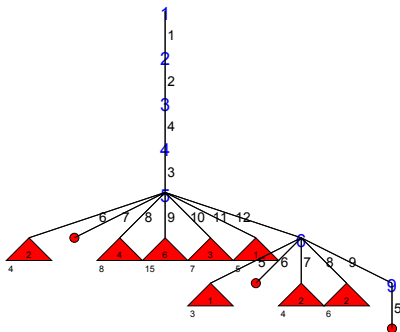
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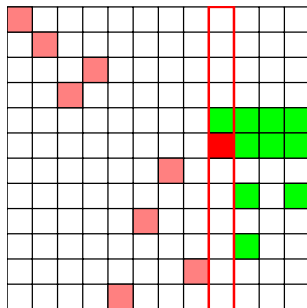
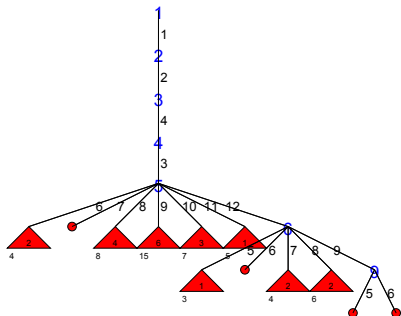
◀ Back to Start

▶▶ Skip Animation

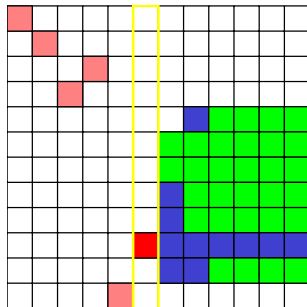
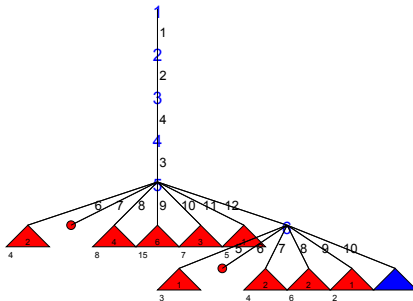
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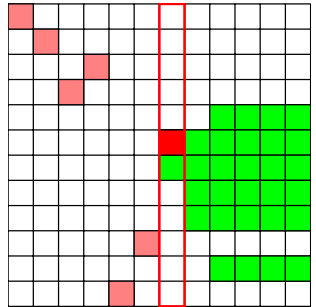
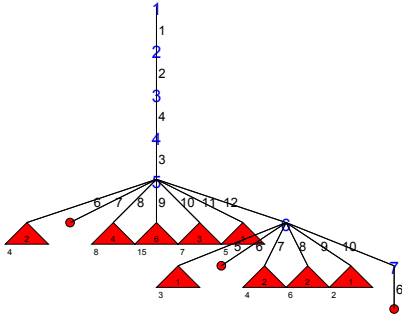
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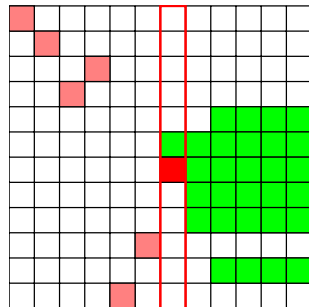
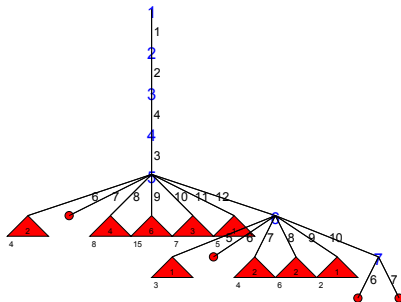
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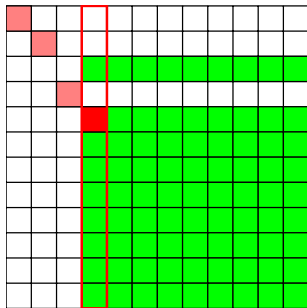
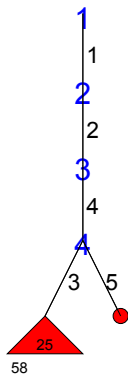
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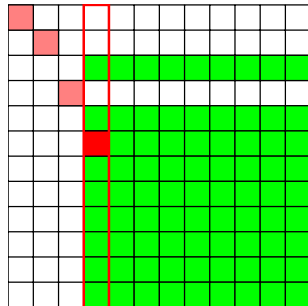
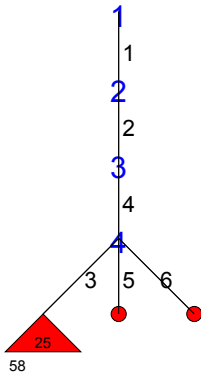
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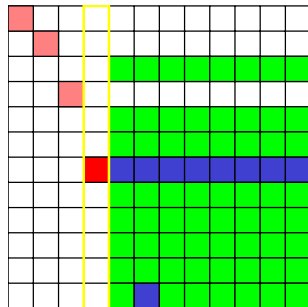
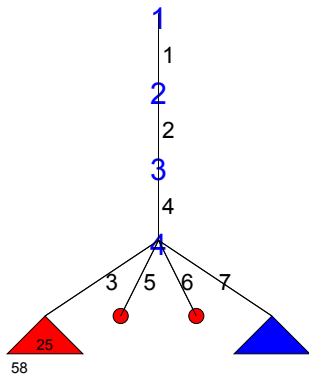
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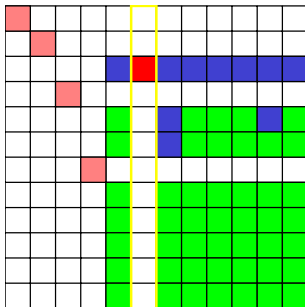
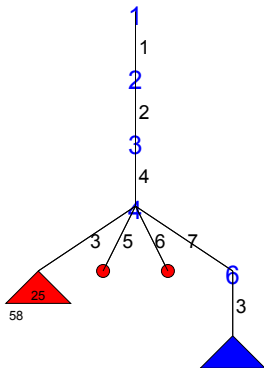
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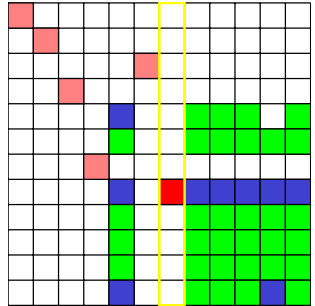
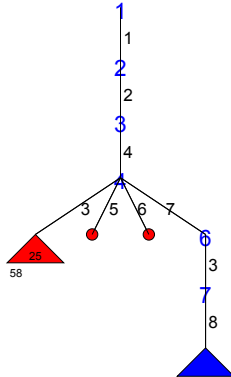
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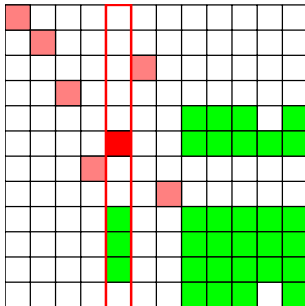
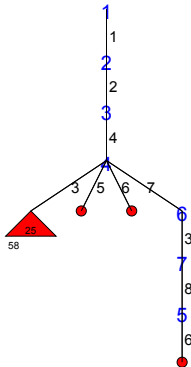
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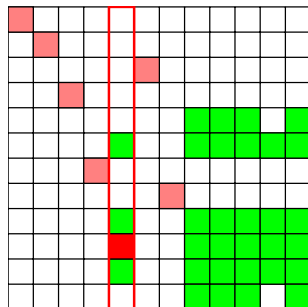
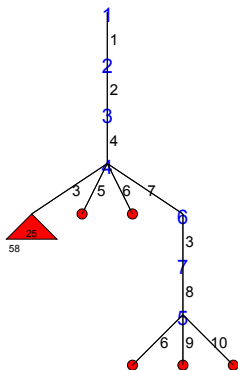
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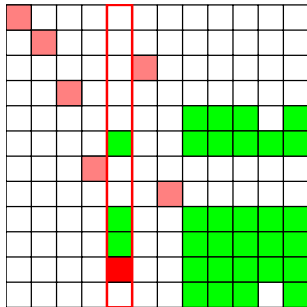
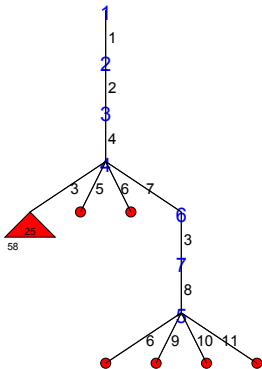
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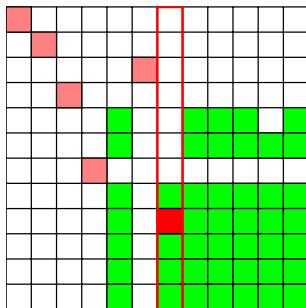
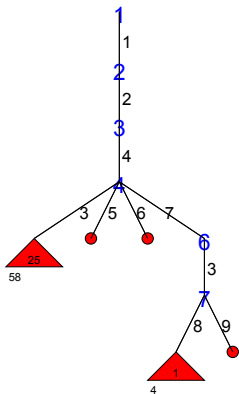
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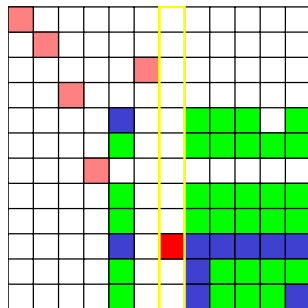
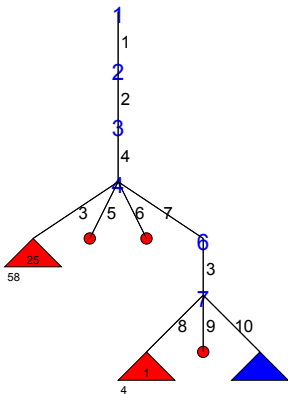
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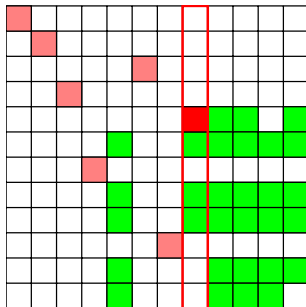
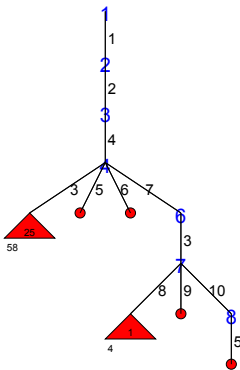
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Basic Model



Basic Model

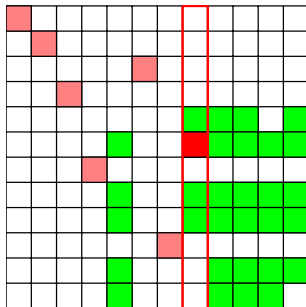
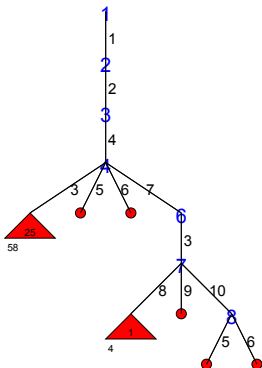


◀ Back to Start

▶▶ Skip Animation



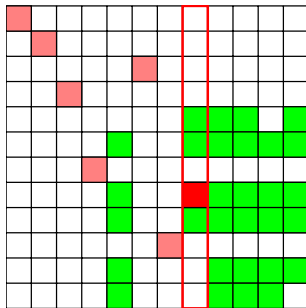
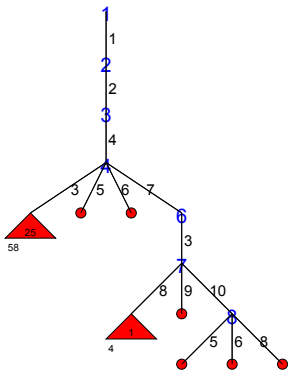
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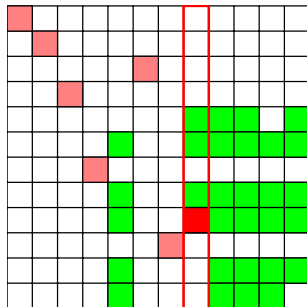
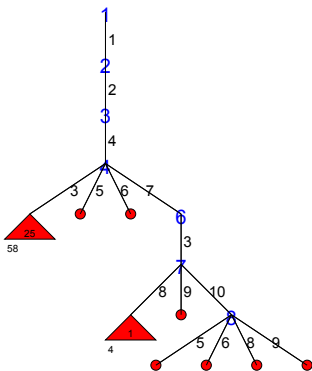
◀ Back to Start ▶ Skip Animation



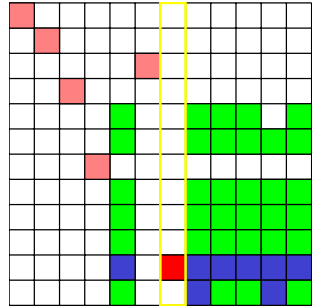
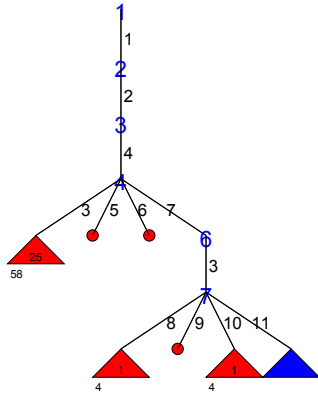
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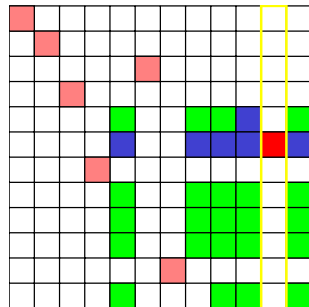
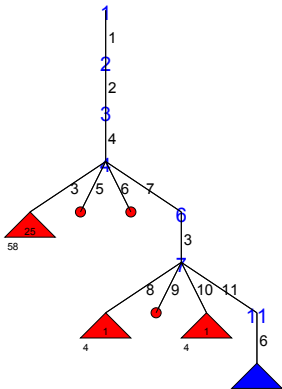
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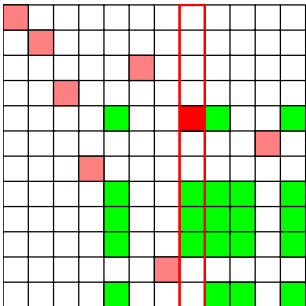
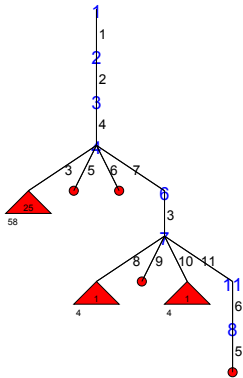
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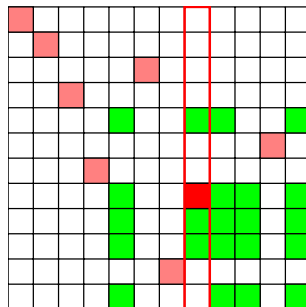
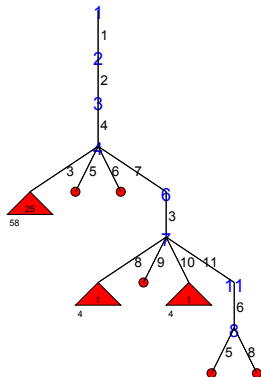
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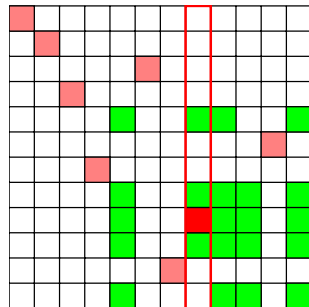
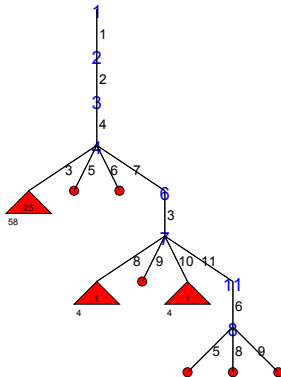
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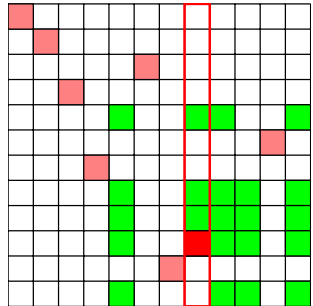
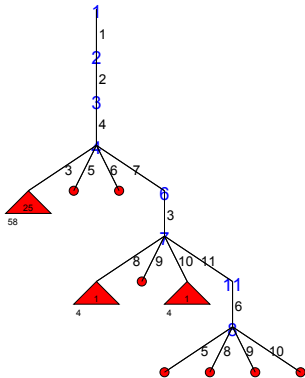
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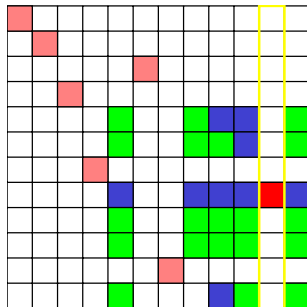
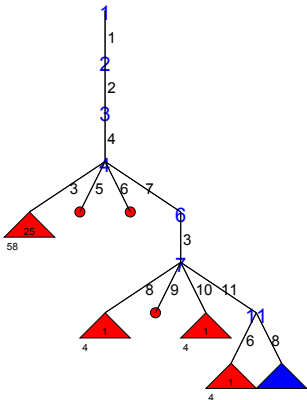
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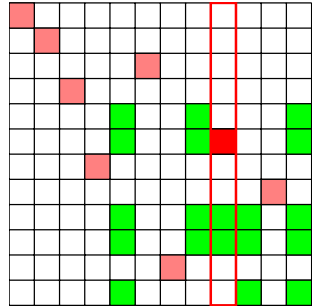
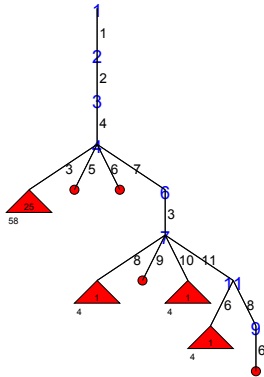
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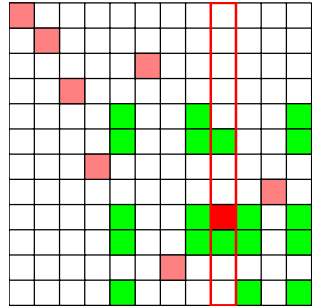
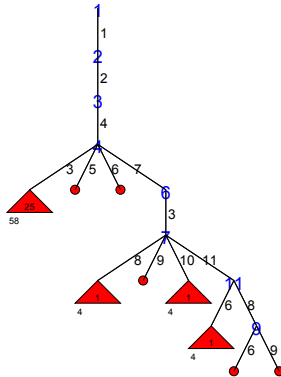
◀ Back to Start

▶▶ Skip Animation

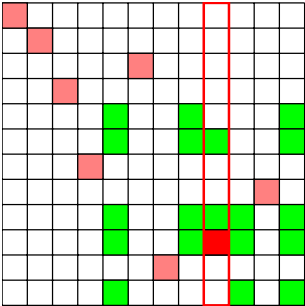
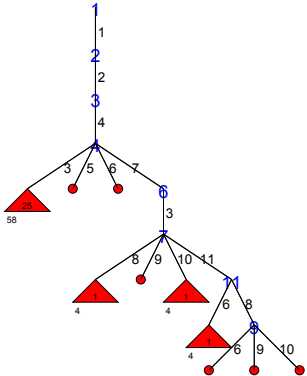
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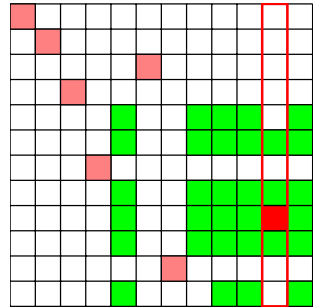
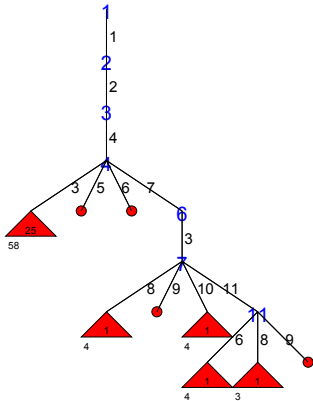
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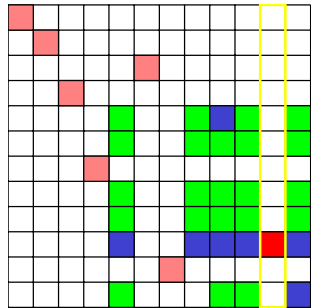
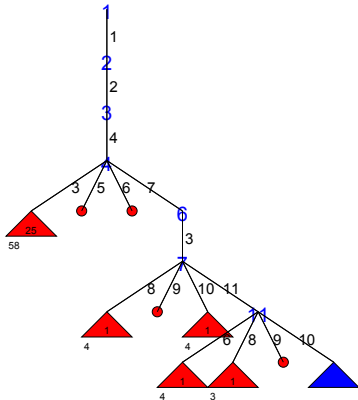
Basic Model



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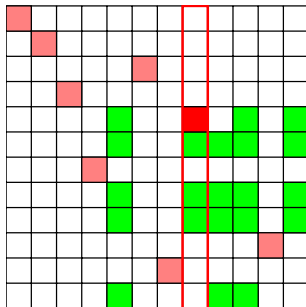
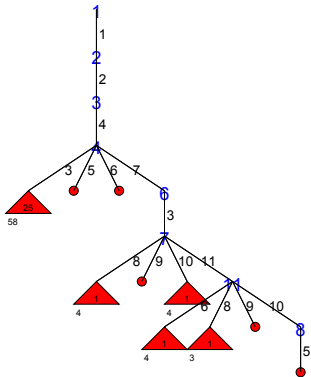
Basic Model



◀ Back to Start

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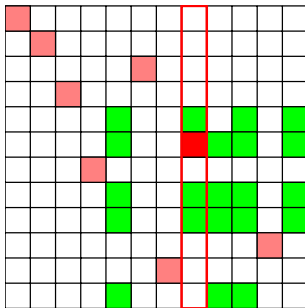
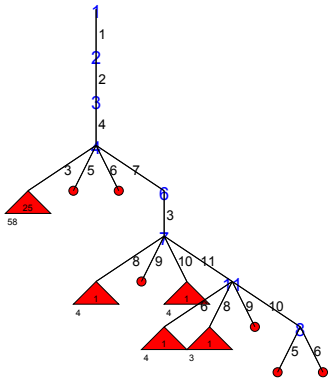
Basic Model



◀ Back to Start

▶ Skip Animation

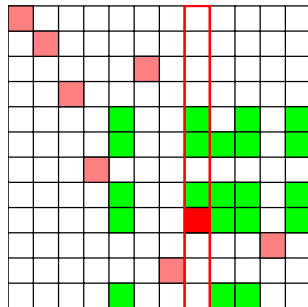
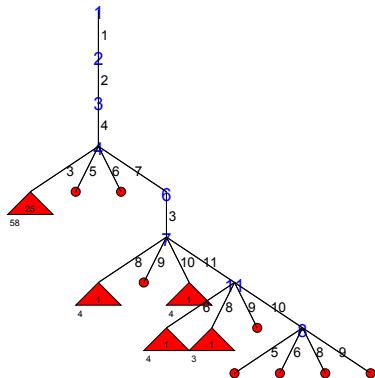
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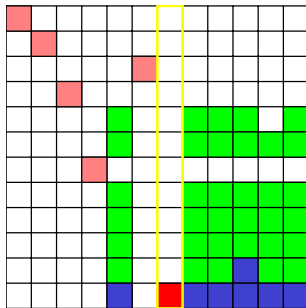
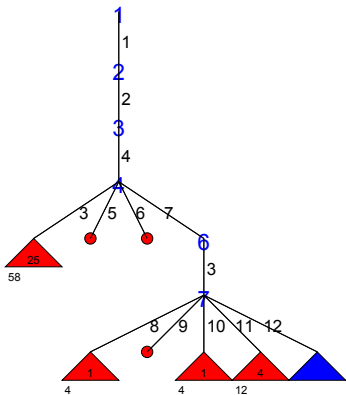
◀ Back to Start

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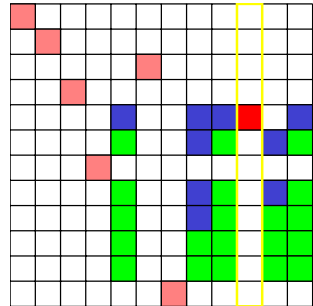
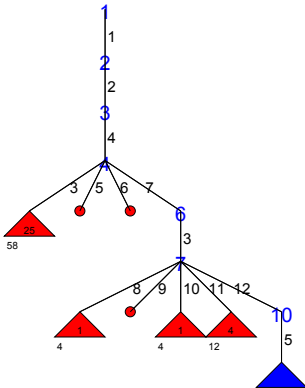
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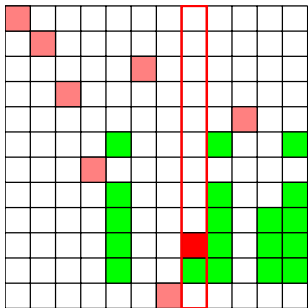
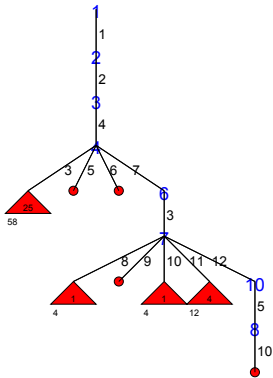
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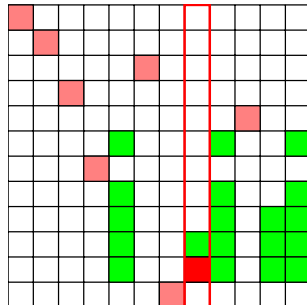
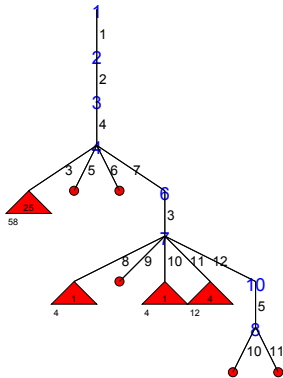
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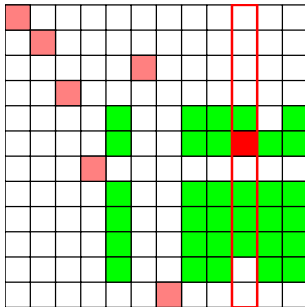
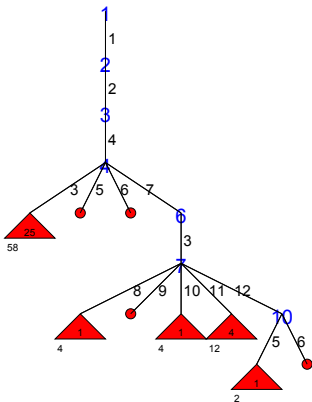
◀ Back to Start

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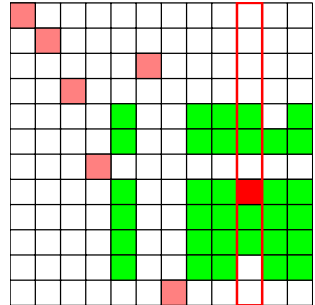
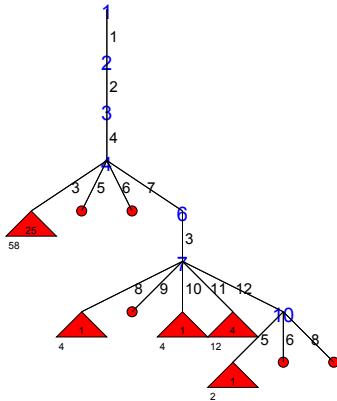
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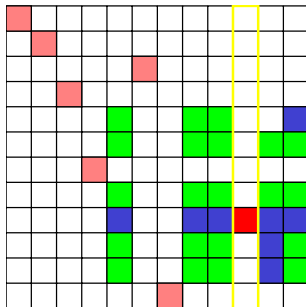
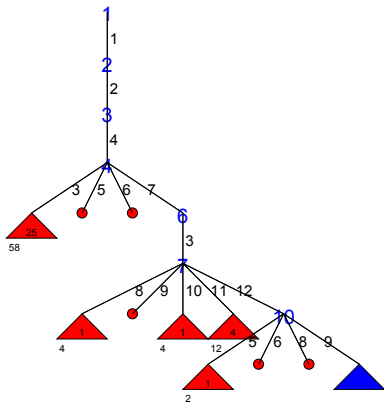
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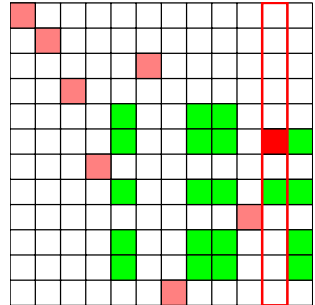
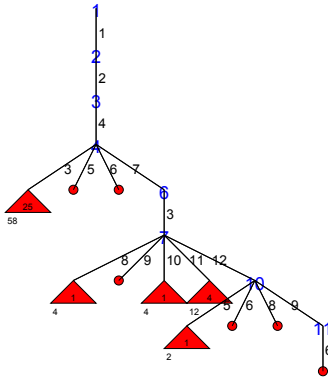
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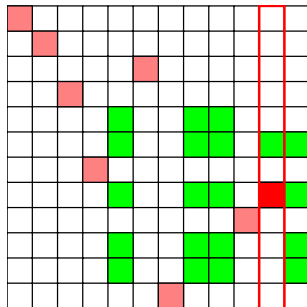
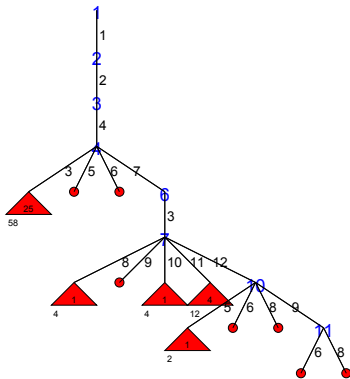
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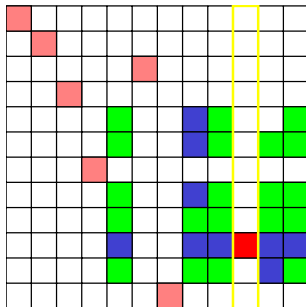
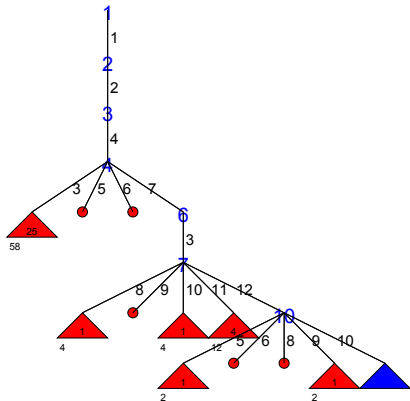
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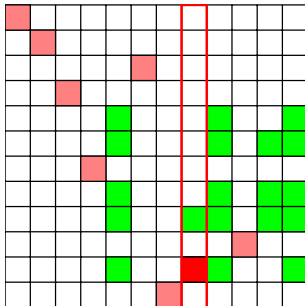
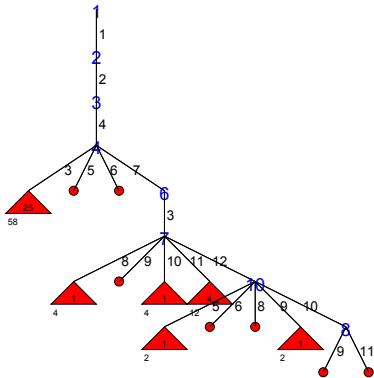
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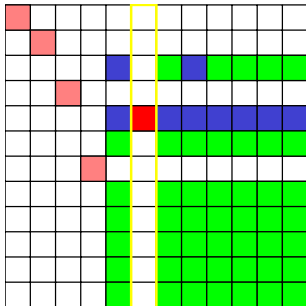
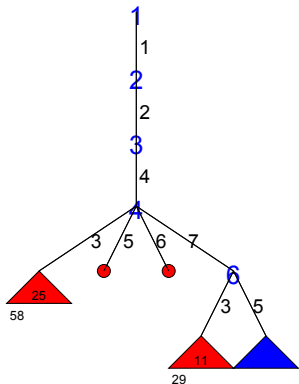
◀ Back to Start

▶▶ Skip Animation

Basic Model



Basic Model

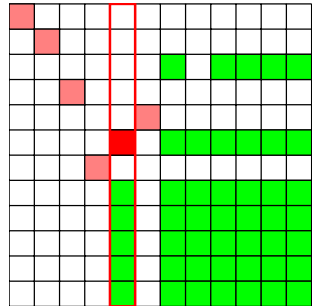
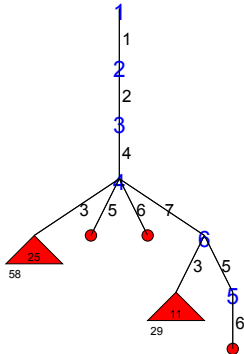


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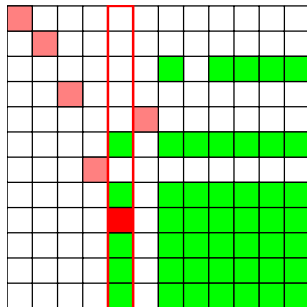
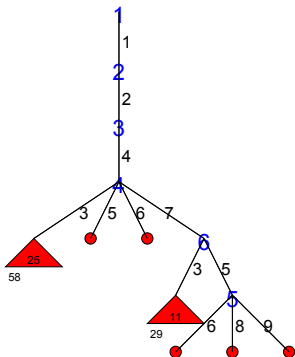
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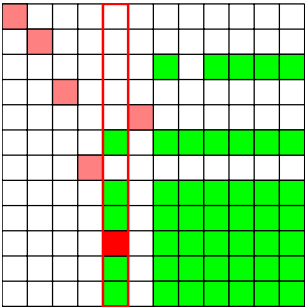
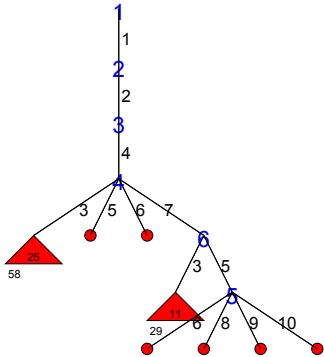
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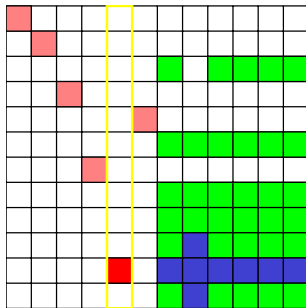
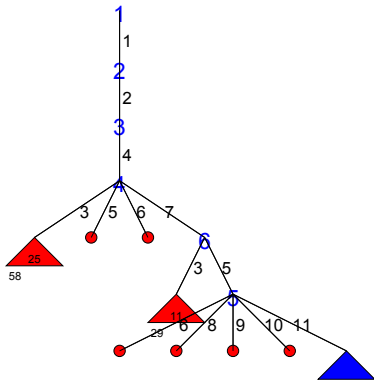
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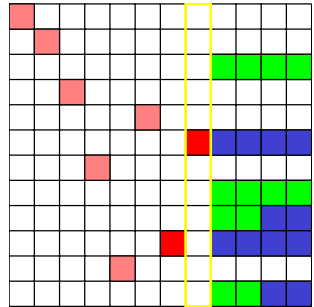
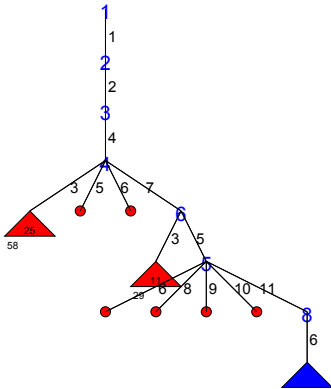
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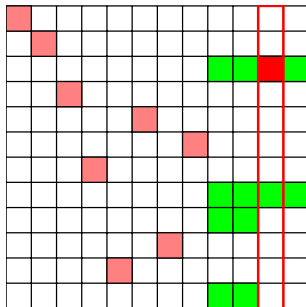
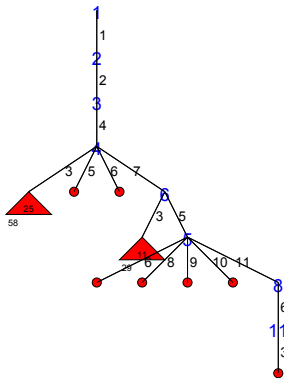
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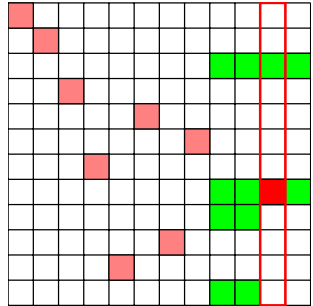
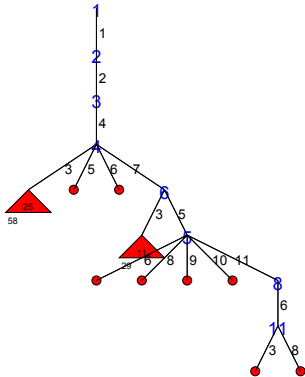
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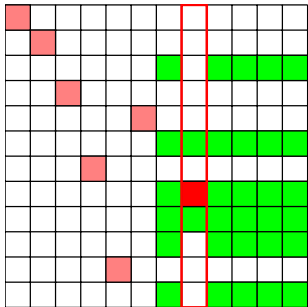
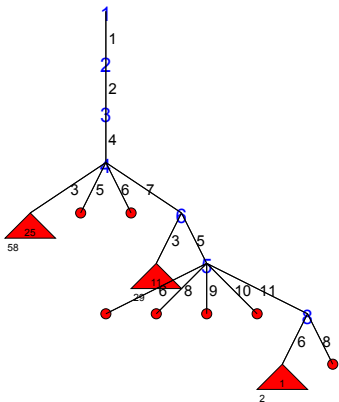
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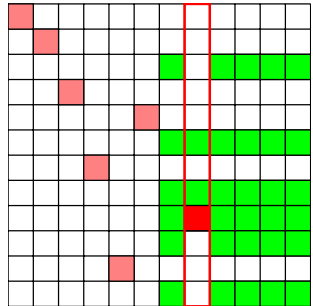
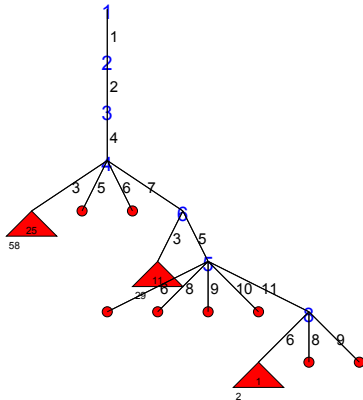
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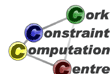
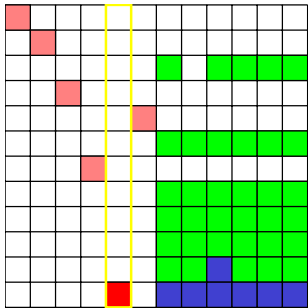
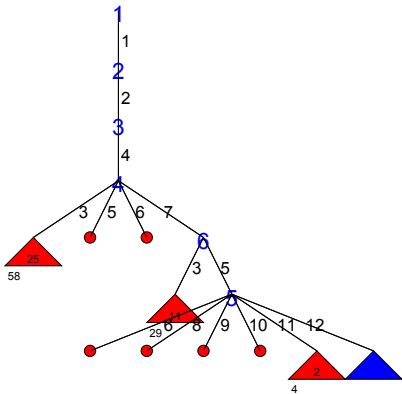
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Basic Model



Basic Model

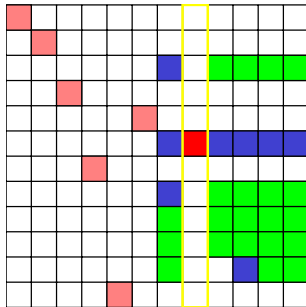
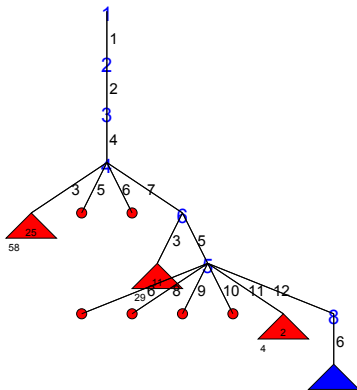


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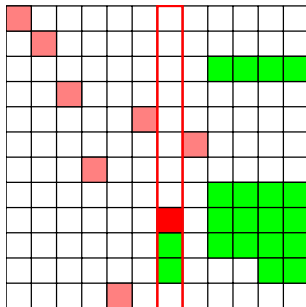
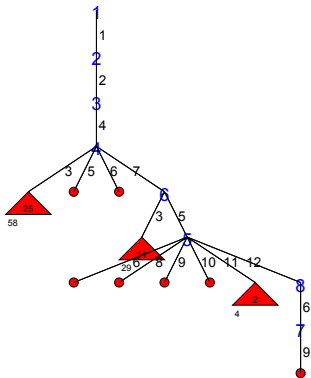
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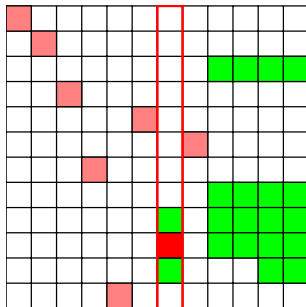
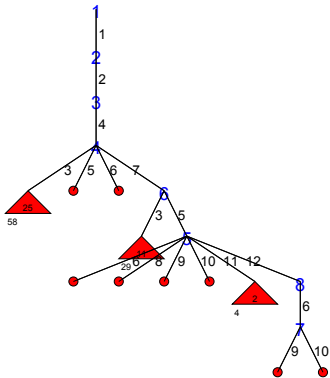
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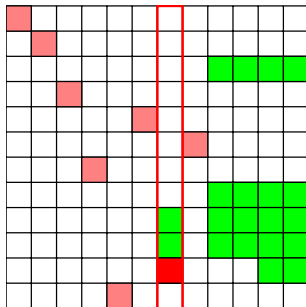
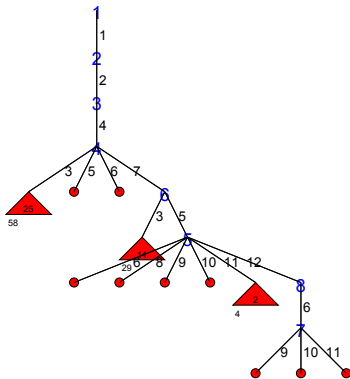
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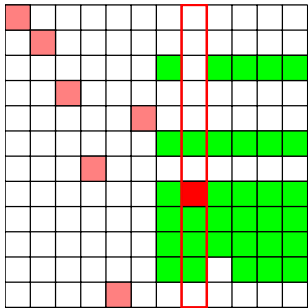
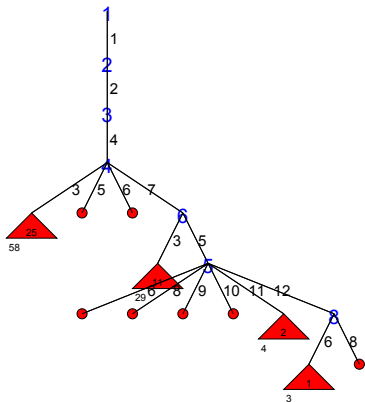
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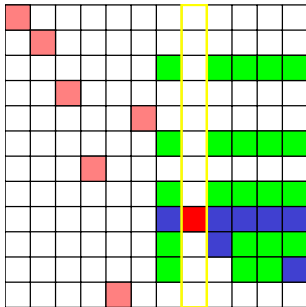
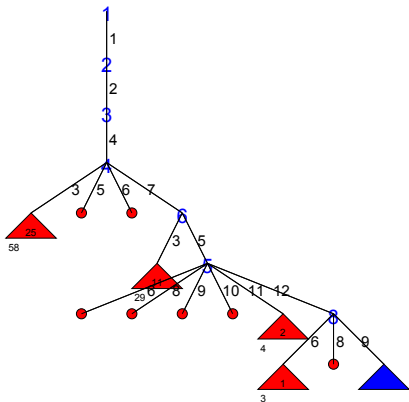
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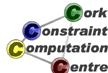


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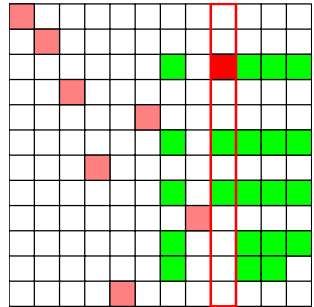
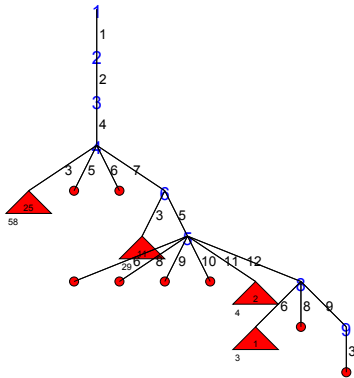


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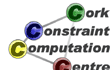
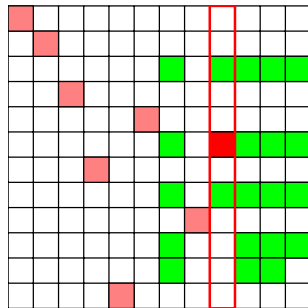
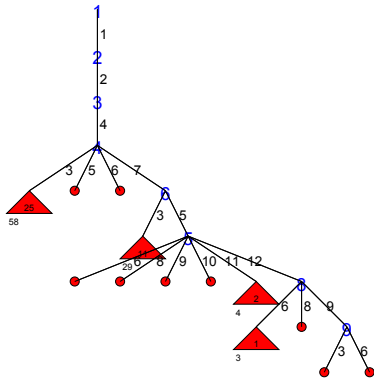
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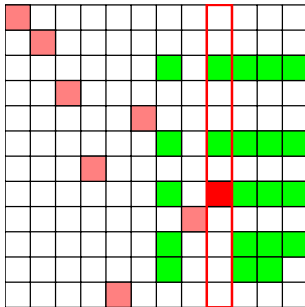
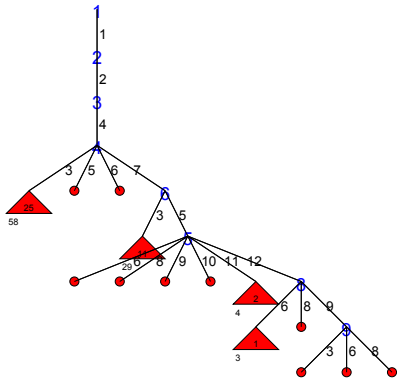
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Basic Model



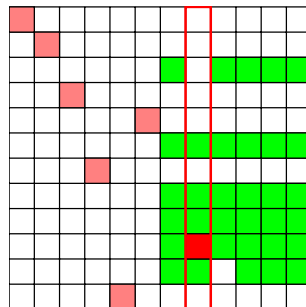
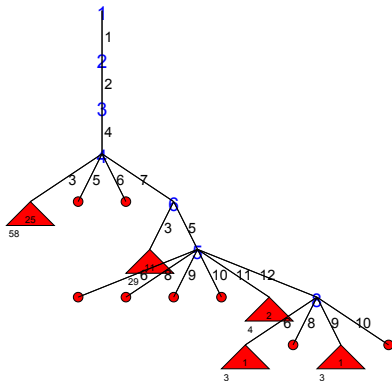
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Basic Model

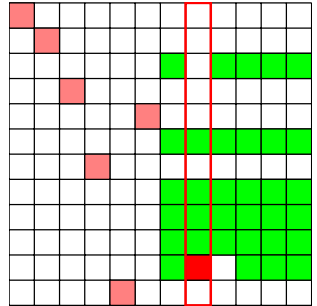
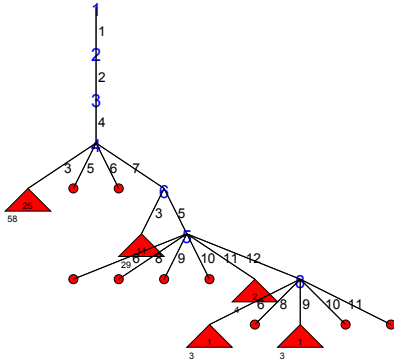


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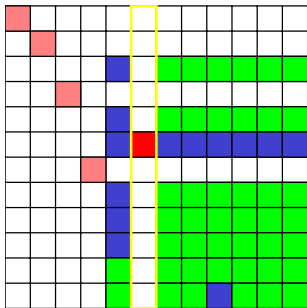
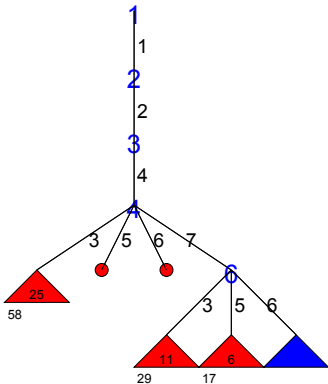
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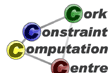
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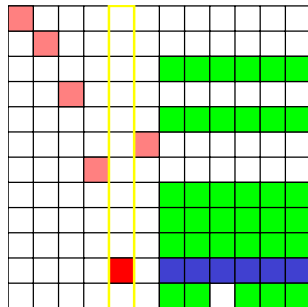
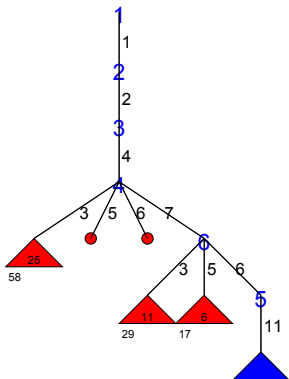


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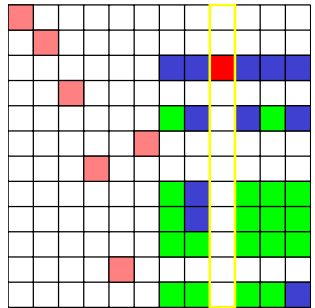
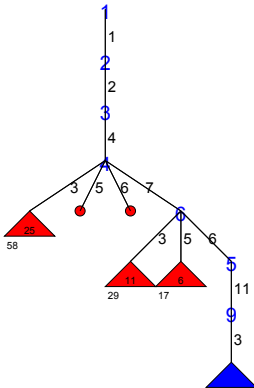
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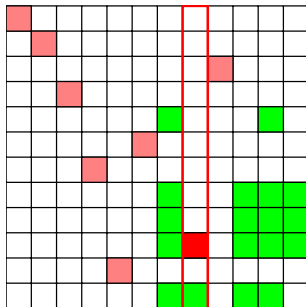
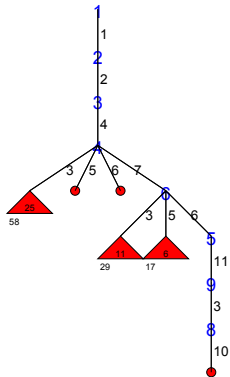
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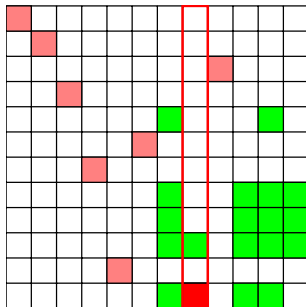
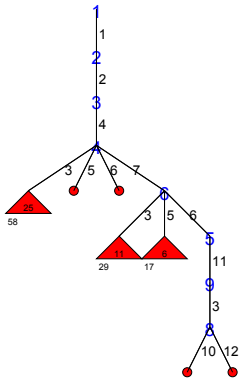
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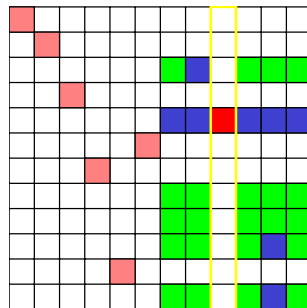
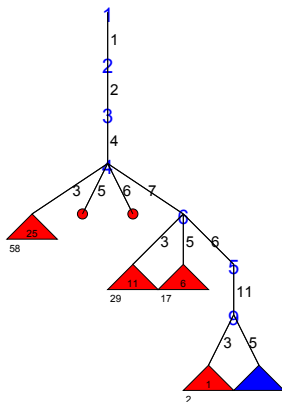
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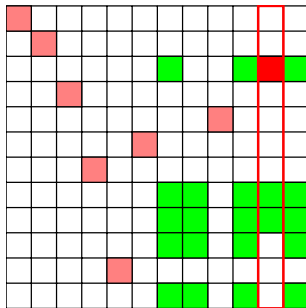
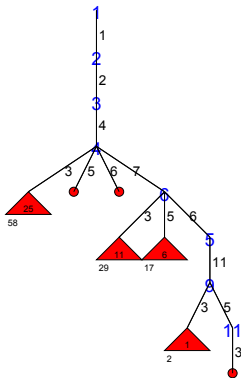
◀ Back to Start

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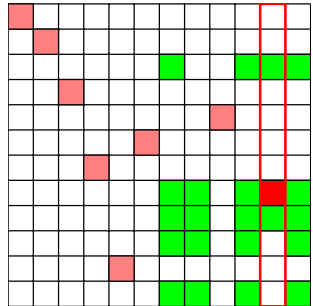
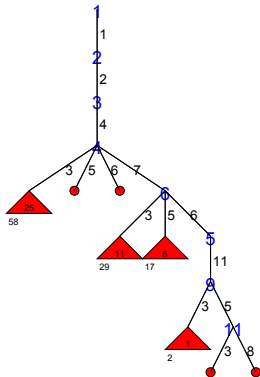
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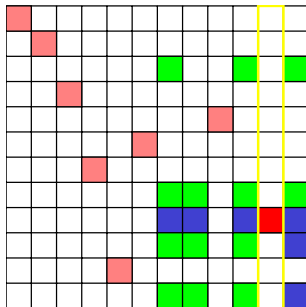
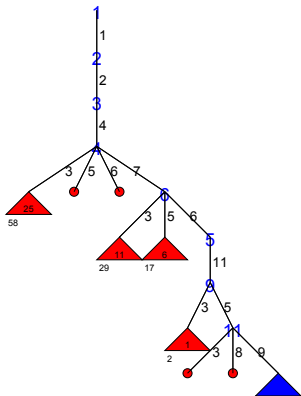
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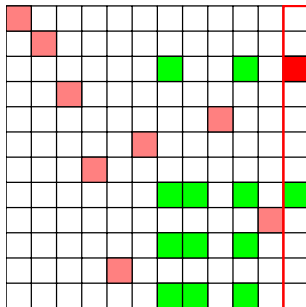
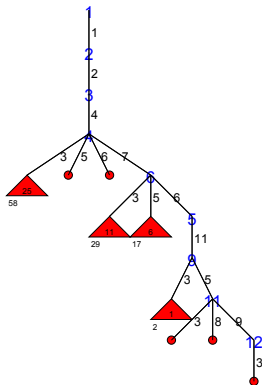
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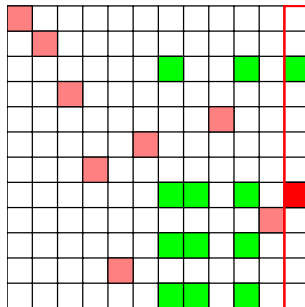
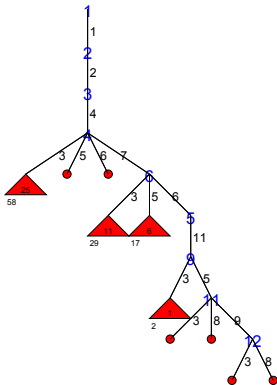


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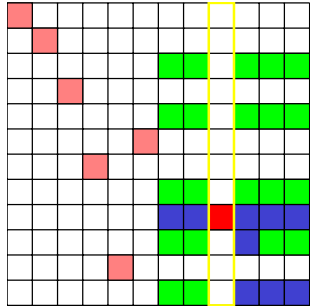
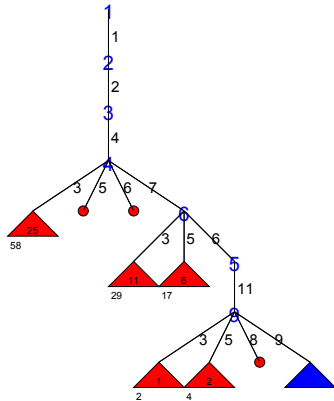
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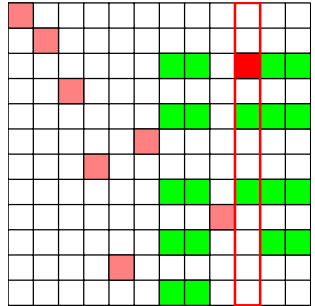
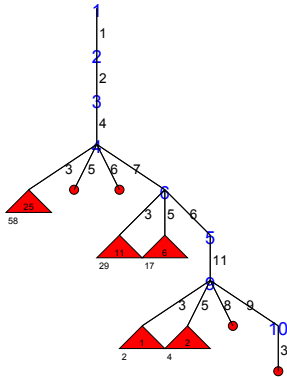
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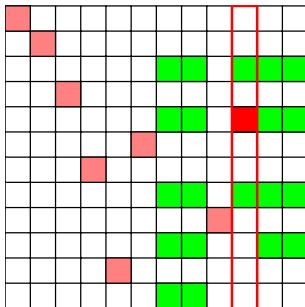
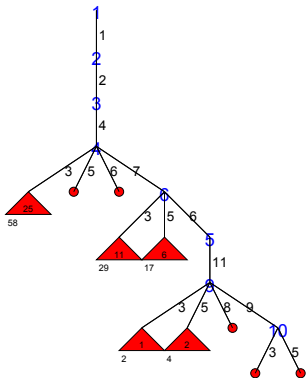
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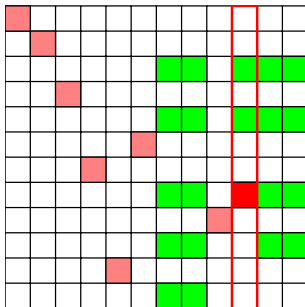
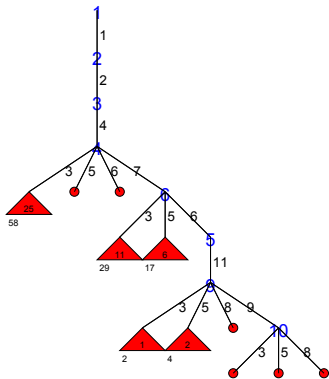
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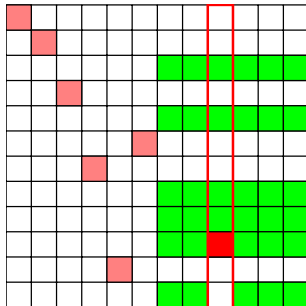
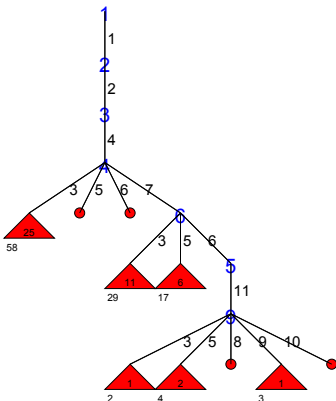
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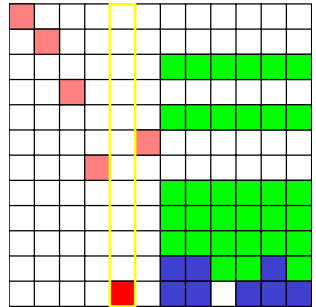
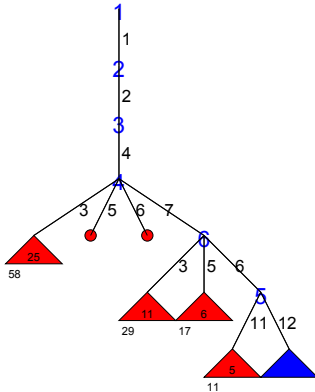
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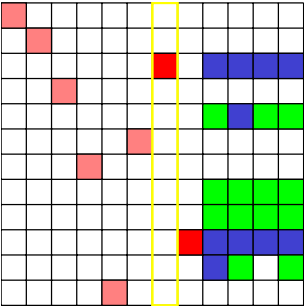
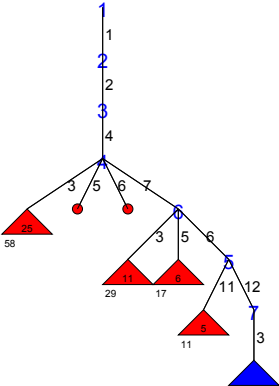
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Basic Model



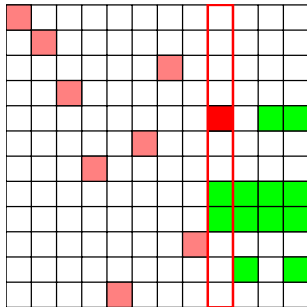
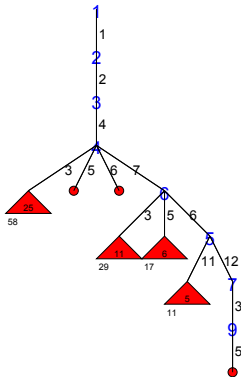
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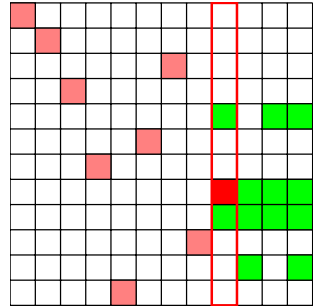
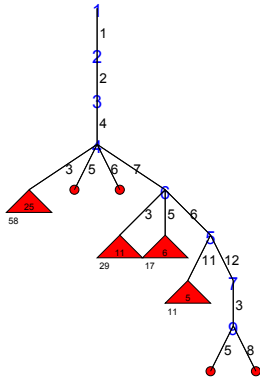
◀ Back to Start

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Basic Model



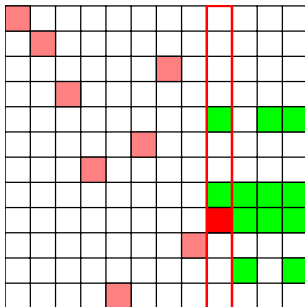
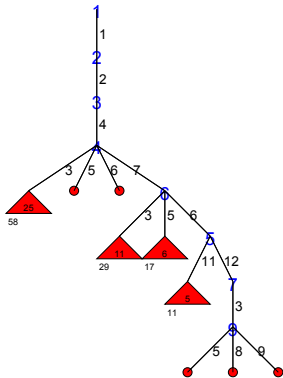
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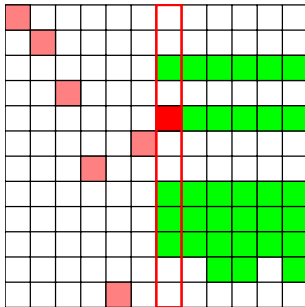
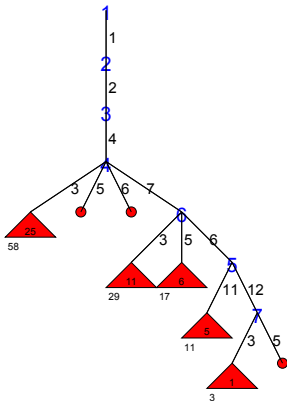
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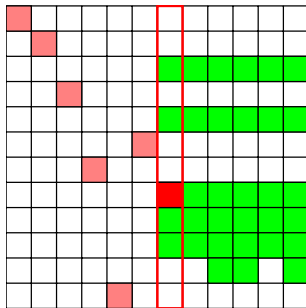
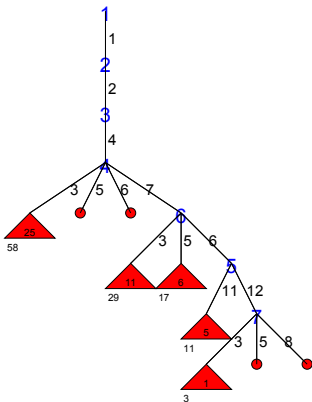
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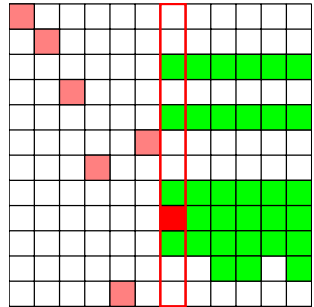
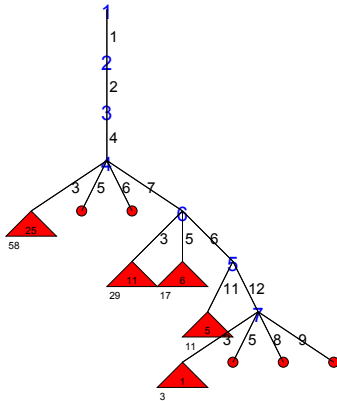
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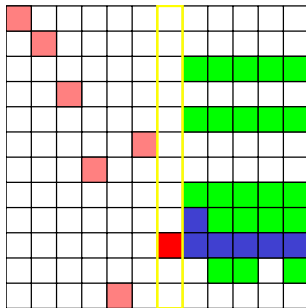
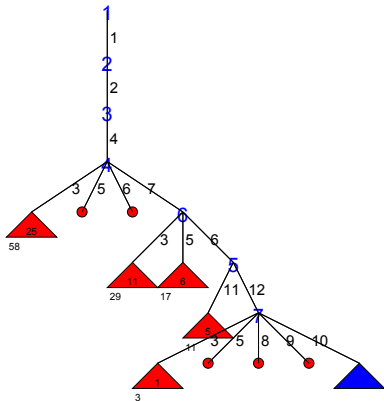
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Basic Model



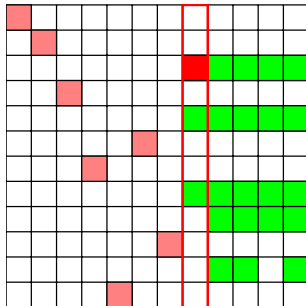
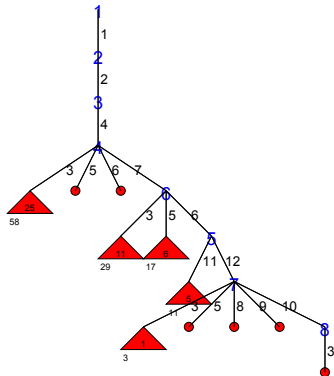
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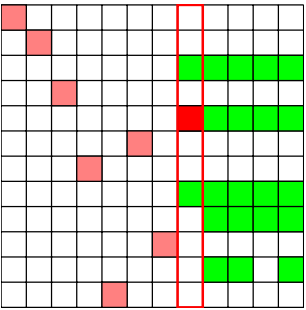
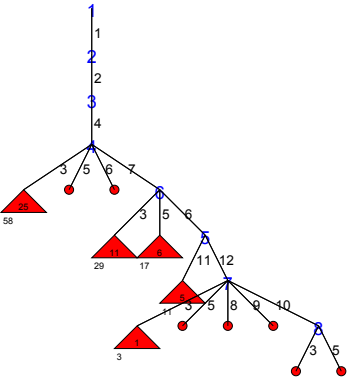
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Basic Model

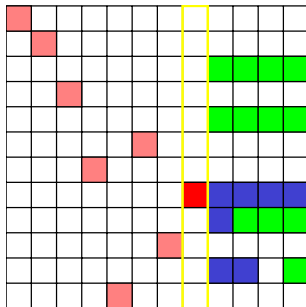
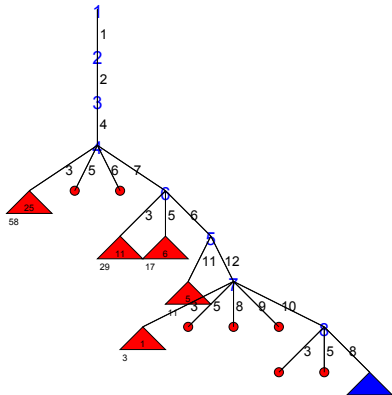


Basic Model



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Basic Model

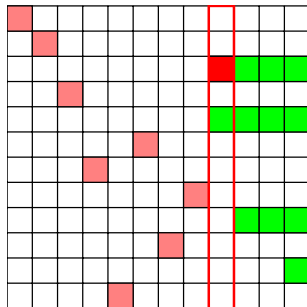
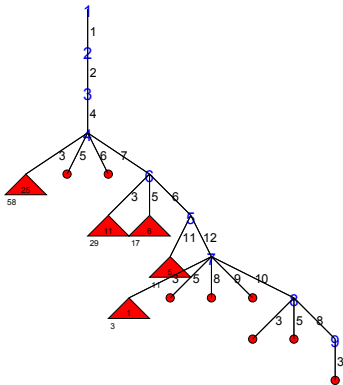


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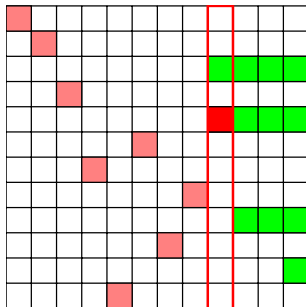
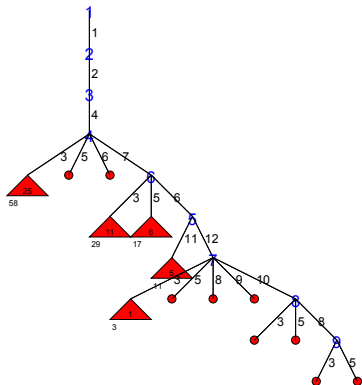
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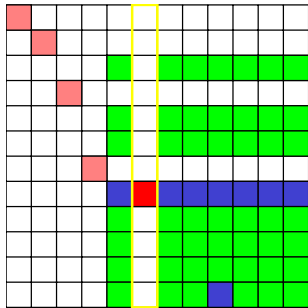
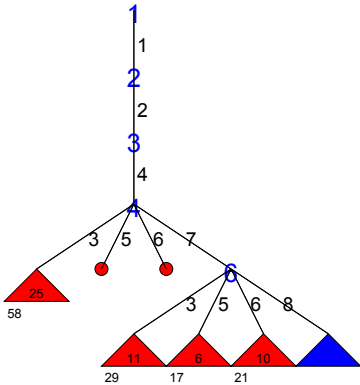
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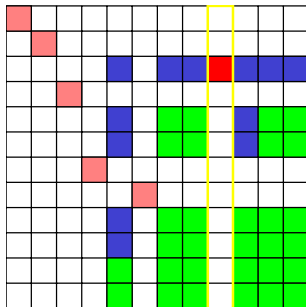
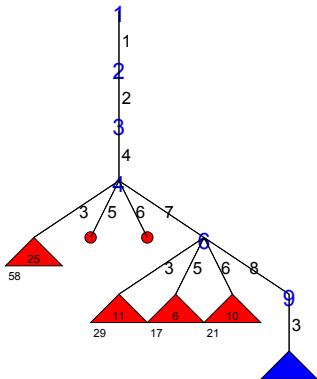
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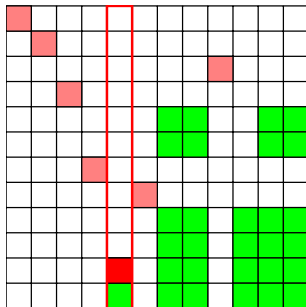
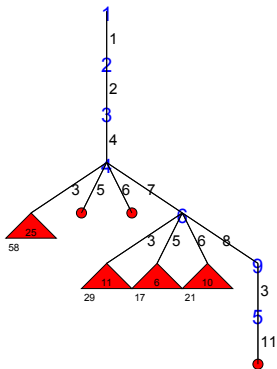
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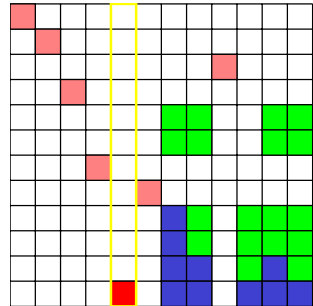
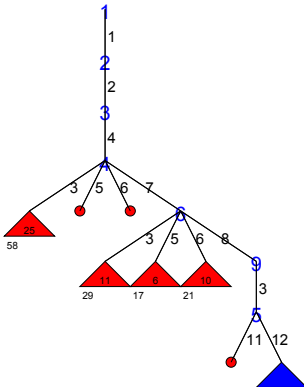
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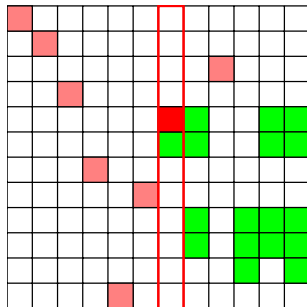
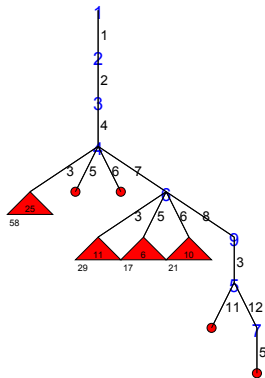
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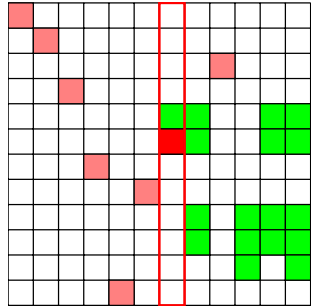
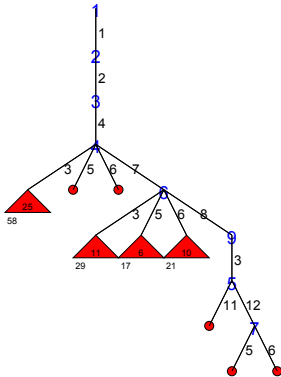
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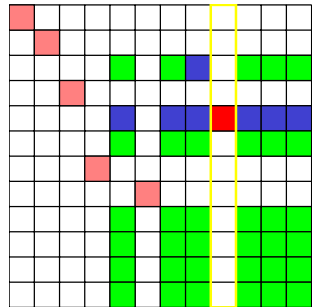
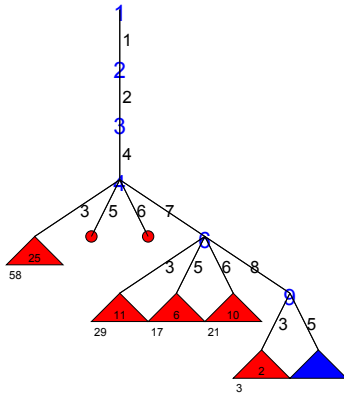
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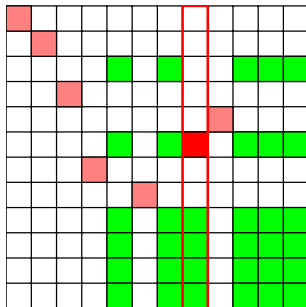
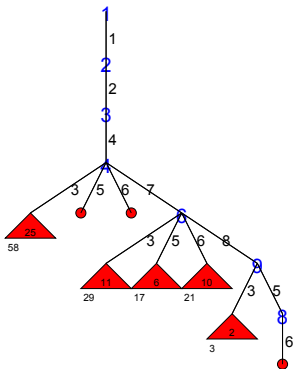


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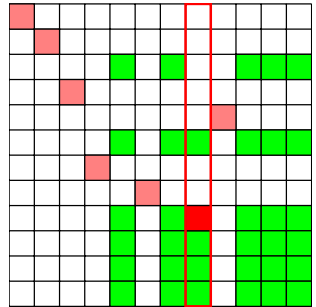
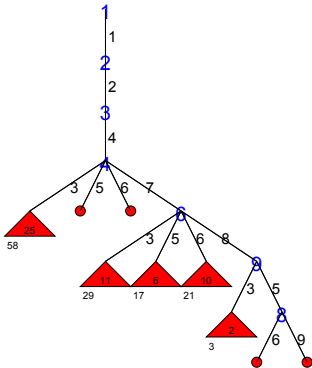
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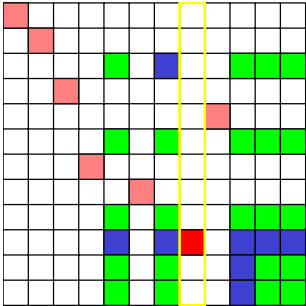
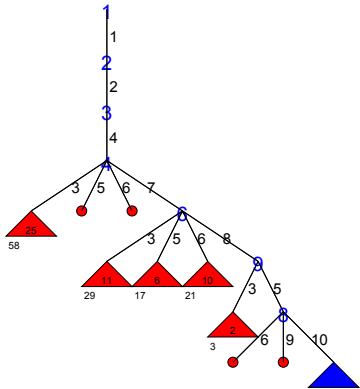
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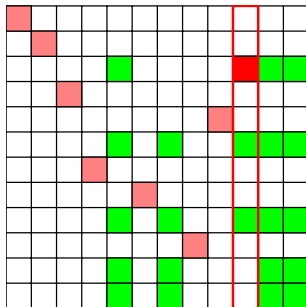
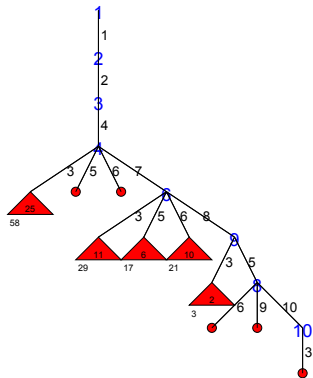
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Basic Model



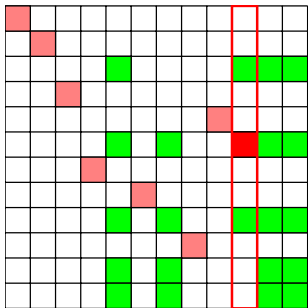
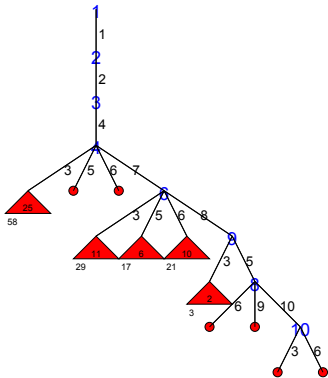
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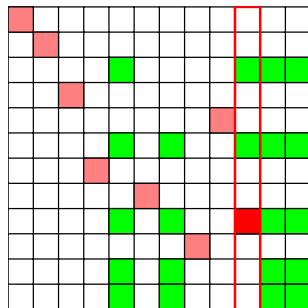
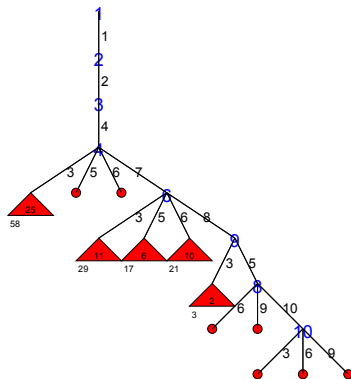
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Basic Model



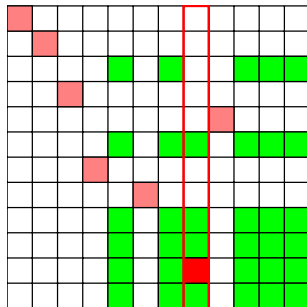
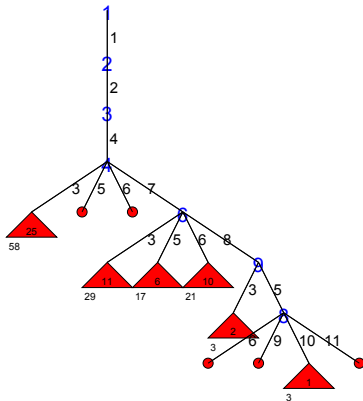
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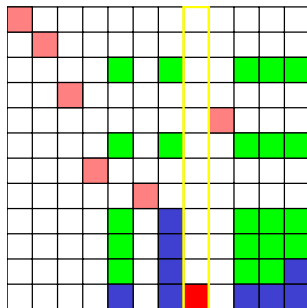
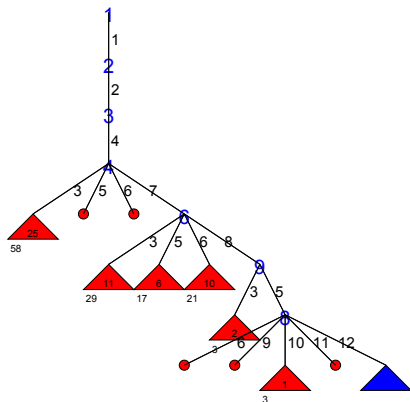
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Basic Model



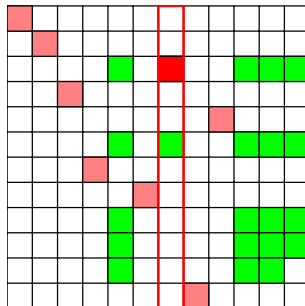
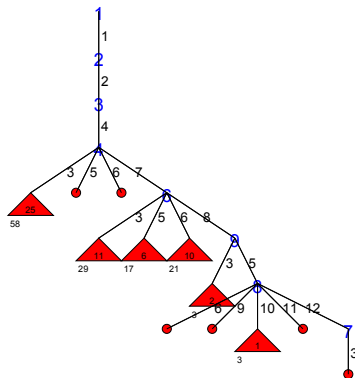
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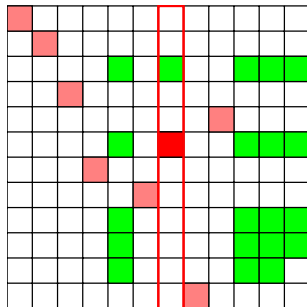
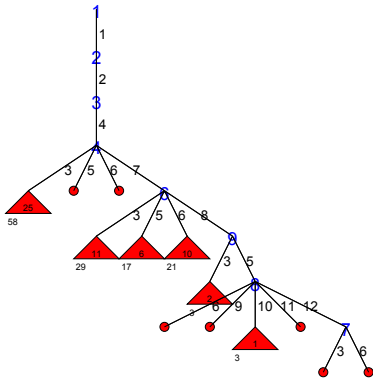
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Basic Model



Basic Model

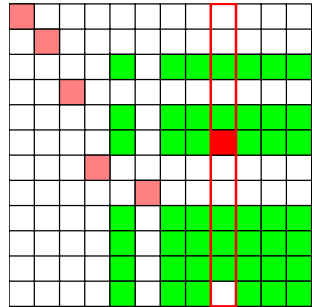
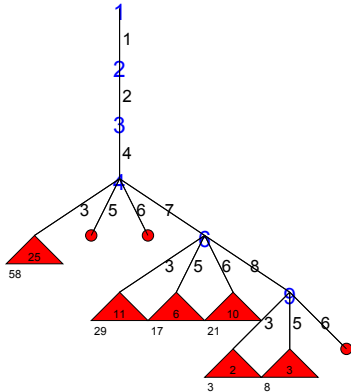


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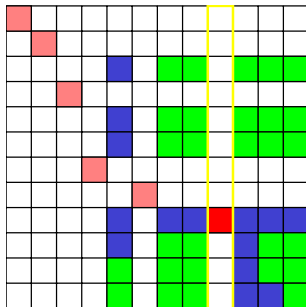
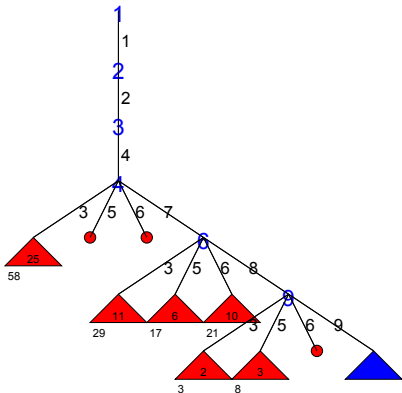
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Basic Model



Basic Model

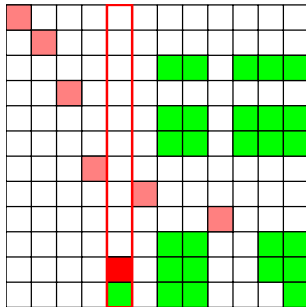
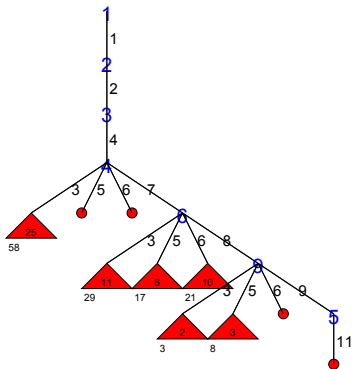


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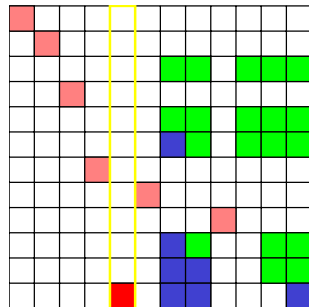
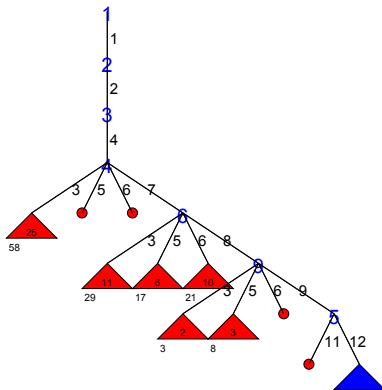
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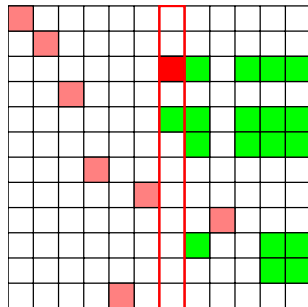
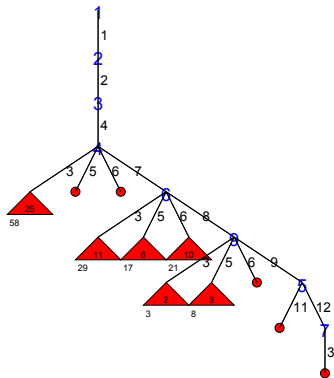
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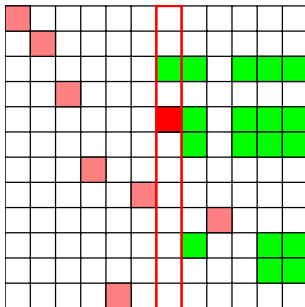
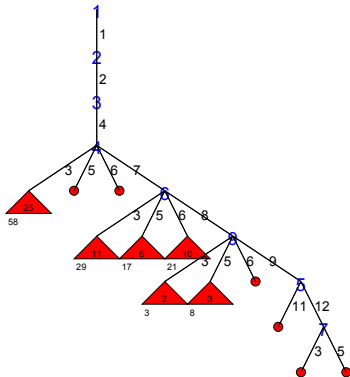
◀ Back to Start

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Basic Model



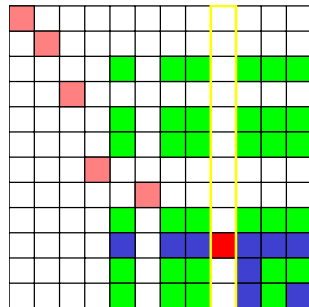
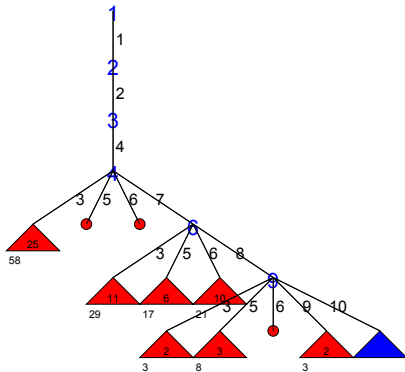
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Basic Model

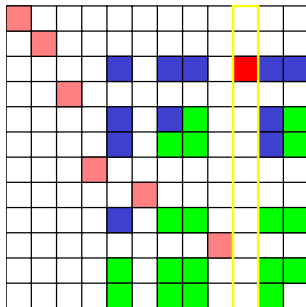
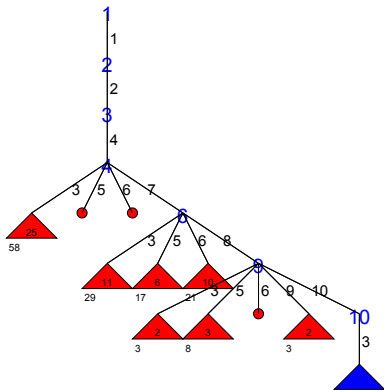


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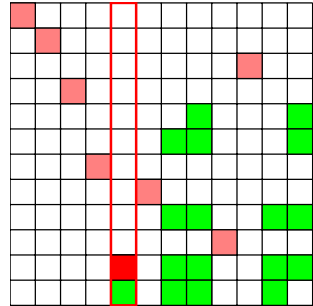
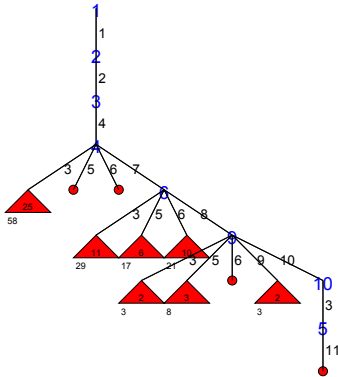
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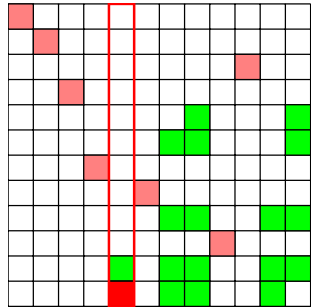
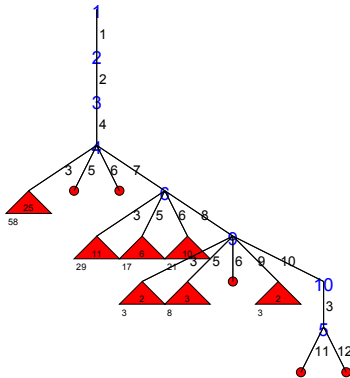
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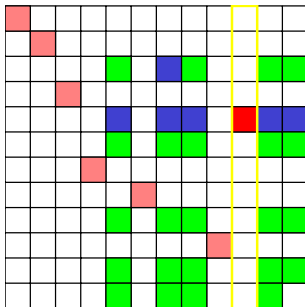
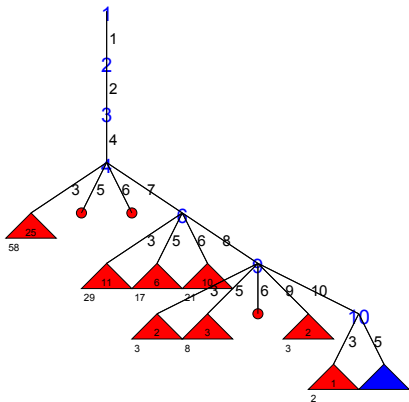
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▶▶ Skip Animation

Basic Model



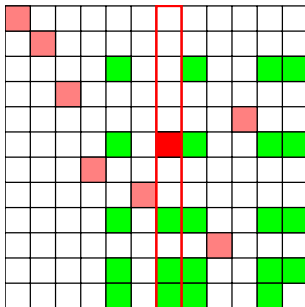
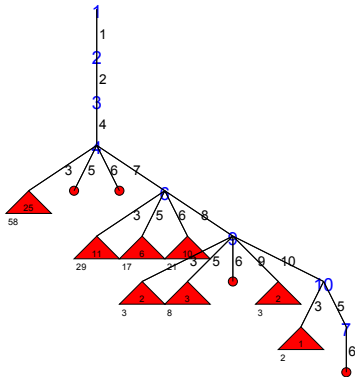
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▶ Skip Animation

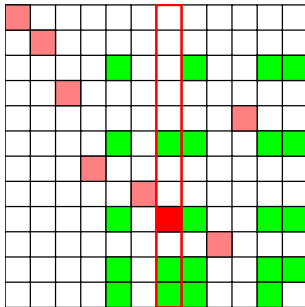
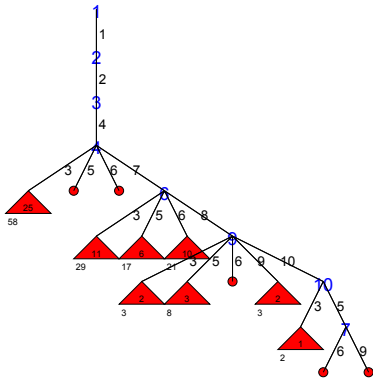
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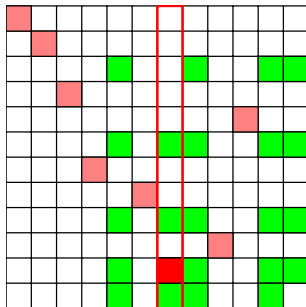
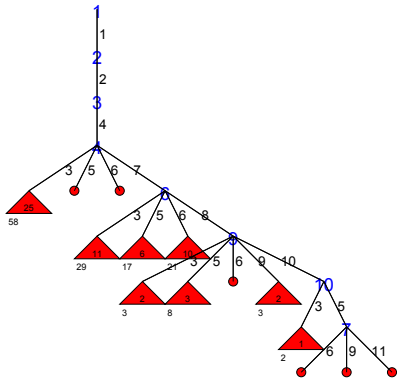
◀ Back to Start

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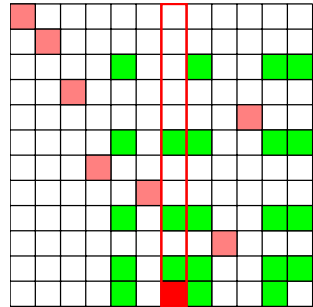
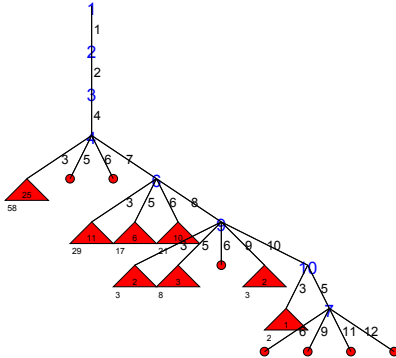
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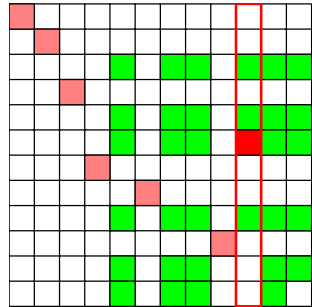
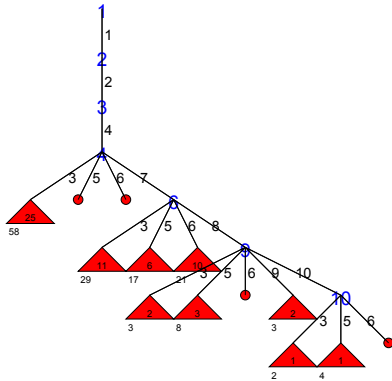
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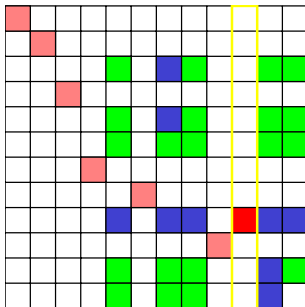
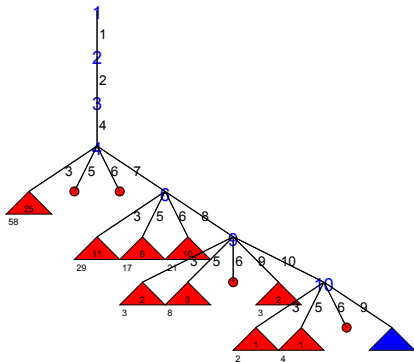
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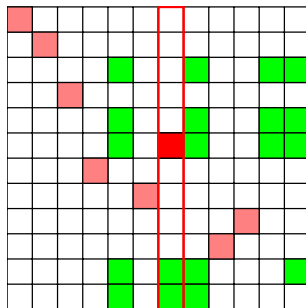
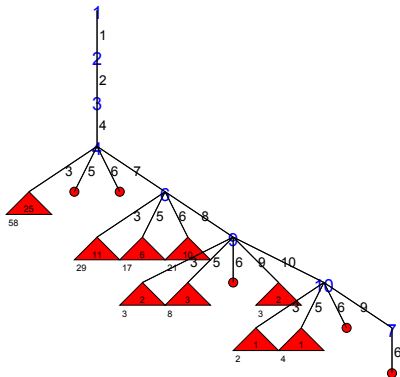
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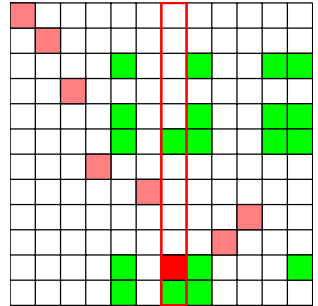
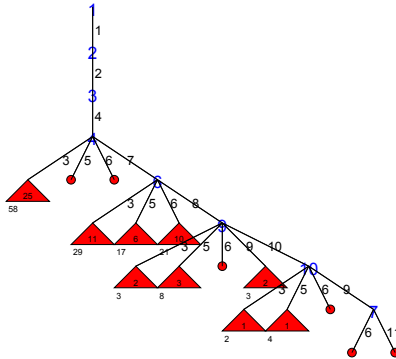
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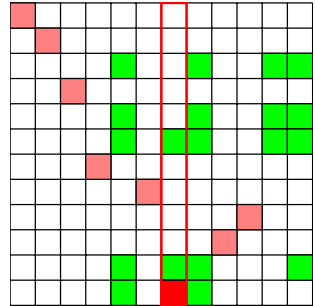
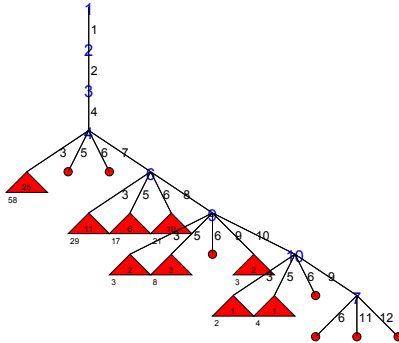
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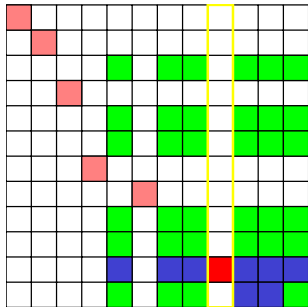
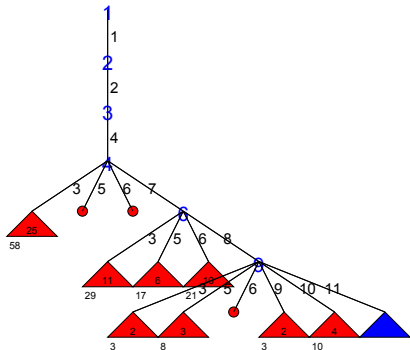
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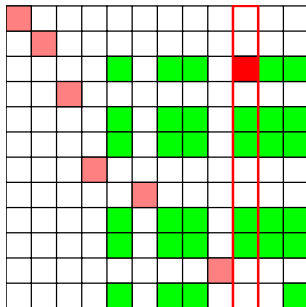
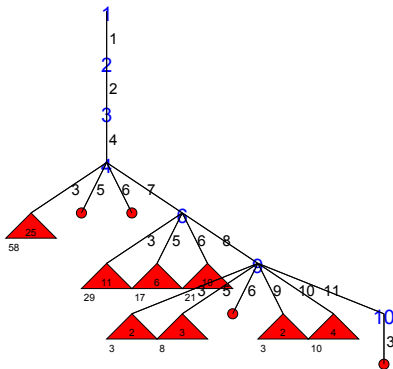
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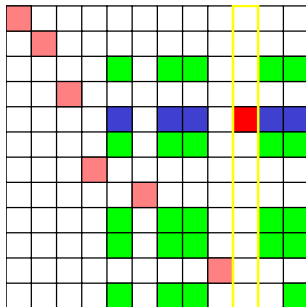
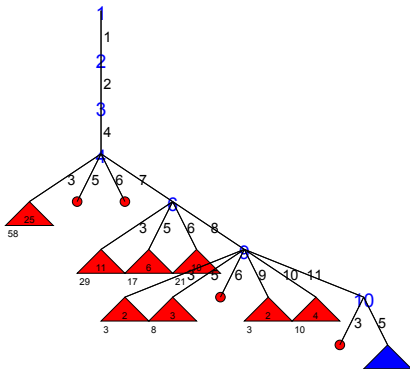
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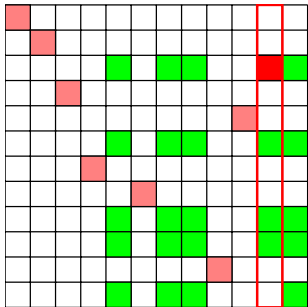
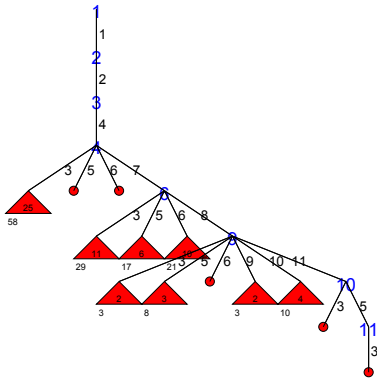
◀ Back to Start

▶▶ Skip Animation

Basic Model



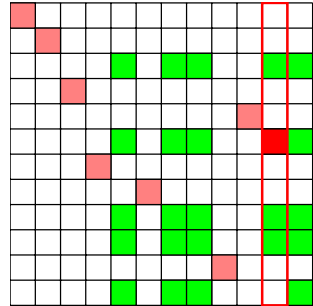
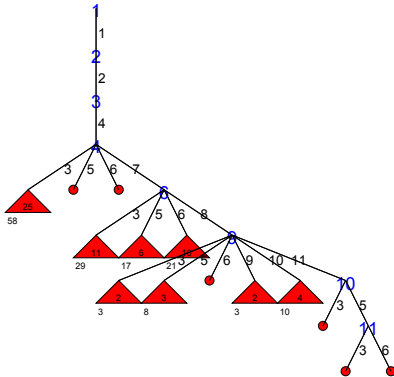
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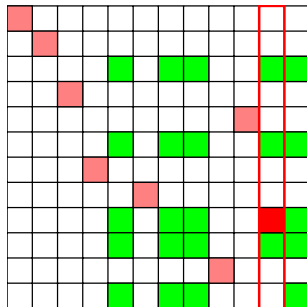
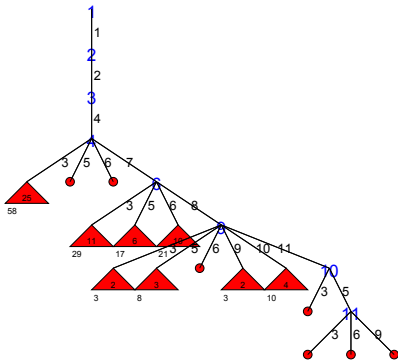
◀ Back to Start

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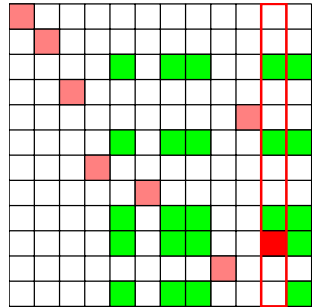
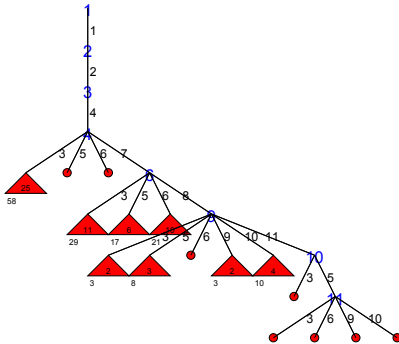
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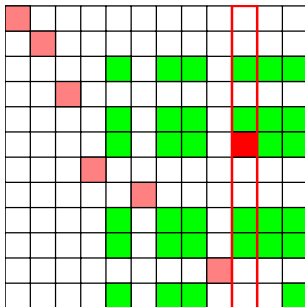
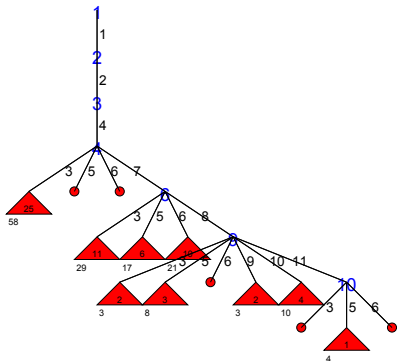
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Basic Model



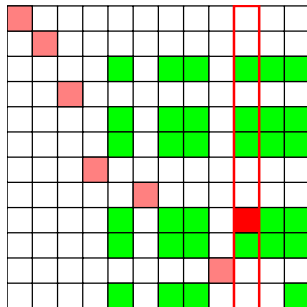
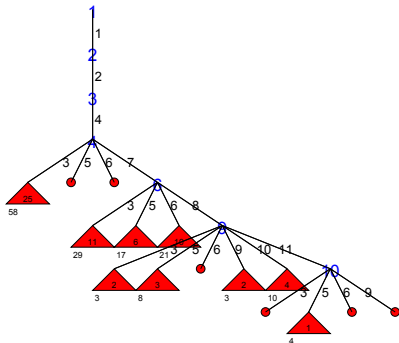
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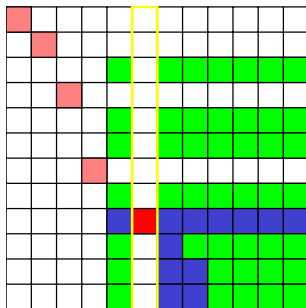
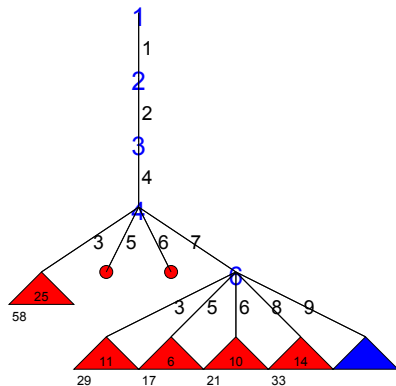
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Basic Model



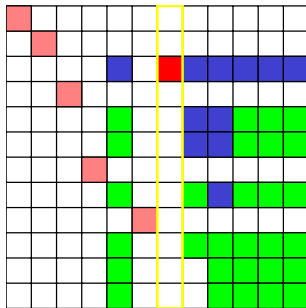
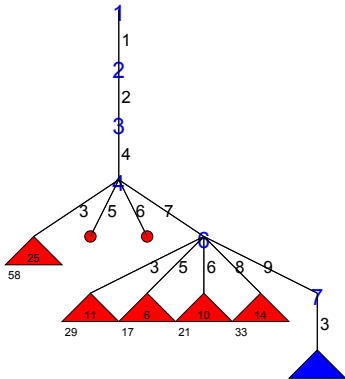
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◀ Back to Start

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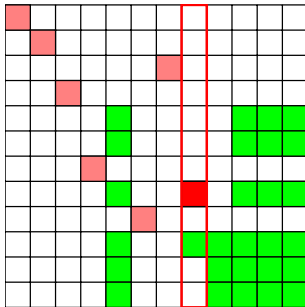
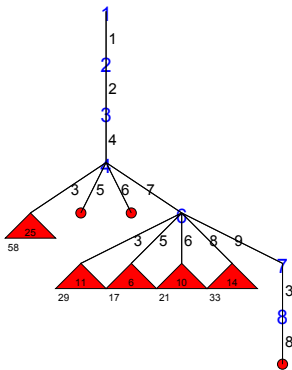
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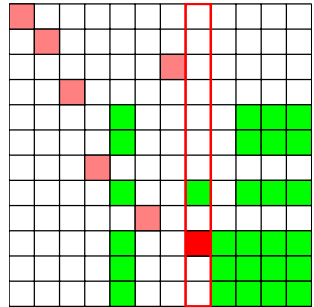
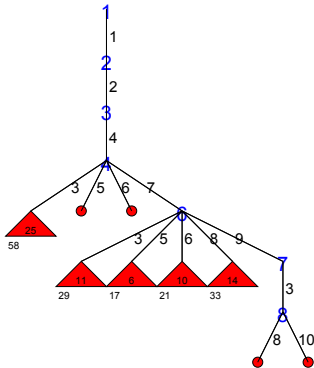
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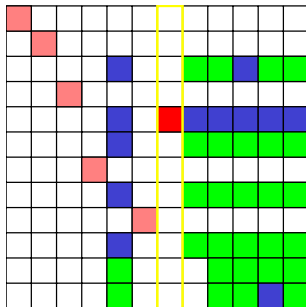
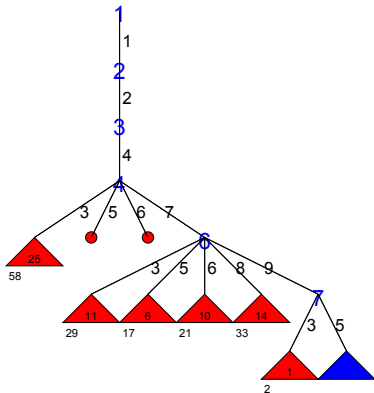
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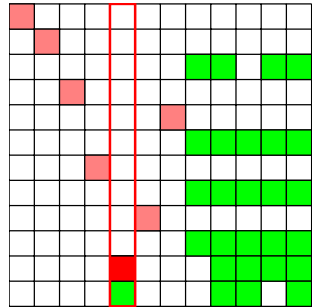
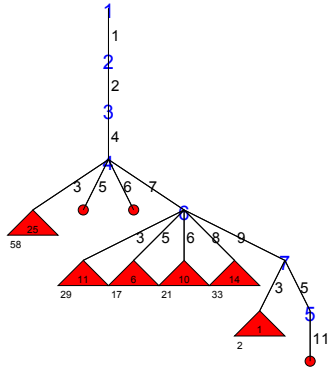
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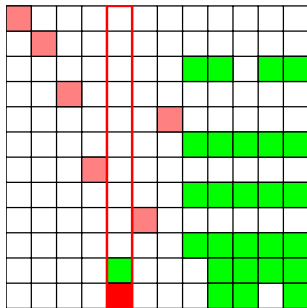
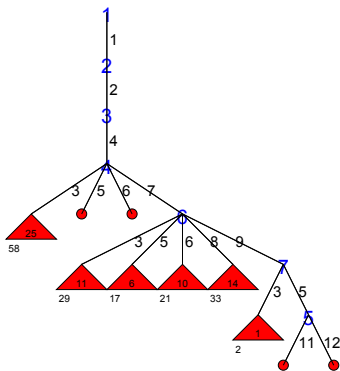
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Basic Model



Basic Model

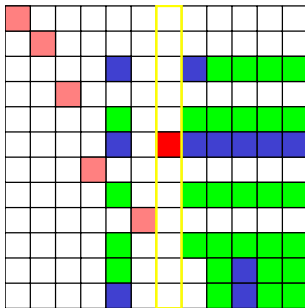
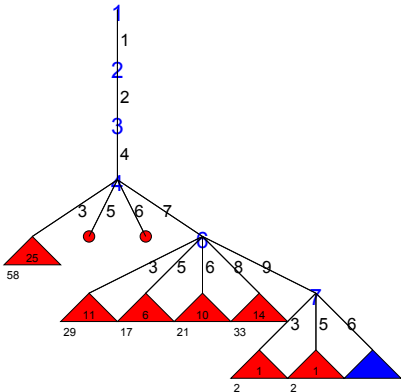


◀ Back to Start

▶ Skip Animation



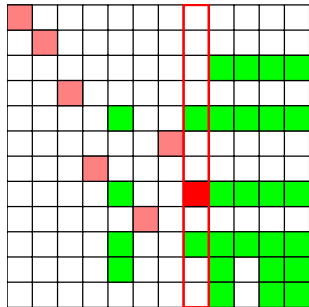
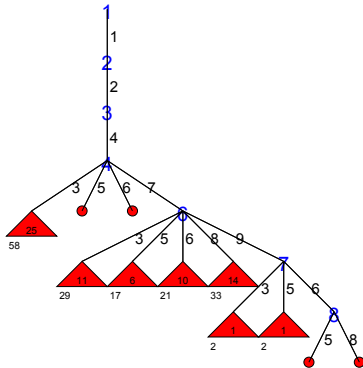
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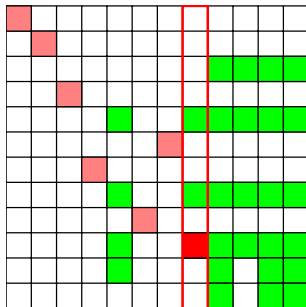
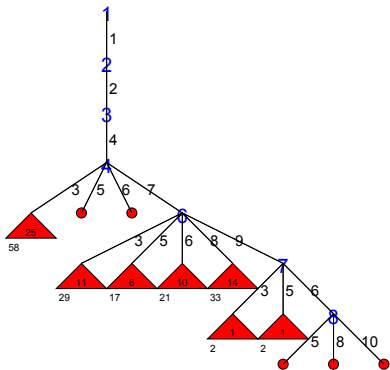
[◀ Back to Start](#)

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Basic Model



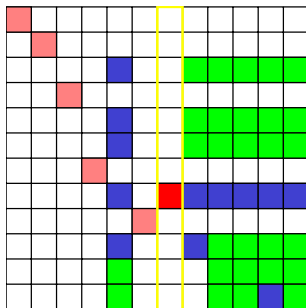
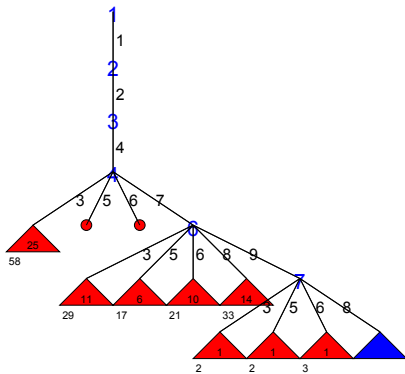
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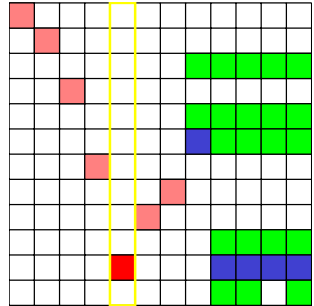
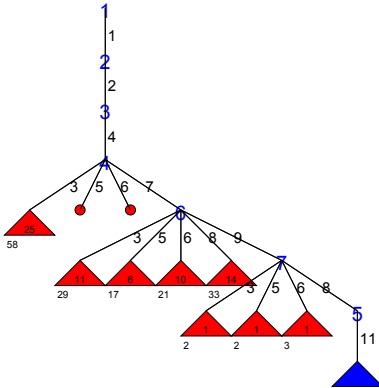
◀ Back to Start

▶ Skip Animation

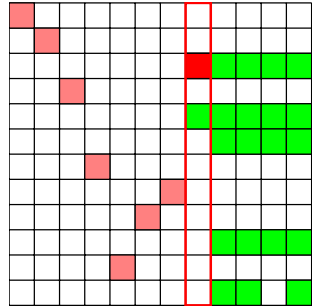
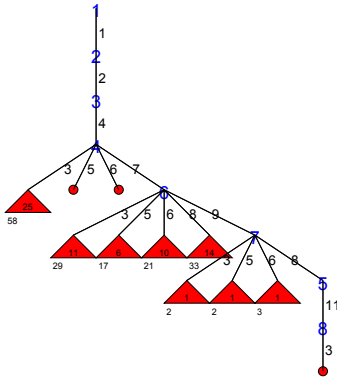
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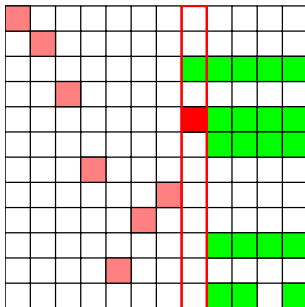
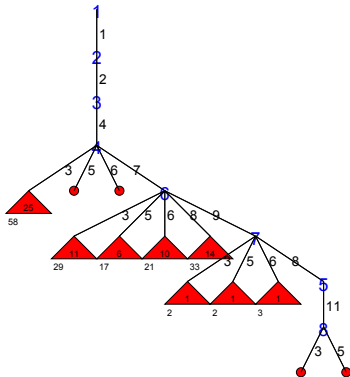
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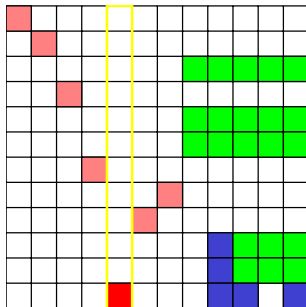
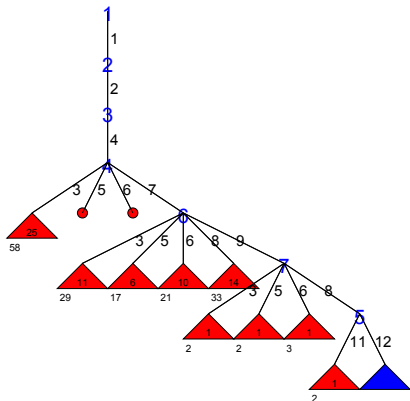
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▶ Skip Animation

Basic Model

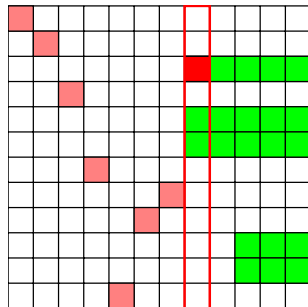
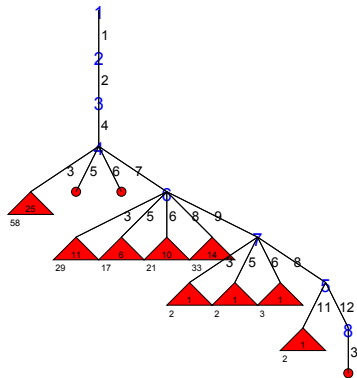


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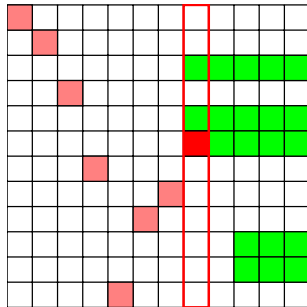
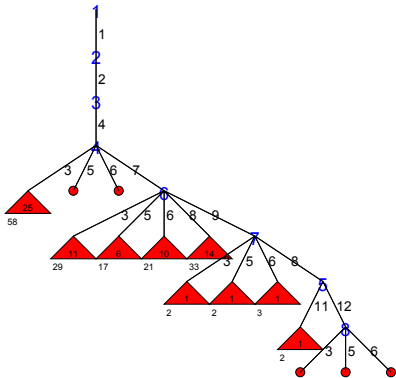
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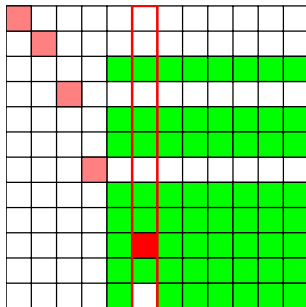
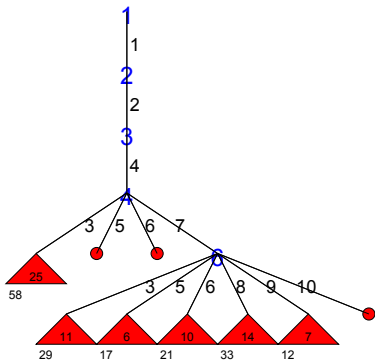
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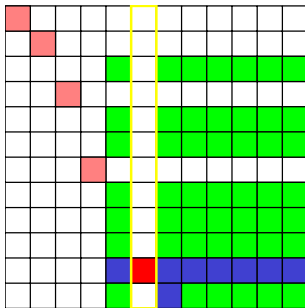
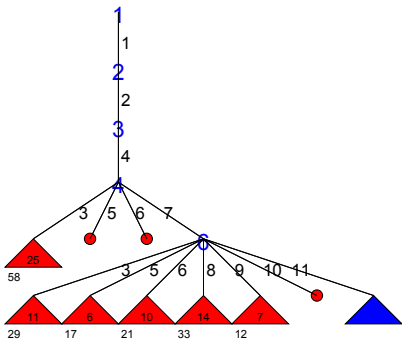
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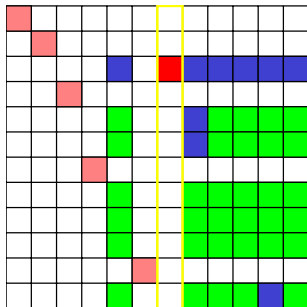
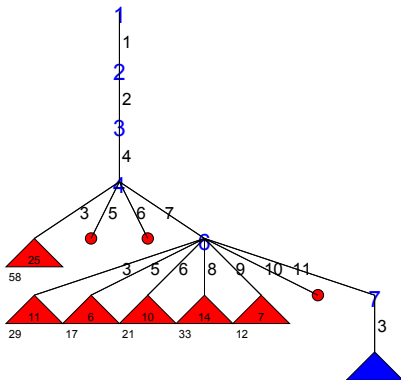
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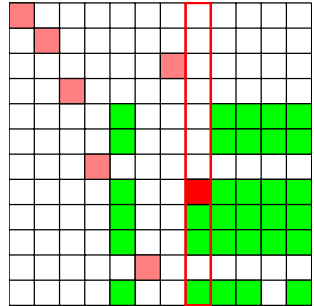
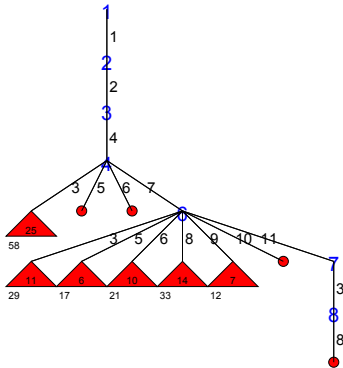
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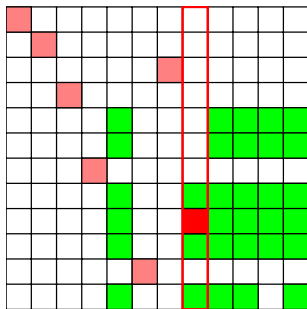
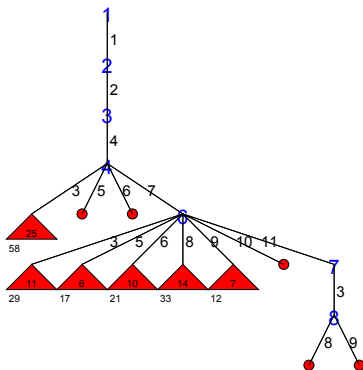
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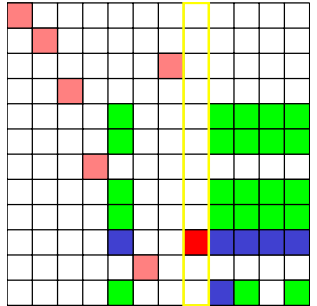
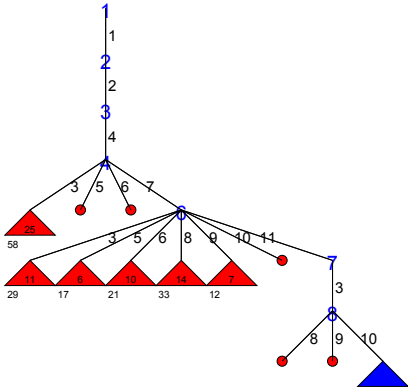
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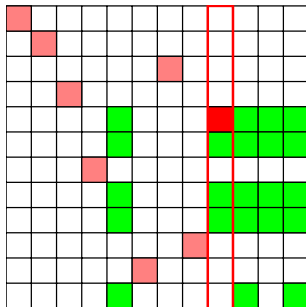
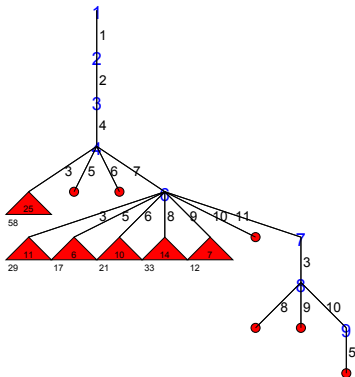
◀ Back to Start

▶▶ Skip Animation

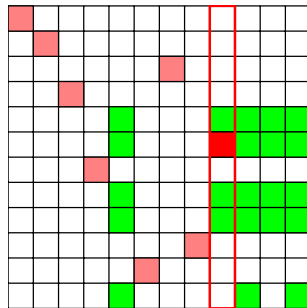
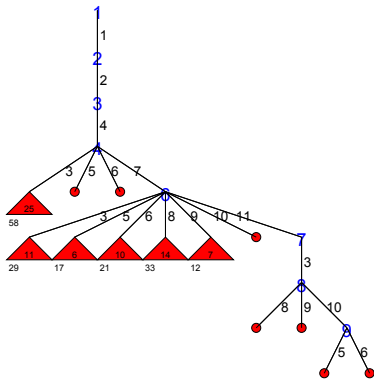
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Basic Model



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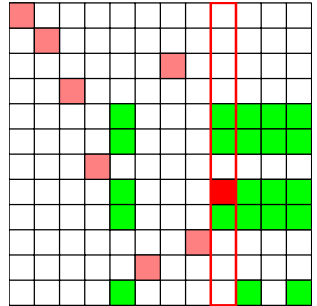
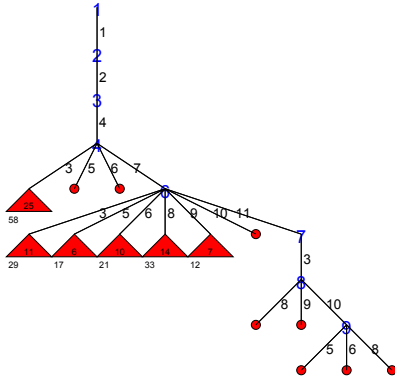


◀ Back to Start

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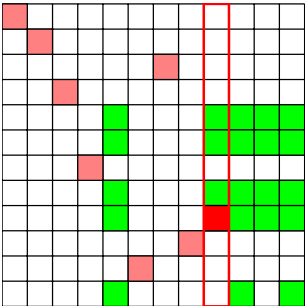
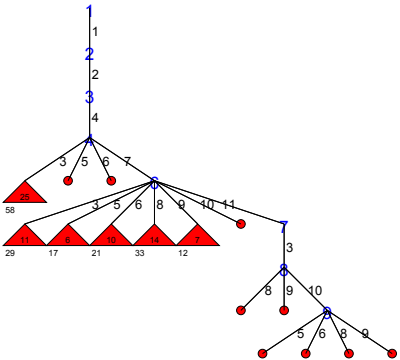


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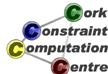
◀ Back to Start ▶▶ Skip Animation

Basic Model

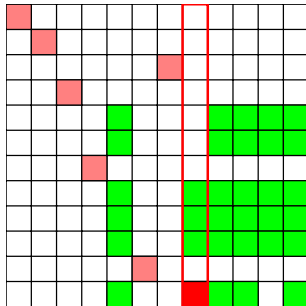
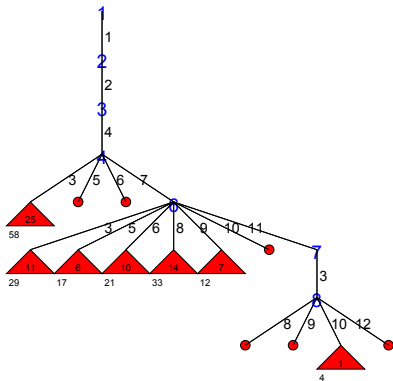


◀ Back to Start

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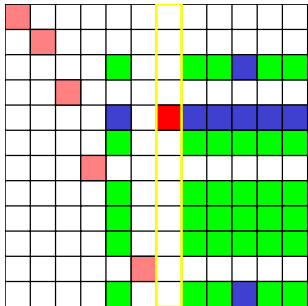
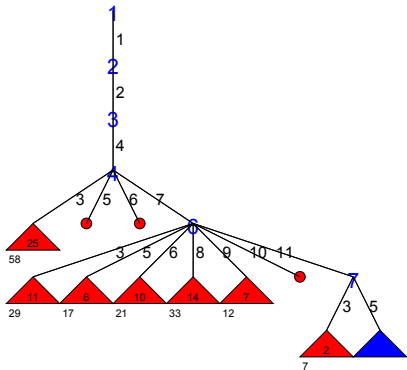
Basic Model



◀ Back to Start

▶ Skip Animation

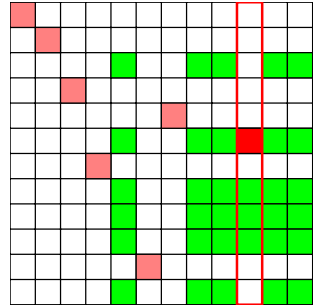
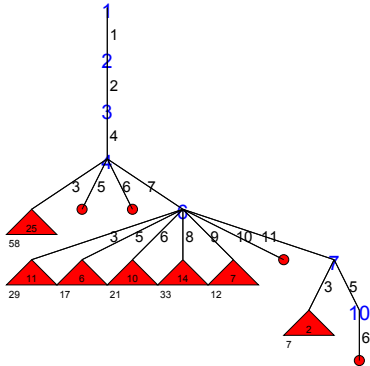
Basic Model



◀ Back to Start

▶ Skip Animation

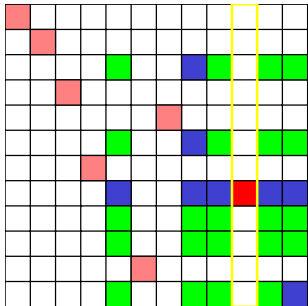
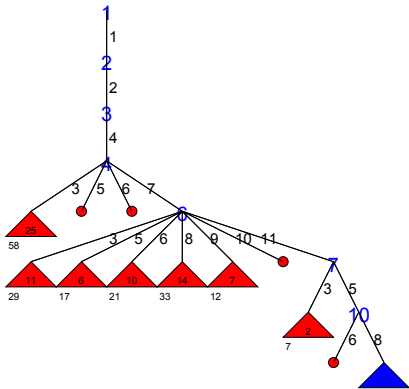
Basic Model



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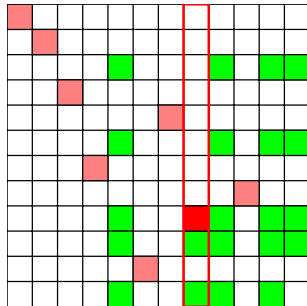
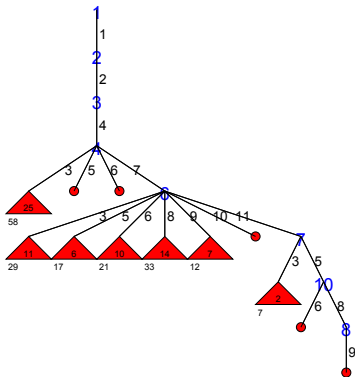
Basic Model



◀ Back to Start

▶ Skip Animation

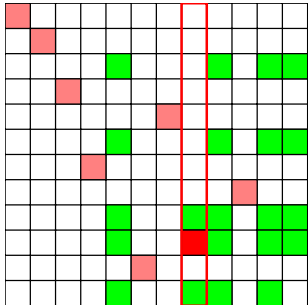
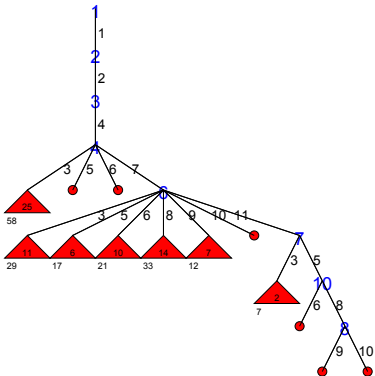
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▶ Skip Animation

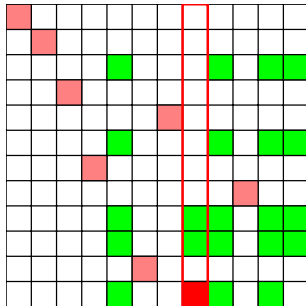
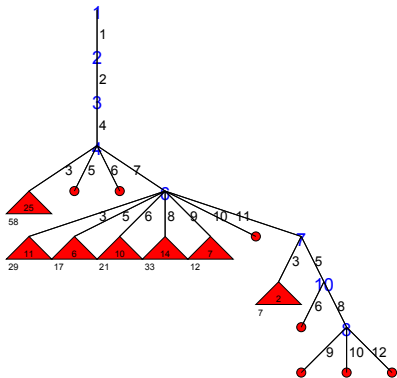
Basic Model



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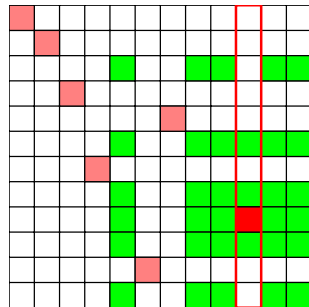
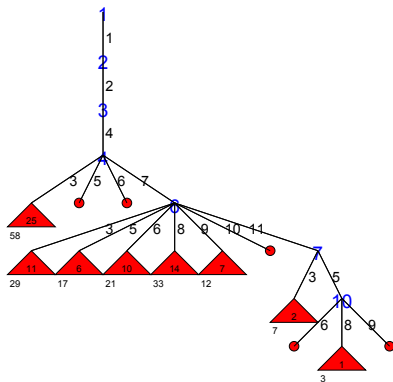
Basic Model



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▶ Skip Animation

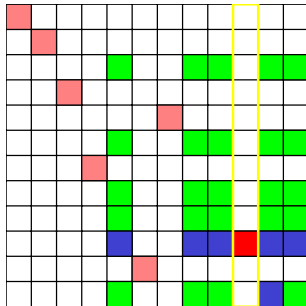
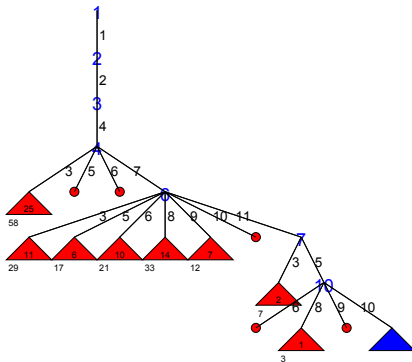
Basic Model



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▶ Skip Animation

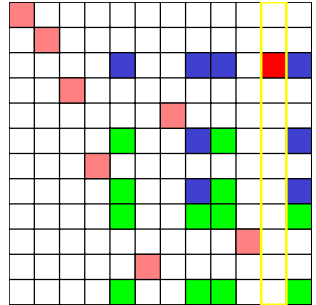
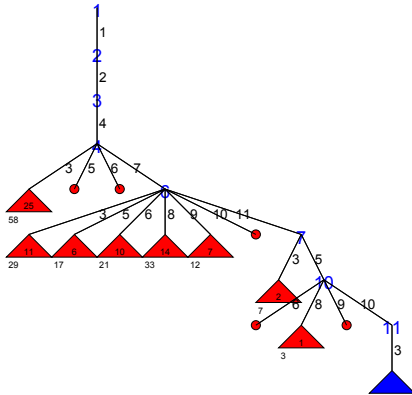
Basic Model



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▶ Skip Animation

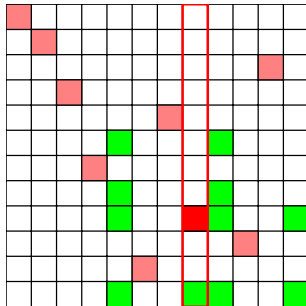
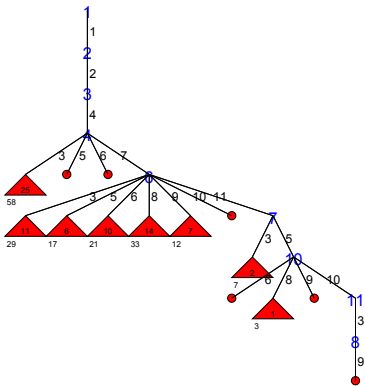
Basic Model



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▶ Skip Animation

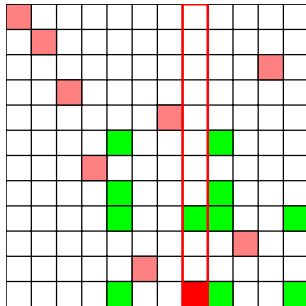
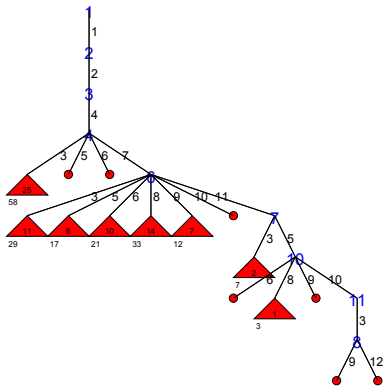
Basic Model



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▶▶ Skip Animation

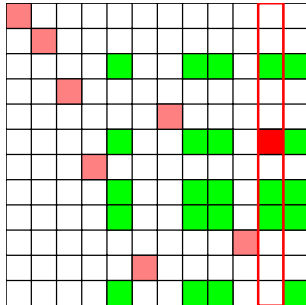
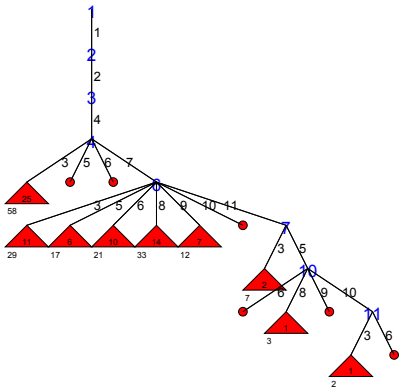
Basic Model



◀ Back to Start

▶ Skip Animation

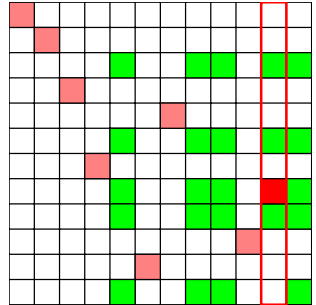
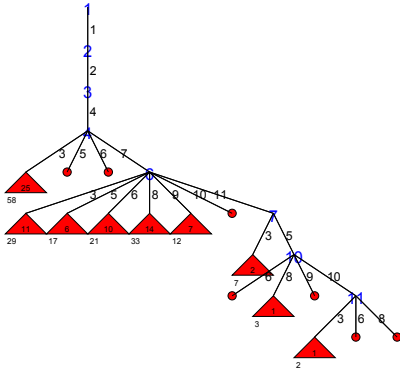
Basic Model



◀ Back to Start

▶ Skip Animation

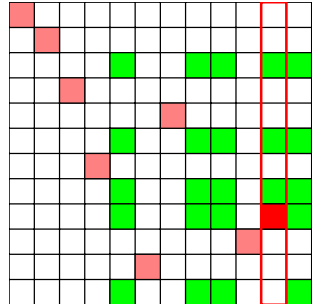
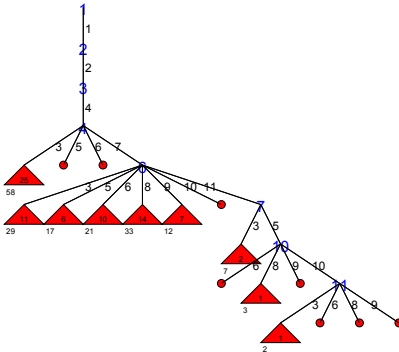
Basic Model



◀ Back to Start

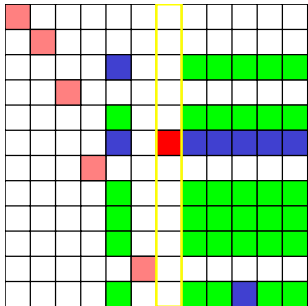
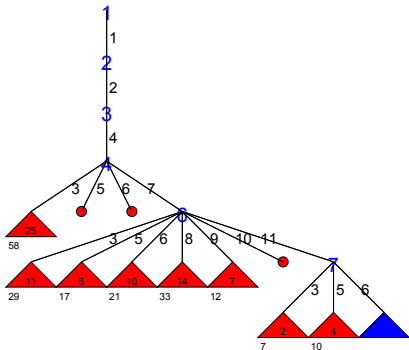
▶ Skip Animation

Basic Model



◀ Back to Start ▶ Skip Animation

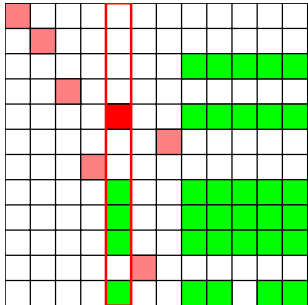
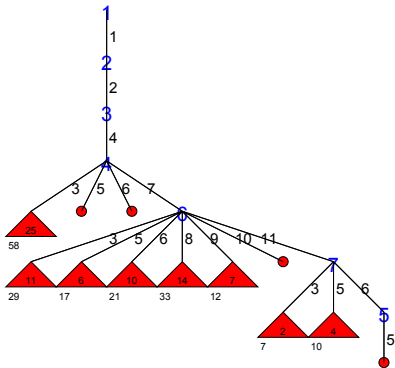
Basic Model



◀ Back to Start

▶▶ Skip Animation

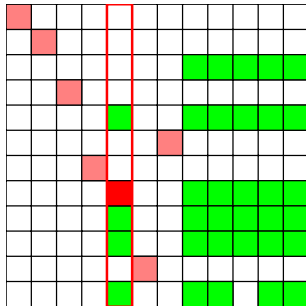
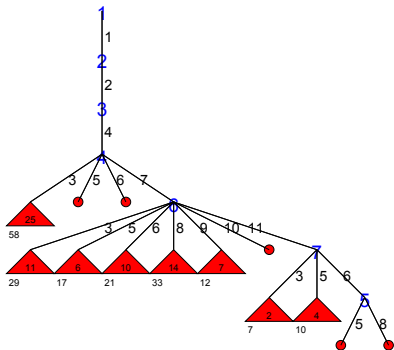
Basic Model



◀ Back to Start

▶ Skip Animation

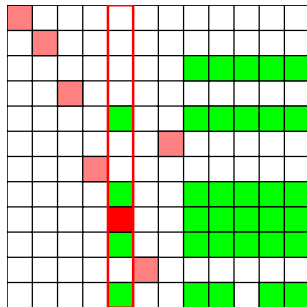
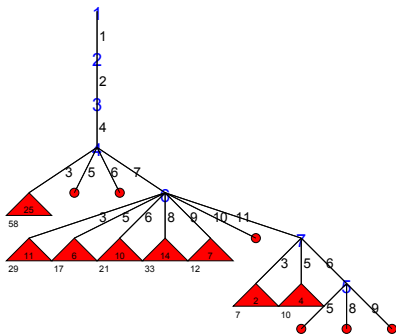
Basic Model



◀ Back to Start

▶ Skip Animation

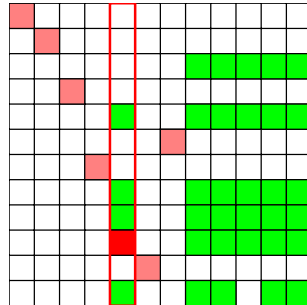
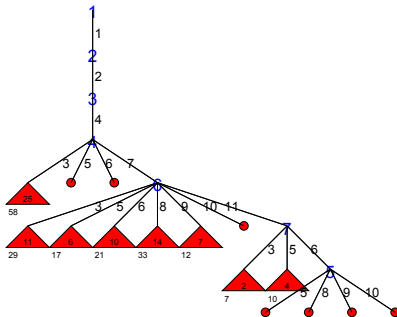
Basic Model



◀ Back to Start

▶ Skip Animation

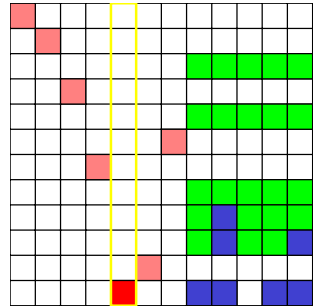
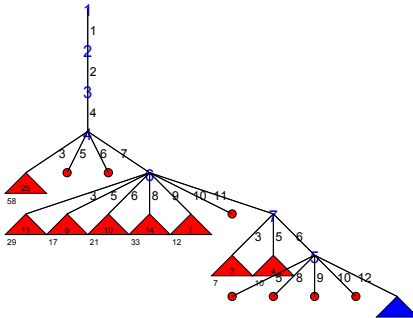
Basic Model



◀ Back to Start

▶ Skip Animation

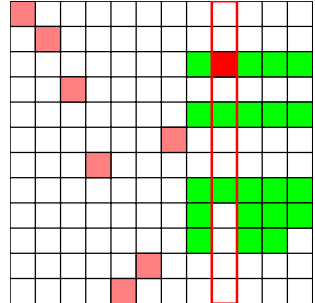
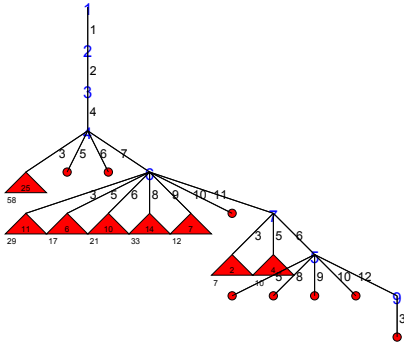
Basic Model



◀ Back to Start

▶ Skip Animation

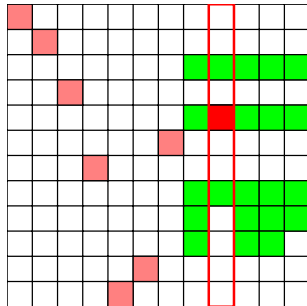
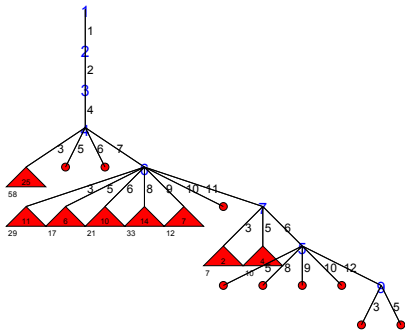
Basic Model



◀ Back to Start ▶ Skip Animation



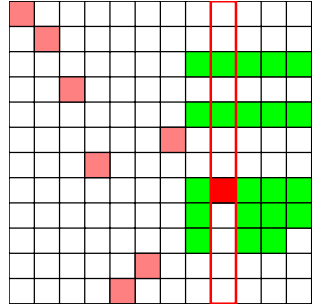
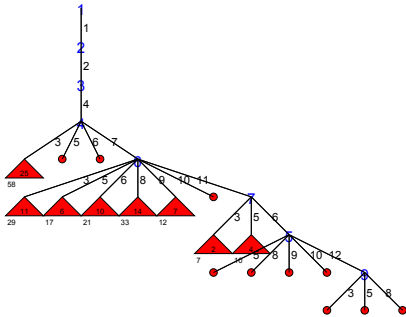
Basic Model



◀ Back to Start

▶ Skip Animation

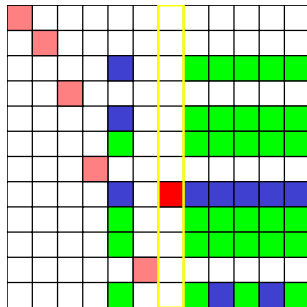
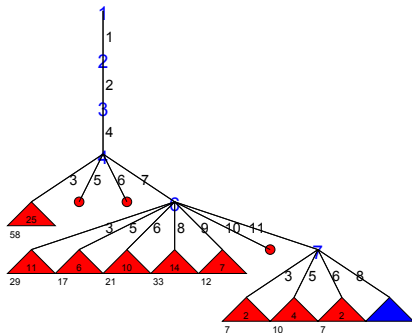
Basic Model



◀ Back to Start

▶ Skip Animation

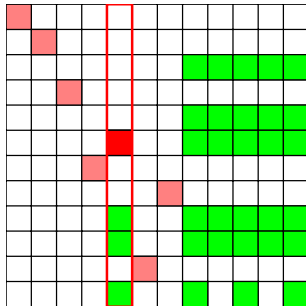
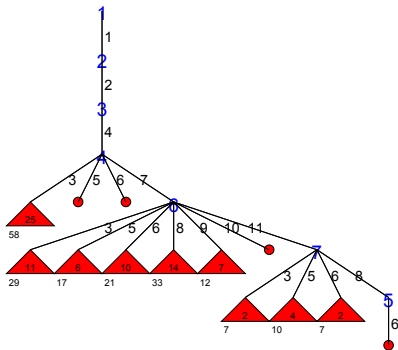
Basic Model



◀ Back to Start

▶ Skip Animation

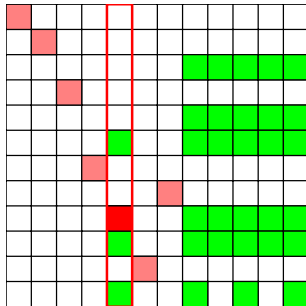
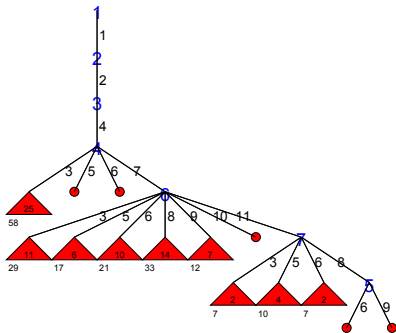
Basic Model



◀ Back to Start

▶ Skip Animation

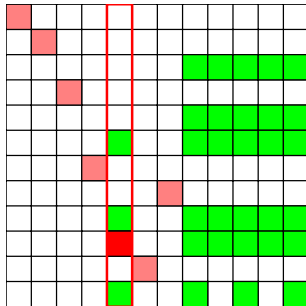
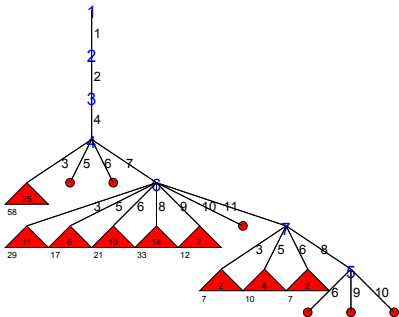
Basic Model



◀ Back to Start

▶ Skip Animation

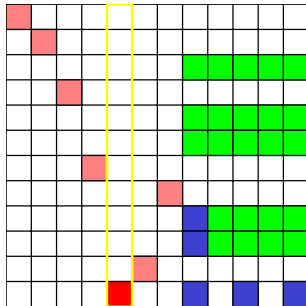
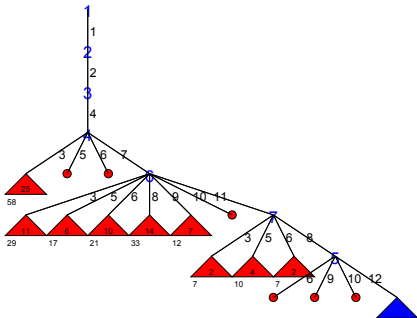
Basic Model



◀ Back to Start

▶ Skip Animation

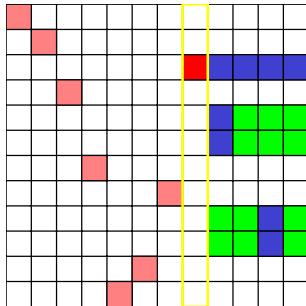
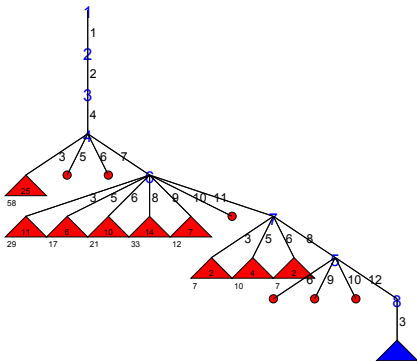
Basic Model



◀ Back to Start

▶ Skip Animation

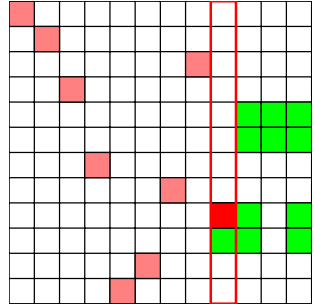
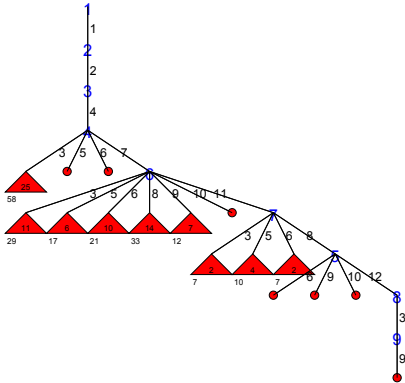
Basic Model



◀ Back to Start

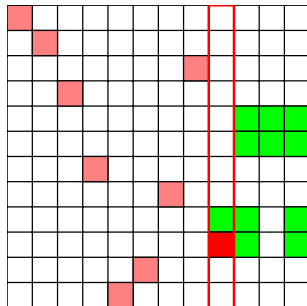
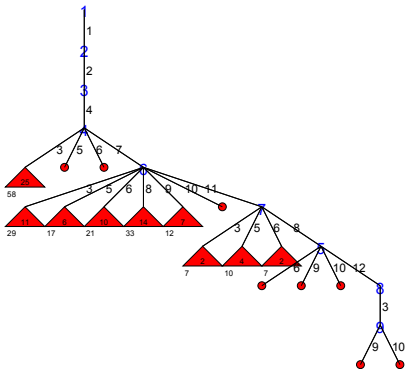
▶ Skip Animation

Basic Model



◀ Back to Start ▶ Skip Animation

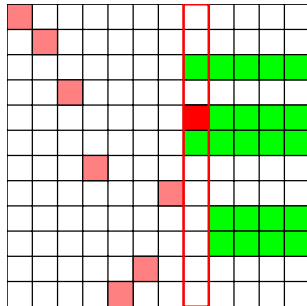
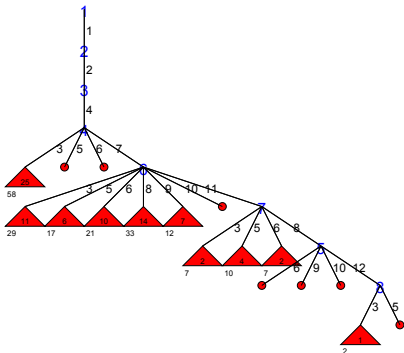
Basic Model



◀ Back to Start

▶ Skip Animation

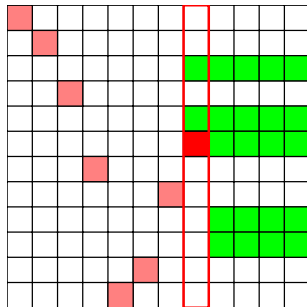
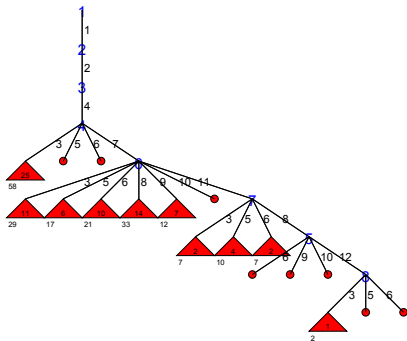
Basic Model



◀ Back to Start

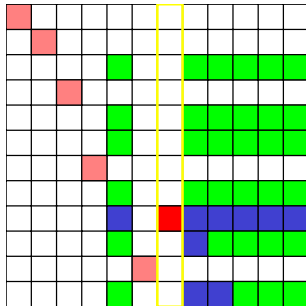
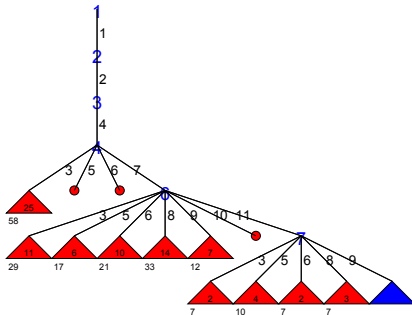
▶ Skip Animation

Basic Model



◀ Back to Start ▶ Skip Animation

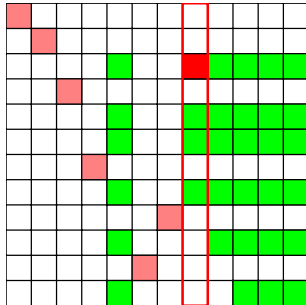
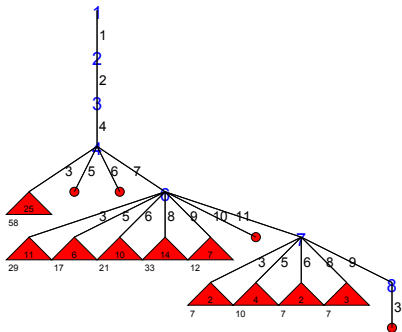
Basic Model



◀ Back to Start

▶ Skip Animation

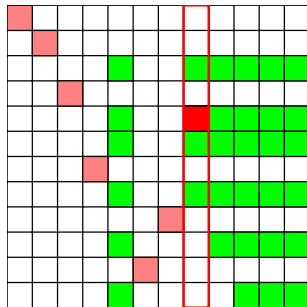
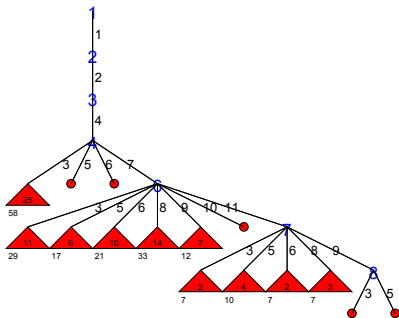
Basic Model



◀ Back to Start

▶ Skip Animation

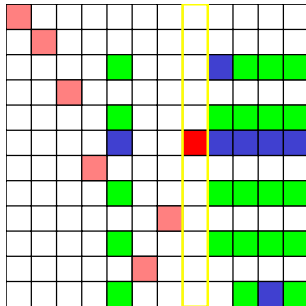
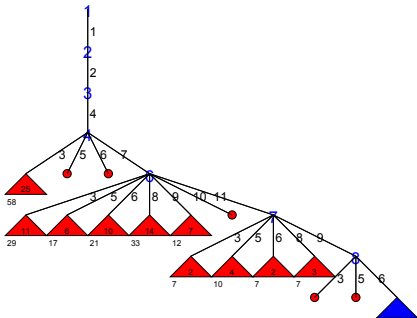
Basic Model



◀ Back to Start

▶ Skip Animation

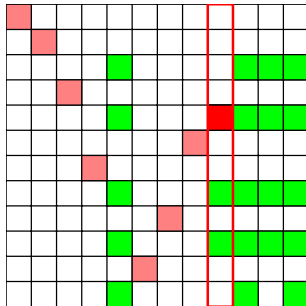
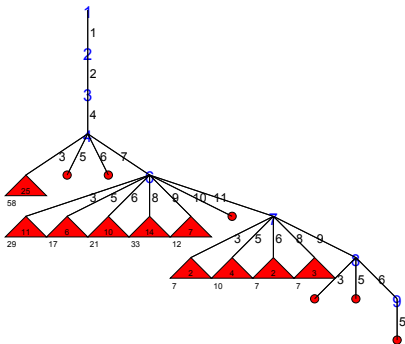
Basic Model



◀ Back to Start

▶ Skip Animation

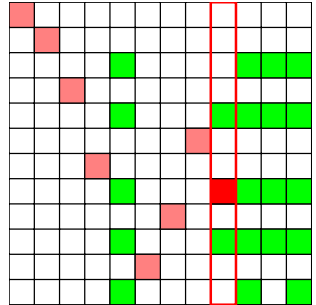
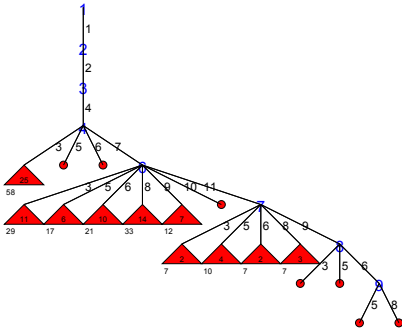
Basic Model



◀ Back to Start

▶ Skip Animation

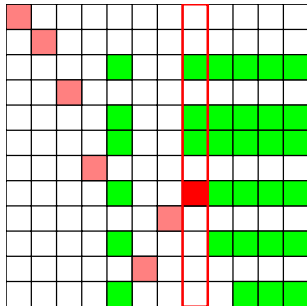
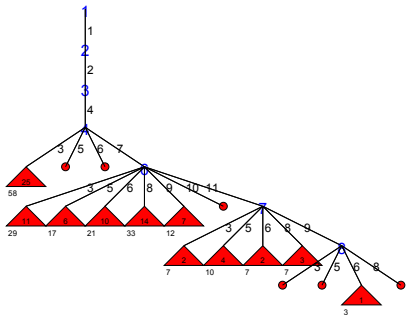
Basic Model



◀ Back to Start

▶ Skip Animation

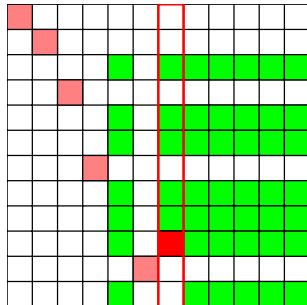
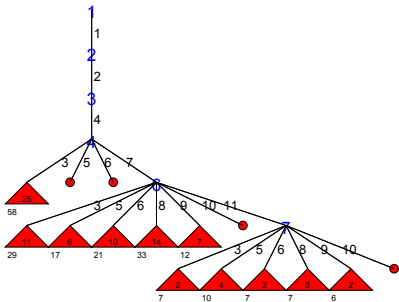
Basic Model



◀ Back to Start

▶ Skip Animation

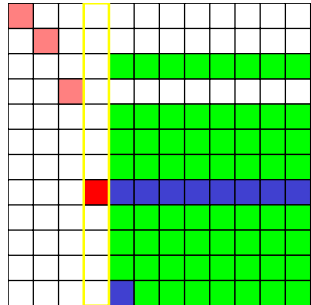
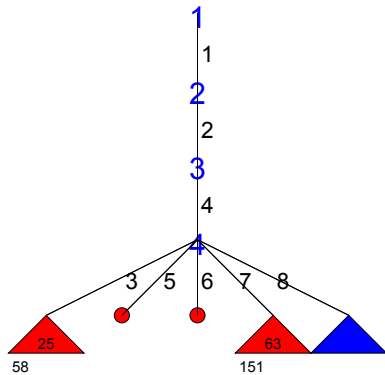
Basic Model



◀ Back to Start

▶ Skip Animation

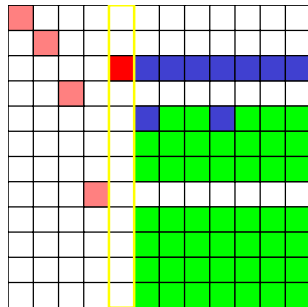
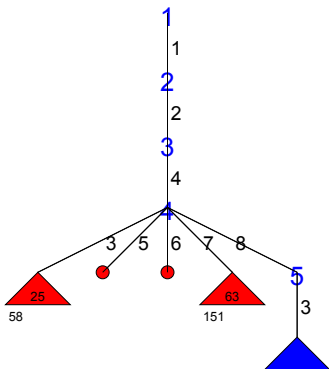
Basic Model



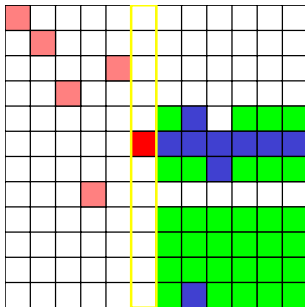
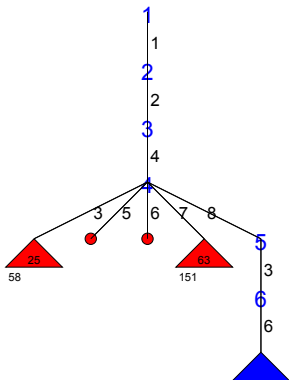
◀ Back to Start

▶ Skip Animation

Basic Model



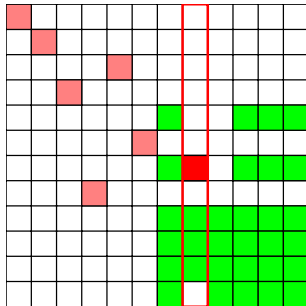
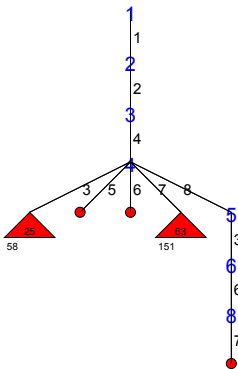
Basic Model



◀ Back to Start

▶ Skip Animation

Basic Model

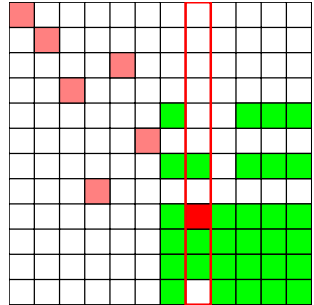
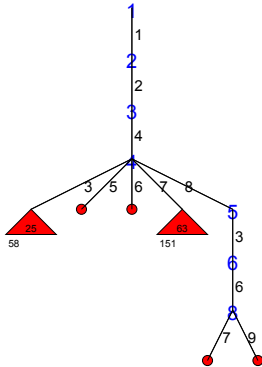


◀ Back to Start

▶ Skip Animation



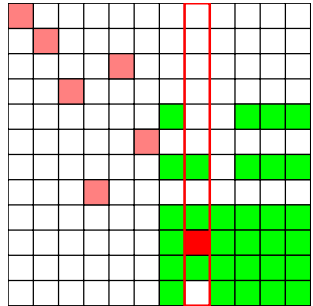
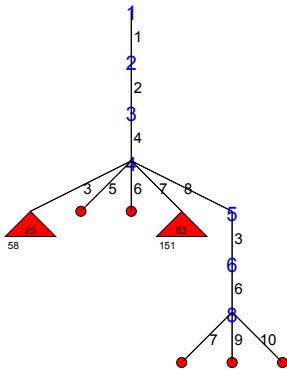
Basic Model



◀ Back to Start ▶ Skip Animation

Navigation icons: home, back, forward, search, refresh, etc.

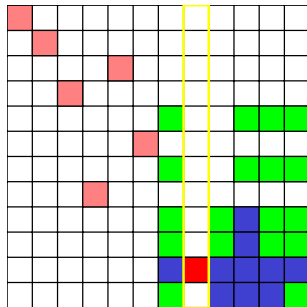
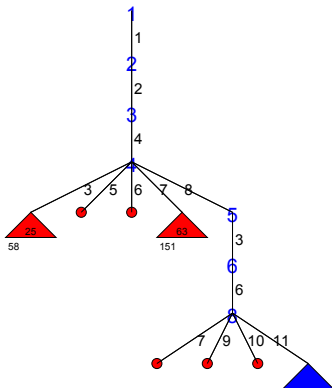
Basic Model



◀ Back to Start

▶ Skip Animation

Basic Model

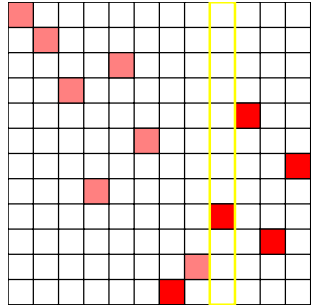
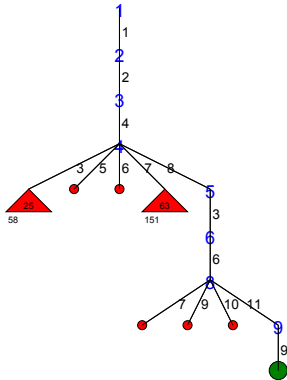


◀ Back to Start

▶ Skip Animation



Basic Model

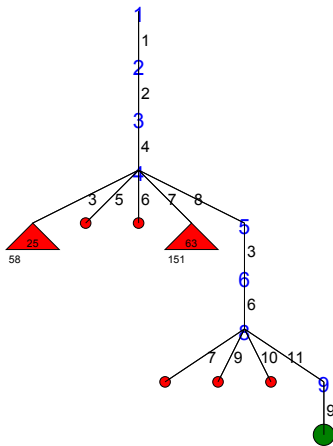


← Back to Start

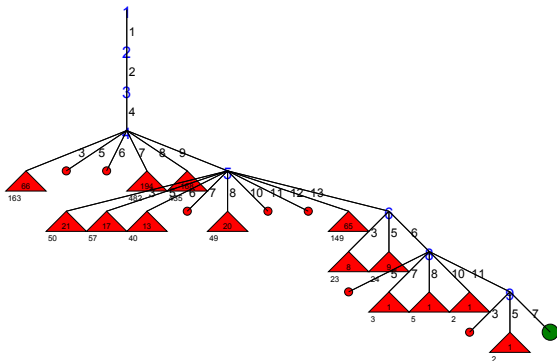
Other Problem Sizes

Size	Basic Model	
	Backtrack	Time
10	4	0.00
11	118	0.08
12	50	0.05
13	335	0.36
14	5008	6.23
15	47332	68.92
16	157773	271.22
17	1641685	3278.19
18	115745	283.97

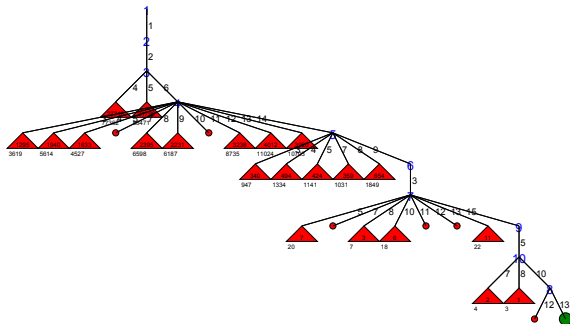
Search tree (Size 12)



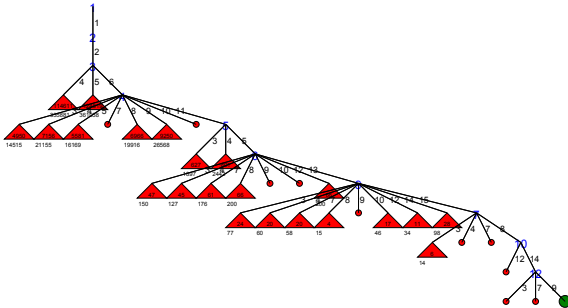
Search tree (Size 13)



Search tree (Size 15)

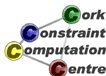


Search tree (Size 16)

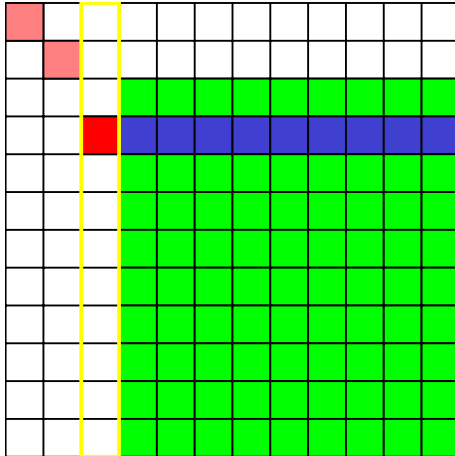


Observation

- Problem becomes harder with increasing size
- Failures occur from level 3 down
- Deep backtracking required to undo wrong choices
- Value selection not working, have to explore all choices
- Increase not uniform

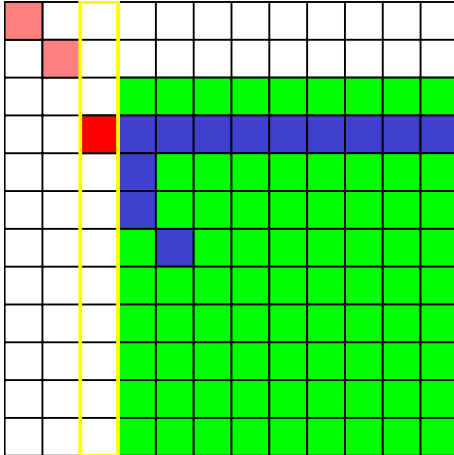


Missing Propagation



The model is
doing this

Missing Propagation



It could be doing that!

Outline

- 1 Problem
- 2 Program
- 3 Search
- 4 Improvements
 - Adding Constraints
 - Change of Search Strategy

Changed Differences

```
diffs (_, []).
diffs (L, [H|T]) :-
    diff_pairs (L, [H|T], Diffs, Triples),
    impose_triples (Triples, []),
    alldifferent (Diffs),
    diffs (L, T).

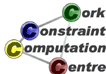
diff_pairs (_, [], [], []).
diff_pairs ([X|X1], [Y|Y1], [D|D1], [t (X, Y, D) | T]) :-
    X #= Y+D,
    diff_pairs (X1, Y1, D1, T).
```

Changed Differences

```
impose_triples([],_).  
impose_triples([t(X,Y,D)|R],Others):-  
    suspend(impose_triple(D,R),4,D->inst),  
    suspend(impose_triple(D,Others),4,D->inst),  
    impose_triples(R,[t(X,Y,D)|Others]).
```

Changed Differences

```
impose_triple(_D, []).  
impose_triple(D, [t(X, Y, _) | R]) :-  
    (var(X) ->  
        suspend(impose_one_triple12(D, X, Y),  
                4, X->inst)  
    ;  
        impose_one_triple12(D, X, Y)  
    ),  
%    ...  
    impose_triple(D, R).  
  
impose_one_triple12(D, X, Y) :-  
    V is X-D,  
    Y #\= V.
```

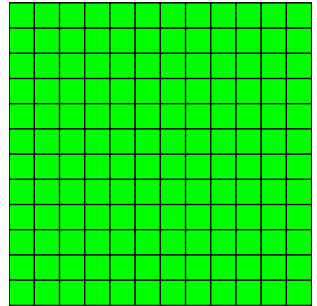


Further Model Improvements

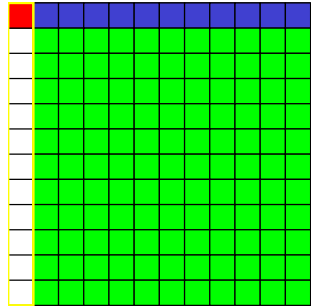
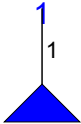
- DC consistent `alldifferent` between variables
- (DC consistent `alldifferent` between differences)
- DC difference constraint



Improved Model



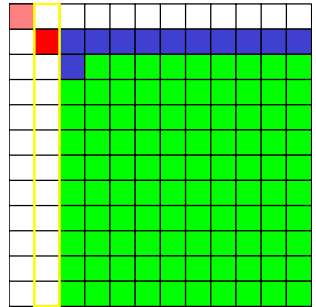
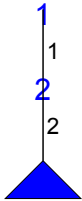
Improved Model



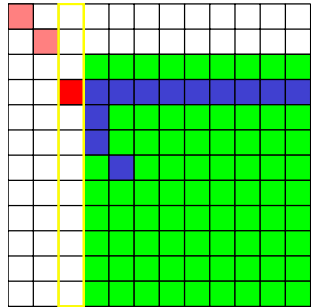
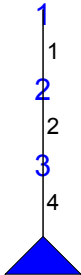
◀ Back to Start ▶ Skip Animation



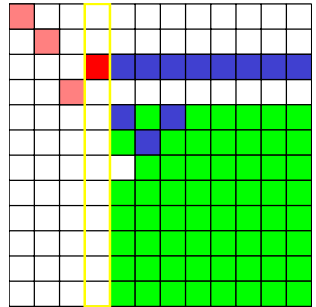
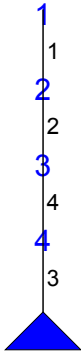
Improved Model



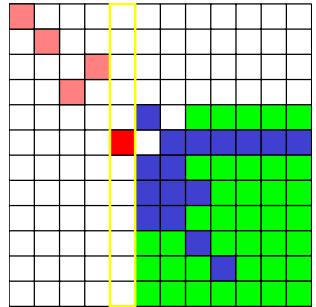
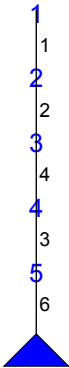
Improved Model



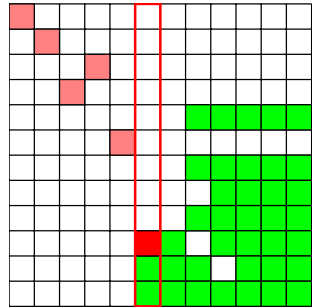
Improved Model



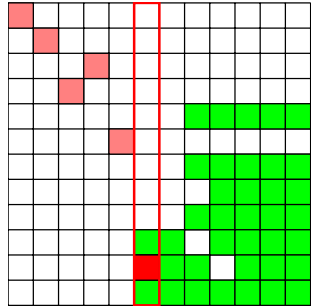
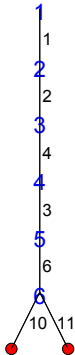
Improved Model



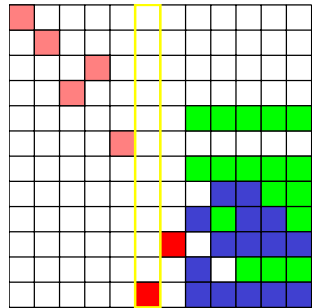
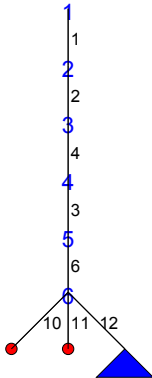
Improved Model



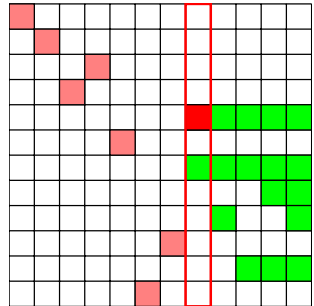
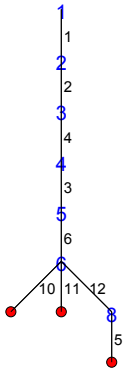
Improved Model



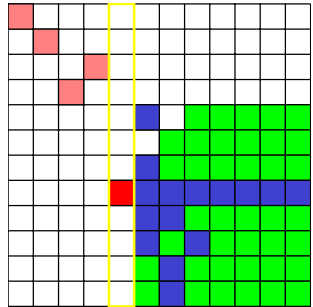
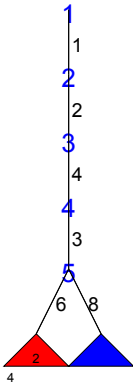
Improved Model



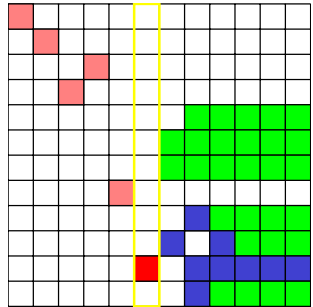
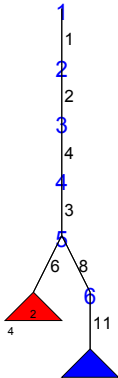
Improved Model



Improved Model



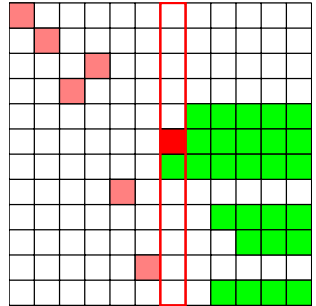
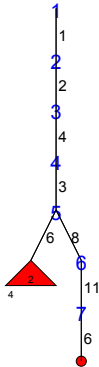
Improved Model



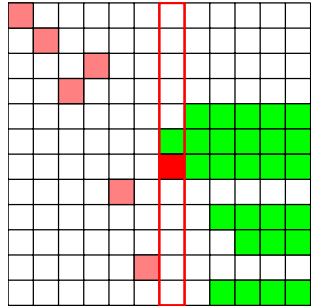
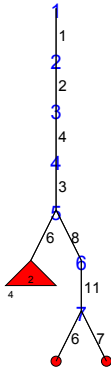
◀ Back to Start

▶▶ Skip Animation

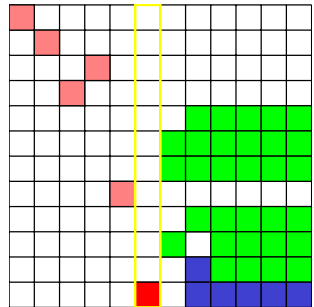
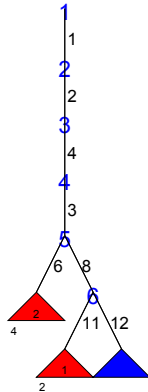
Improved Model



Improved Model



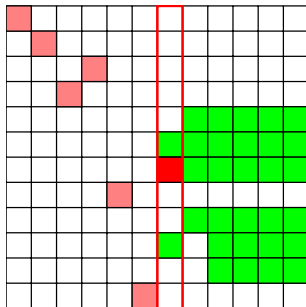
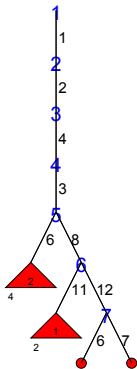
Improved Model



◀ Back to Start

▶▶ Skip Animation

Improved Model

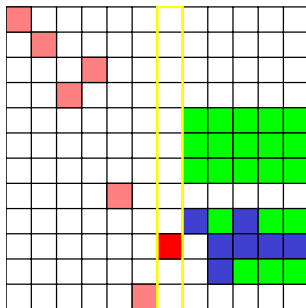
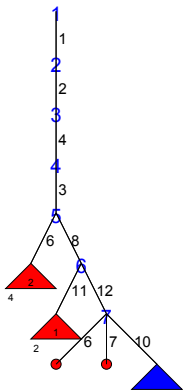


◀ Back to Start

▶▶ Skip Animation



Improved Model

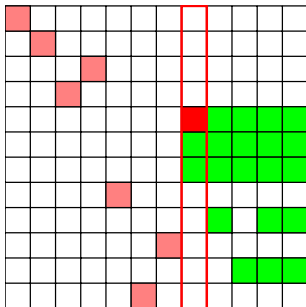
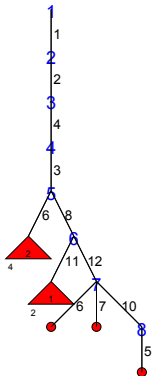


◀ Back to Start

▶▶ Skip Animation



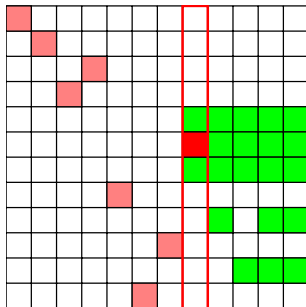
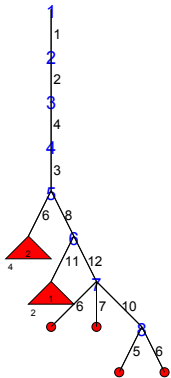
Improved Model



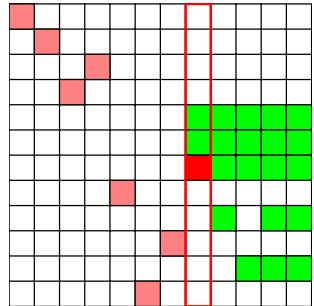
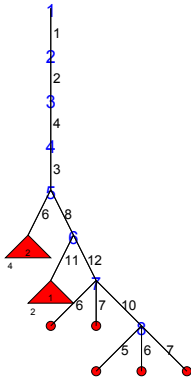
◀ Back to Start

▶▶ Skip Animation

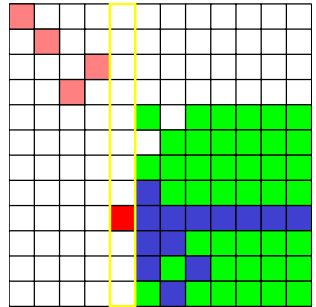
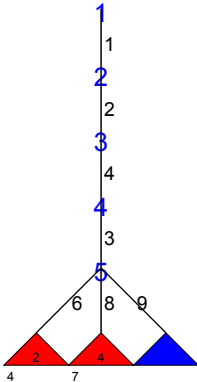
Improved Model



Improved Model



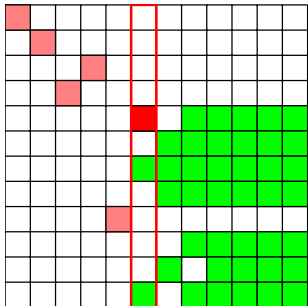
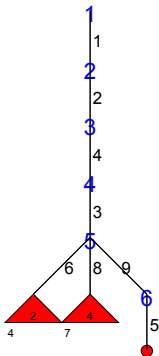
Improved Model



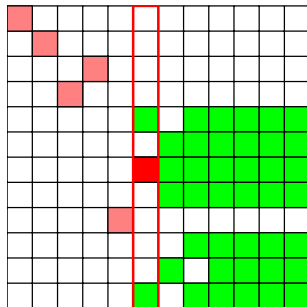
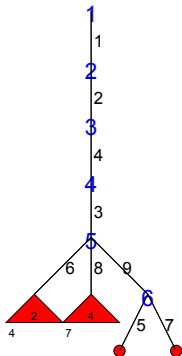
◀ Back to Start

▶▶ Skip Animation

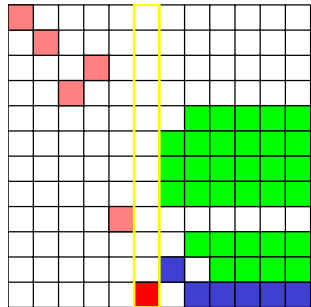
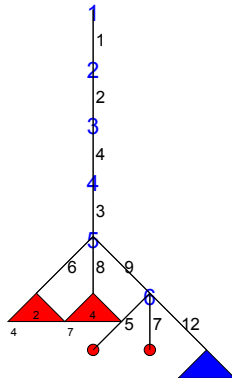
Improved Model



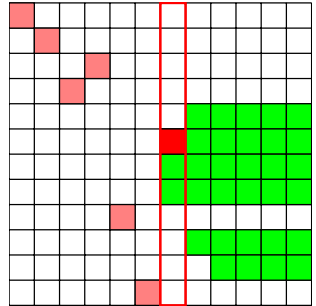
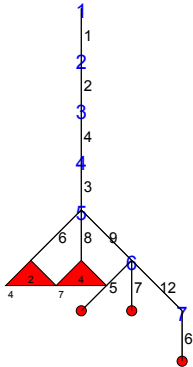
Improved Model



Improved Model



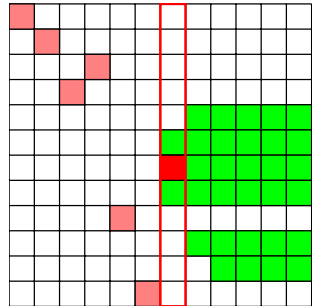
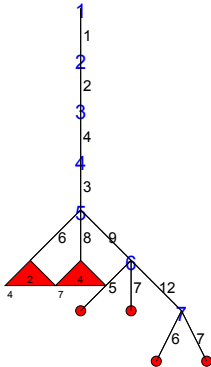
Improved Model



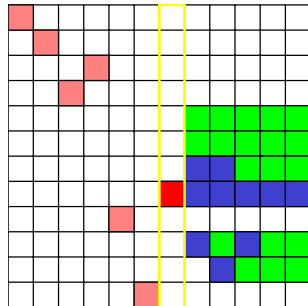
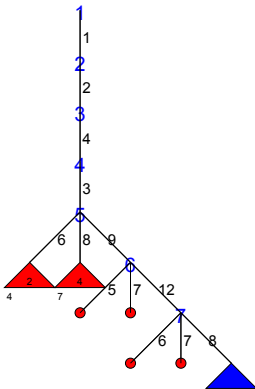
◀ Back to Start

▶▶ Skip Animation

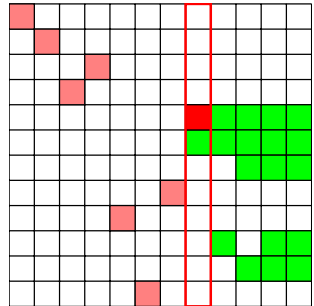
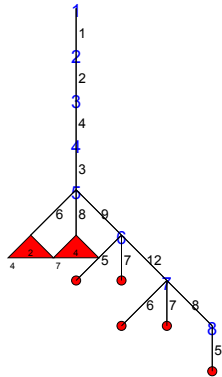
Improved Model



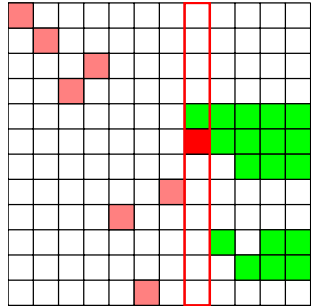
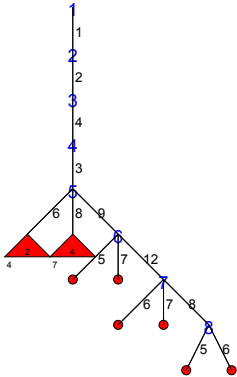
Improved Model



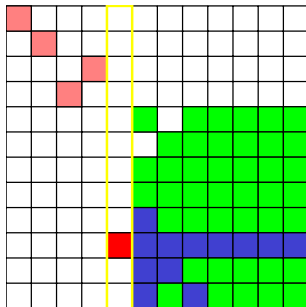
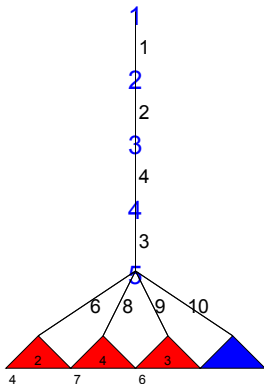
Improved Model



Improved Model



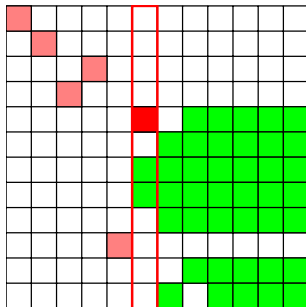
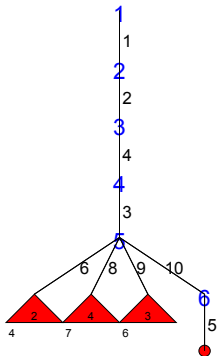
Improved Model



◀ Back to Start ▶ Skip Animation



Improved Model

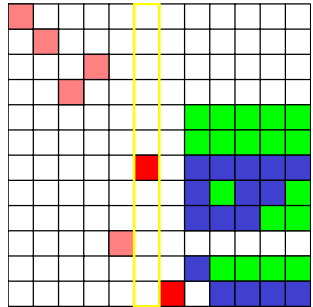
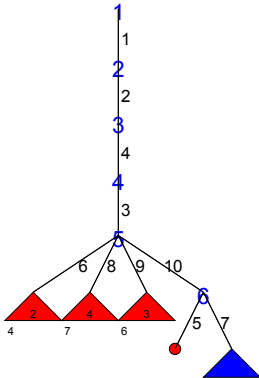


◀ Back to Start

▶▶ Skip Animation



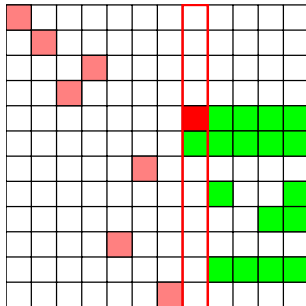
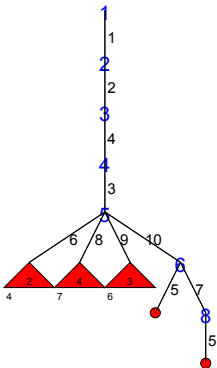
Improved Model



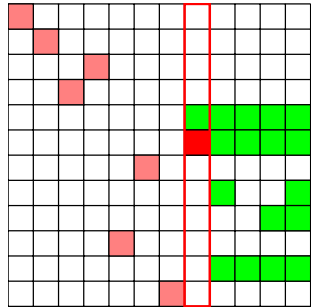
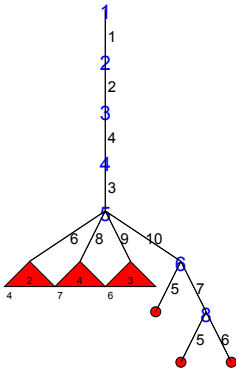
◀ Back to Start

▶▶ Skip Animation

Improved Model



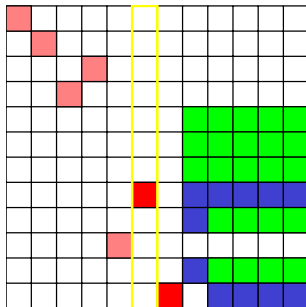
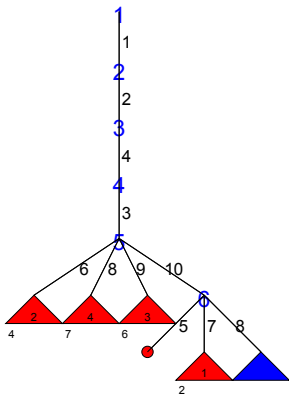
Improved Model



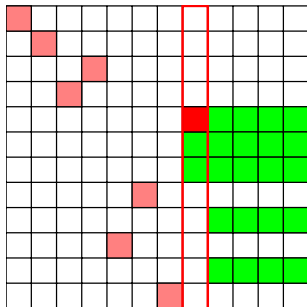
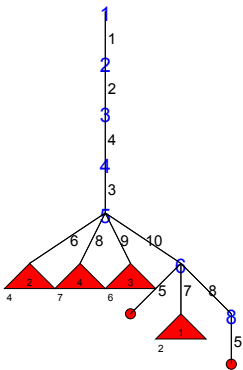
◀ Back to Start

▶▶ Skip Animation

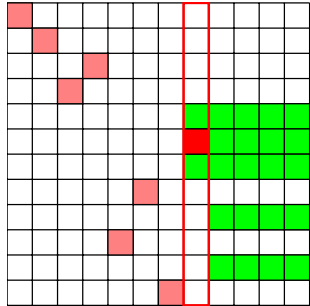
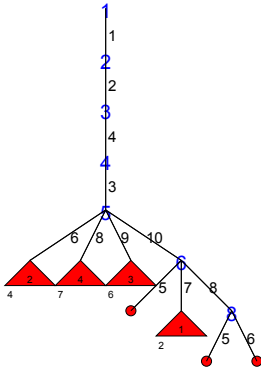
Improved Model



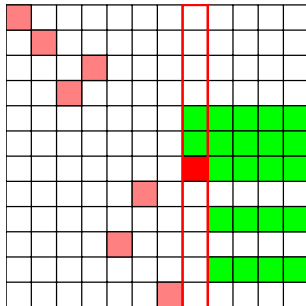
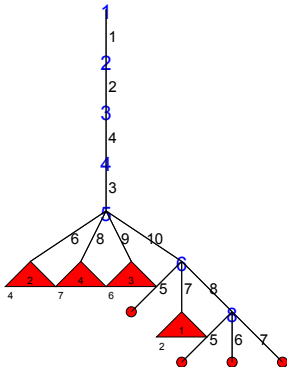
Improved Model



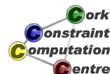
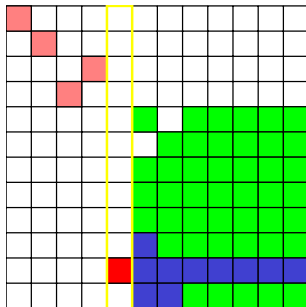
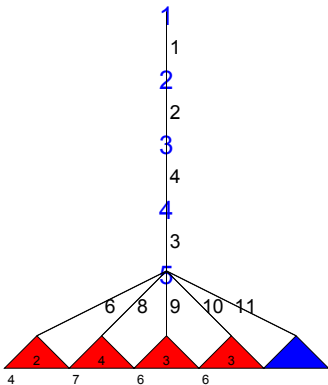
Improved Model



Improved Model



Improved Model

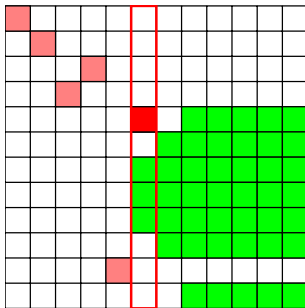
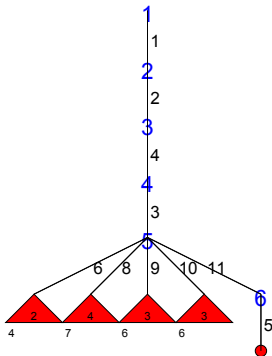


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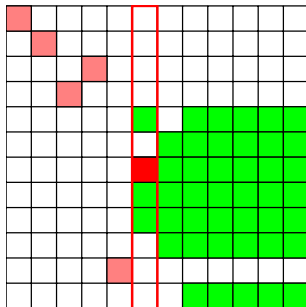
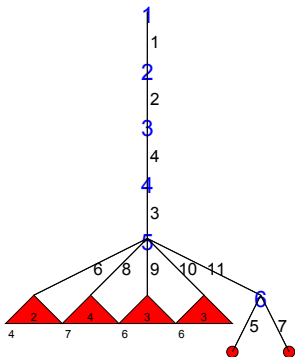
Improved Model



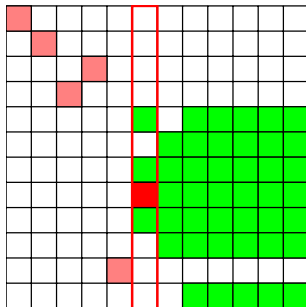
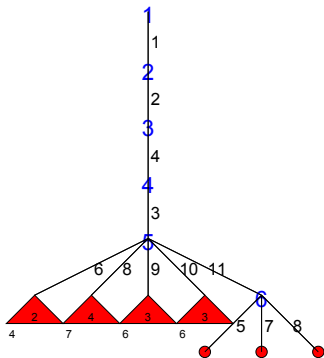
◀ Back to Start

▶▶ Skip Animation

Improved Model



Improved Model

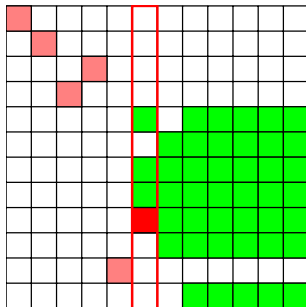
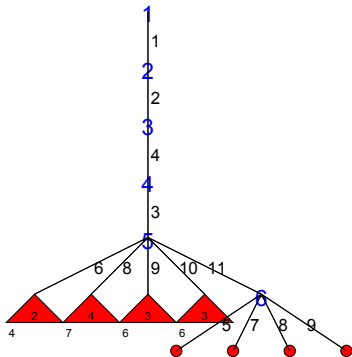


◀ Back to Start

▶▶ Skip Animation



Improved Model

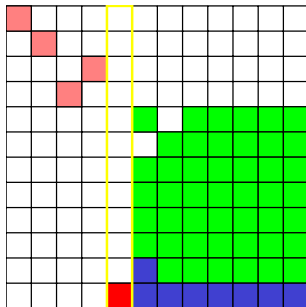
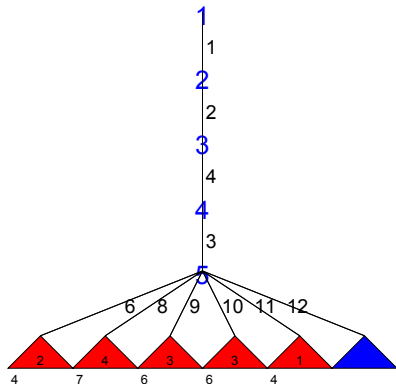


◀ Back to Start

▶▶ Skip Animation



Improved Model

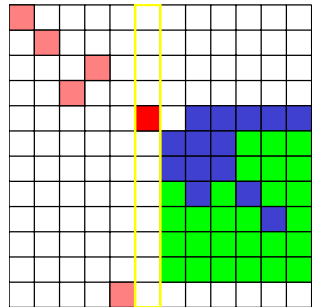
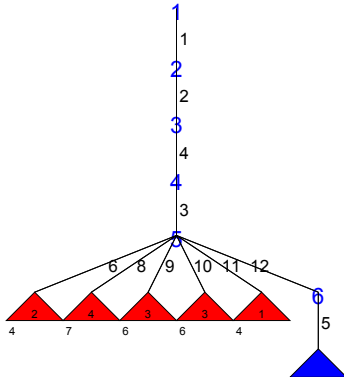


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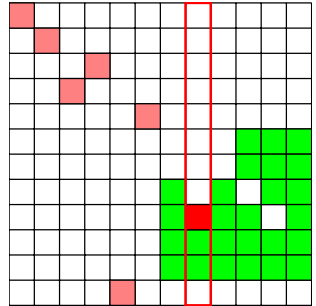
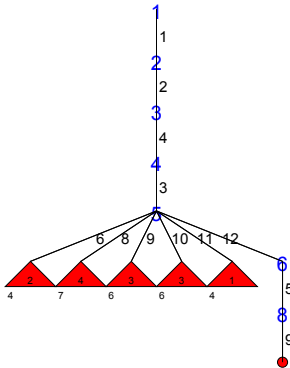
▶▶ Skip Animation



Improved Model



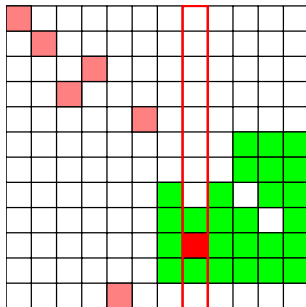
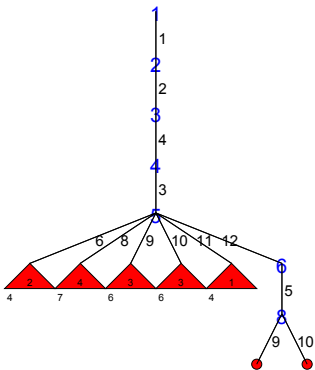
Improved Model



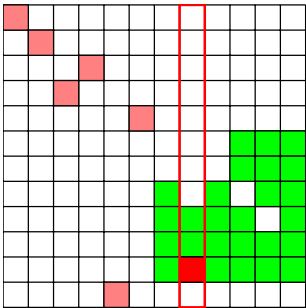
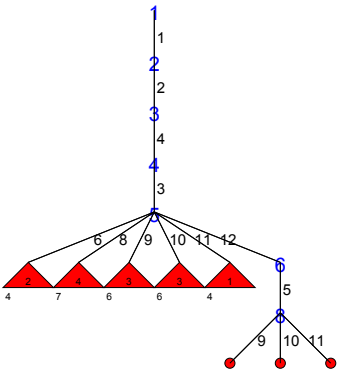
◀ Back to Start

▶▶ Skip Animation

Improved Model



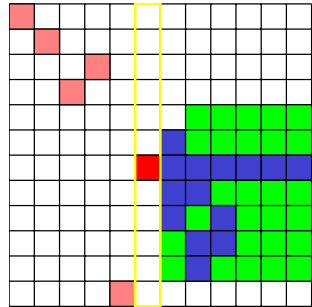
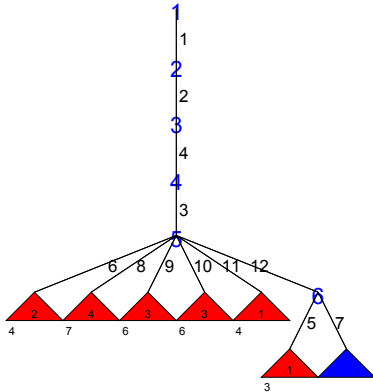
Improved Model



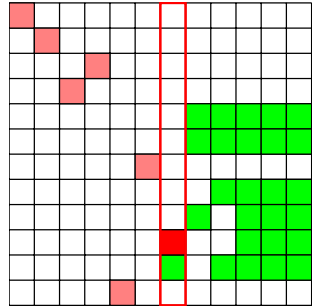
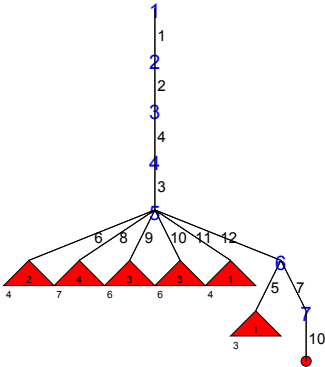
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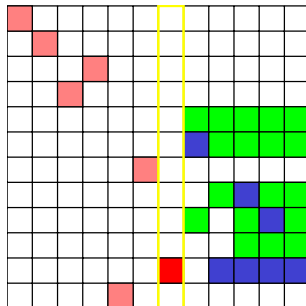
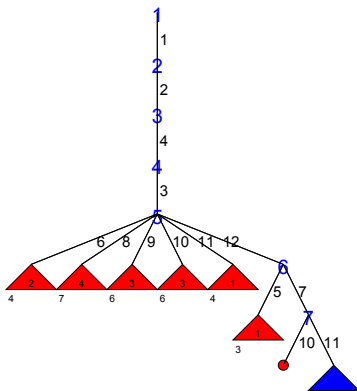
Improved Model



Improved Model



Improved Model

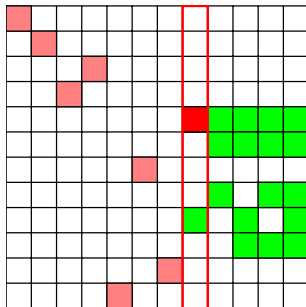
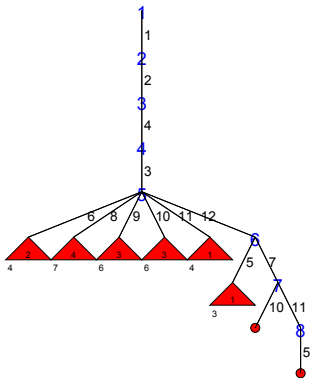


◀ Back to Start

▶▶ Skip Animation



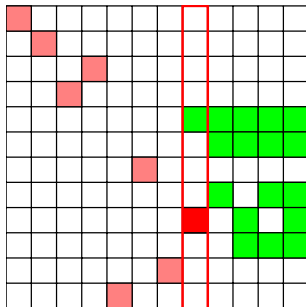
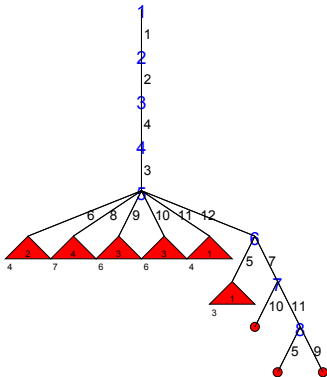
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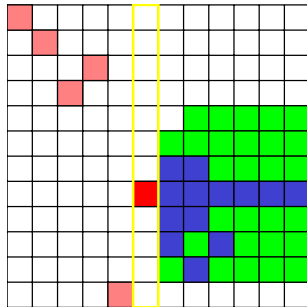
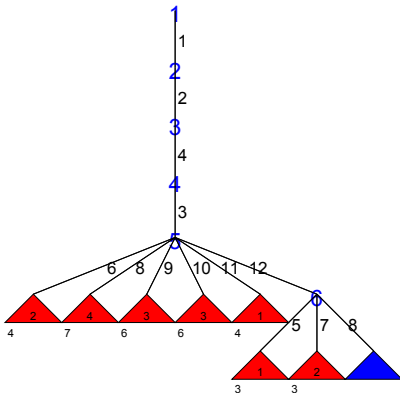
◀ Back to Start

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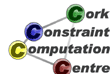
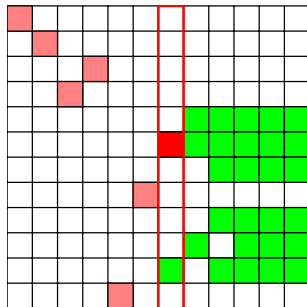
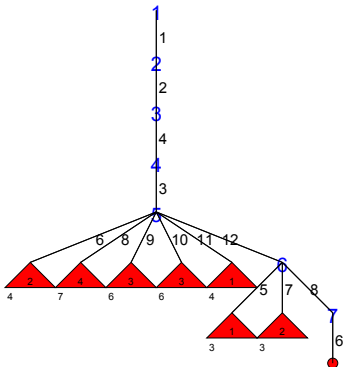
Improved Model



Improved Model



Improved Model

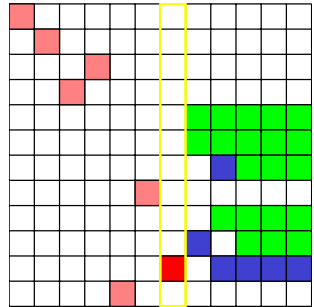
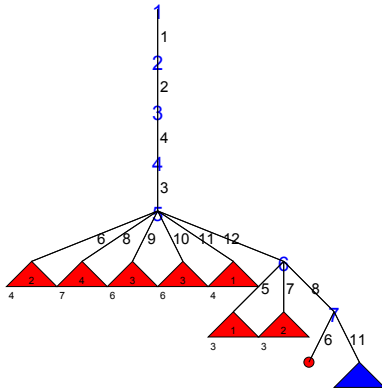


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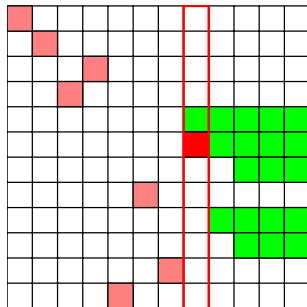
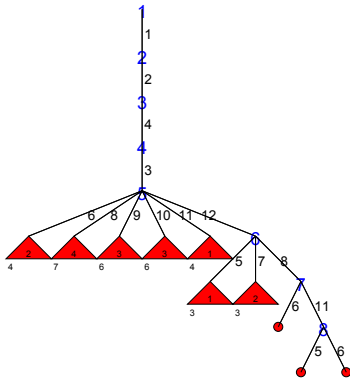
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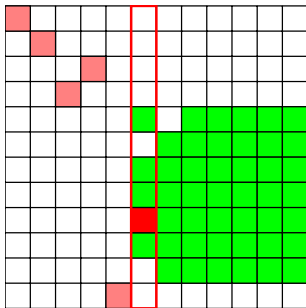
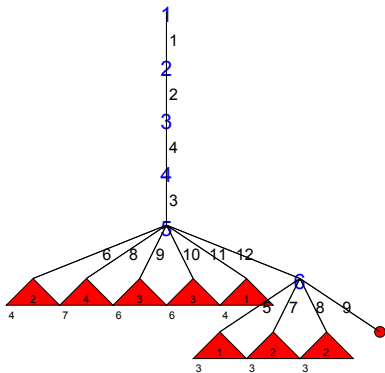
Improved Model



Improved Model



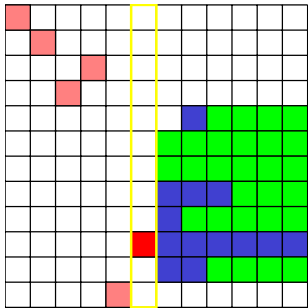
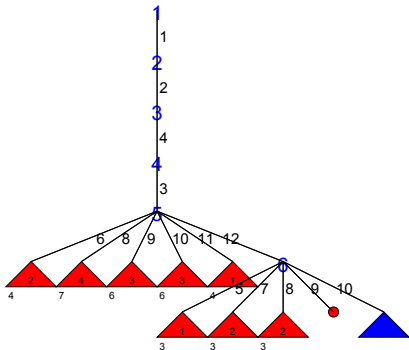
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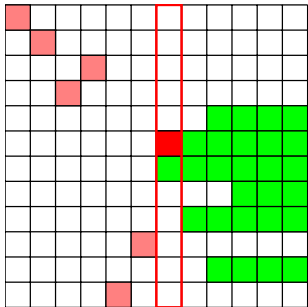
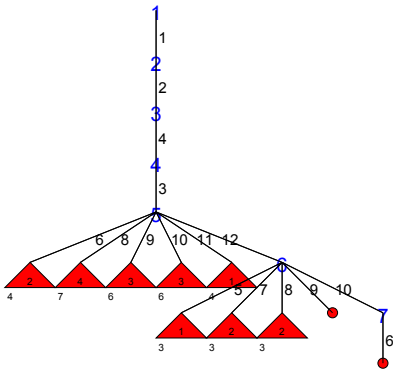
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▶▶ Skip Animation

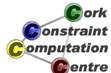
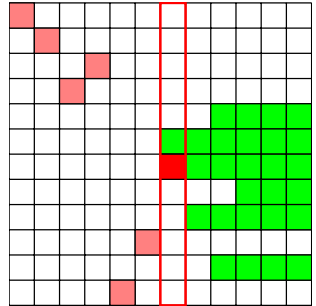
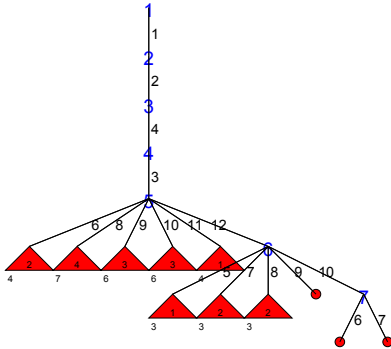
Improved Model



Improved Model



Improved Model

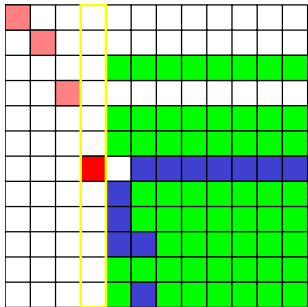
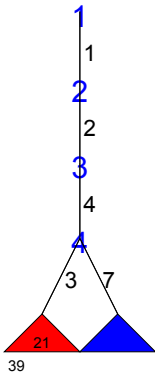


◀ Back to Start

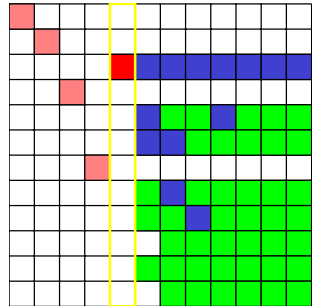
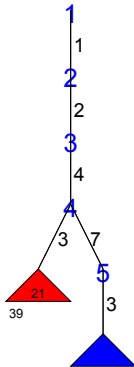
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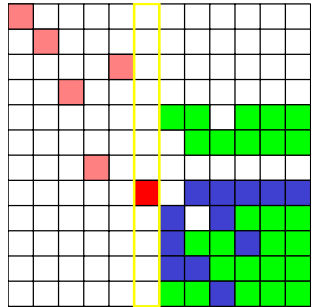
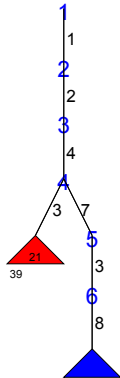
Improved Model



Improved Model



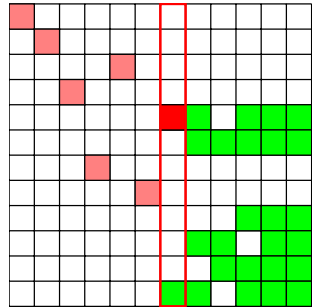
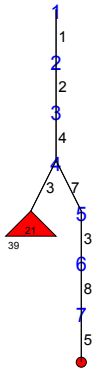
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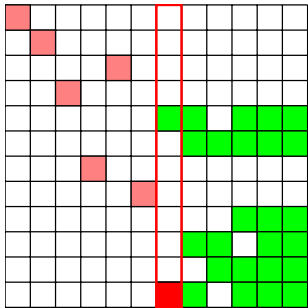
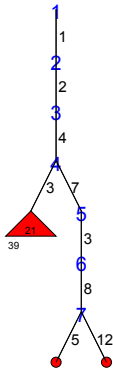
◀ Back to Start

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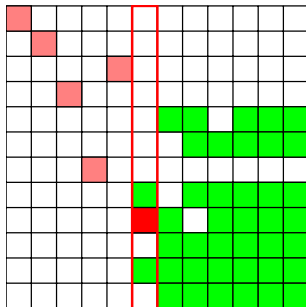
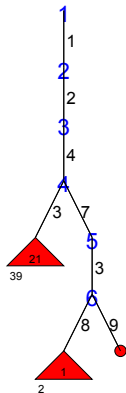
Improved Model



Improved Model

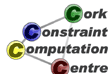


Improved Model

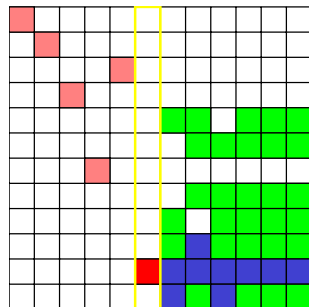
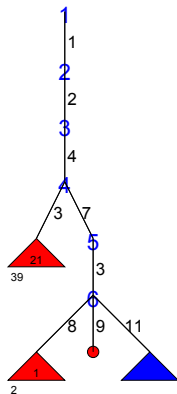


◀ Back to Start

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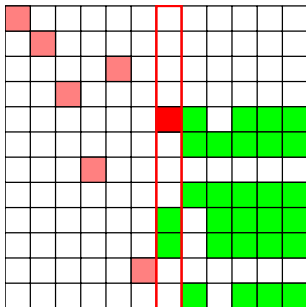
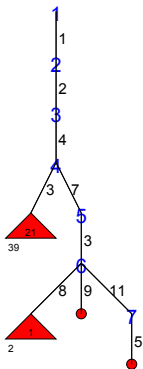
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◀ Back to Start

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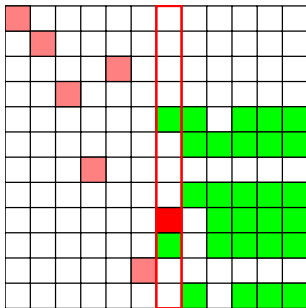
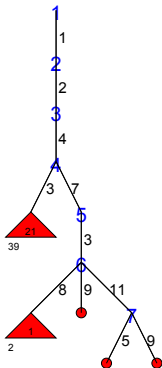
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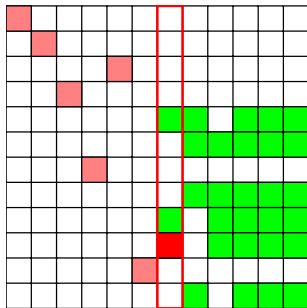
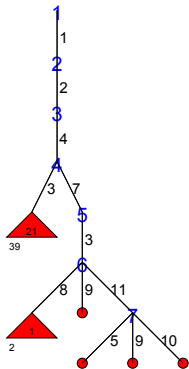
◀ Back to Start

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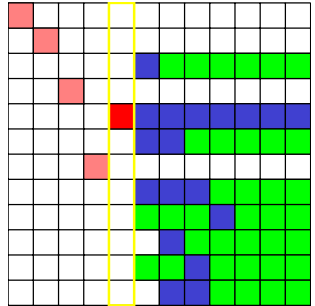
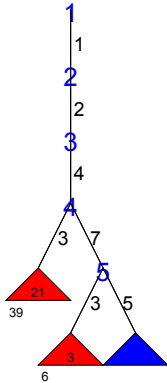
Improved Model



Improved Model



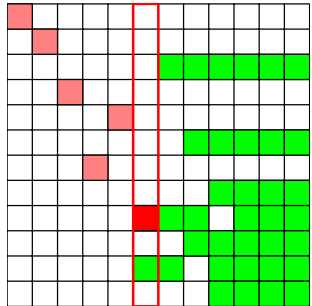
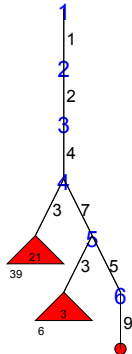
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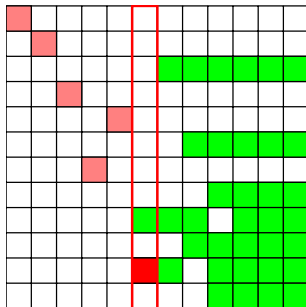
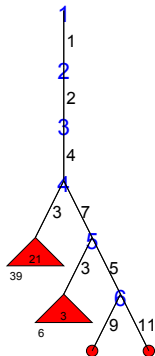
◀ Back to Start

▶ Skip Animation

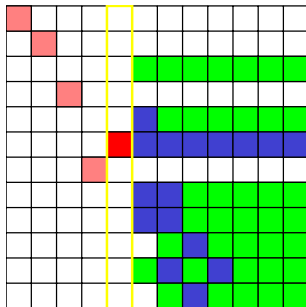
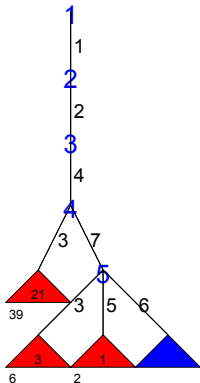
Improved Model



Improved Model



Improved Model

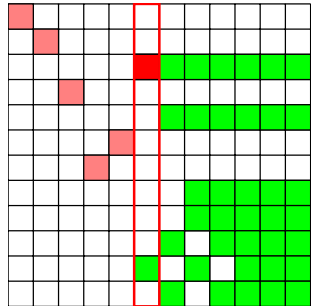
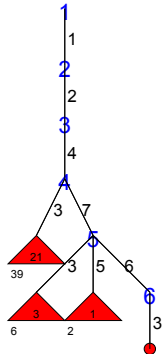


◀ Back to Start

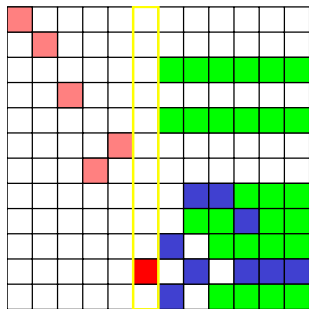
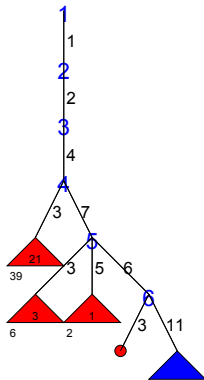
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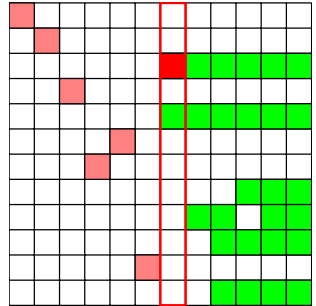
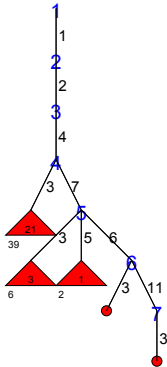
Improved Model



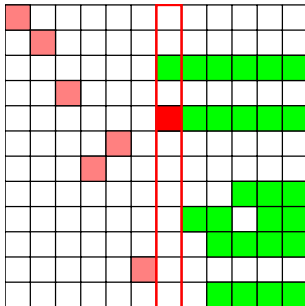
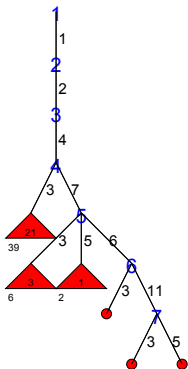
Improved Model



Improved Model



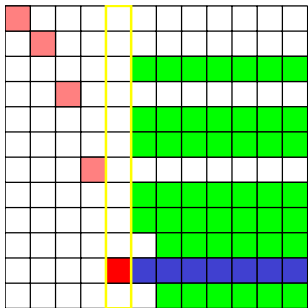
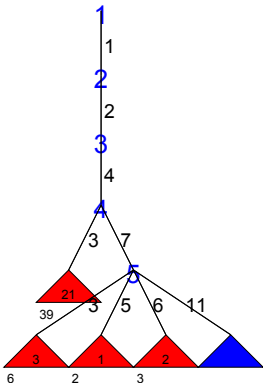
Improved Model



◀ Back to Start

▶▶ Skip Animation

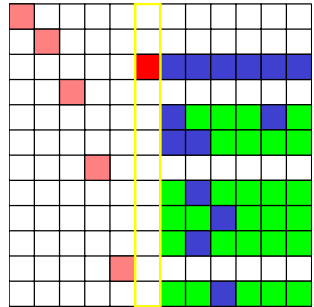
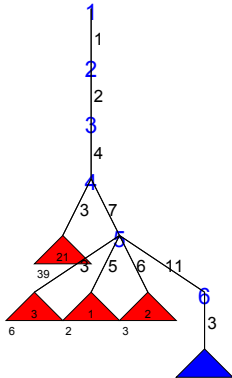
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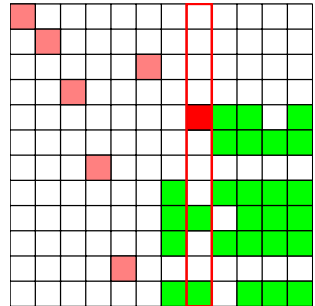
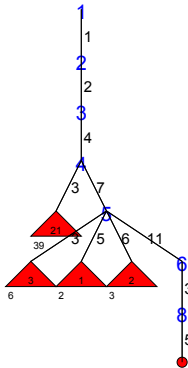
◀ Back to Start

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Improved Model



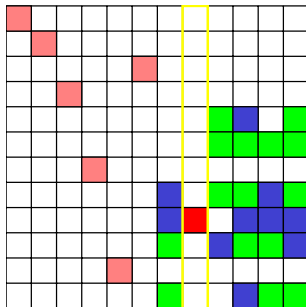
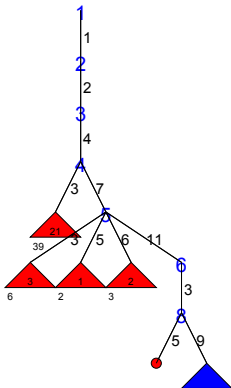
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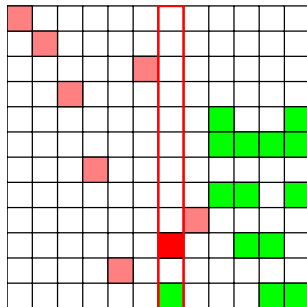
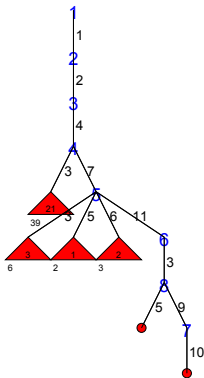
◀ Back to Start

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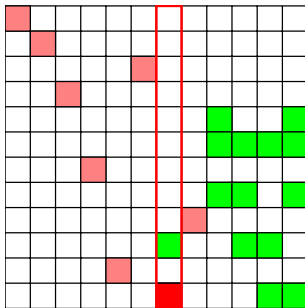
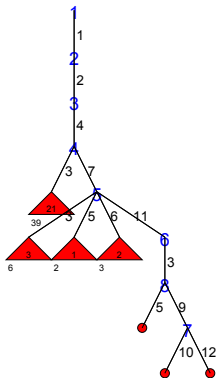
Improved Model



Improved Model



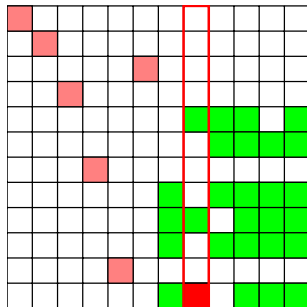
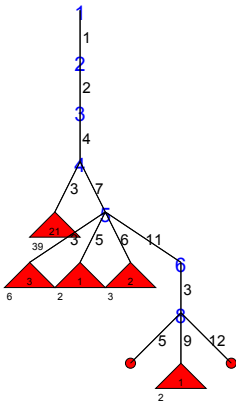
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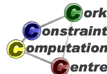
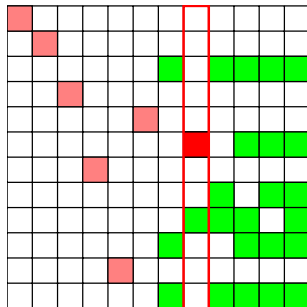
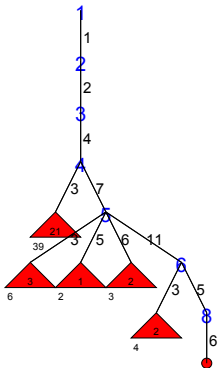
◀ Back to Start

▶▶ Skip Animation

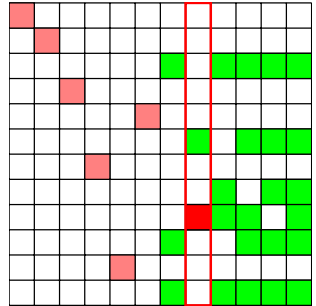
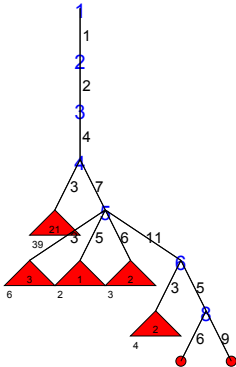
Improved Model



Improved Model



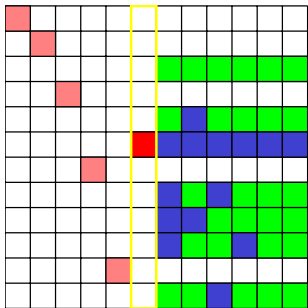
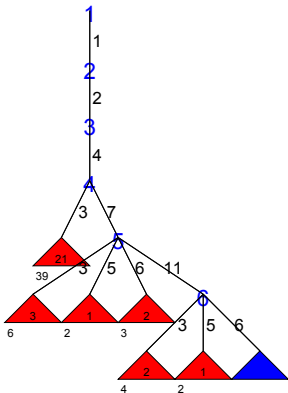
Improved Model



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▶▶ Skip Animation

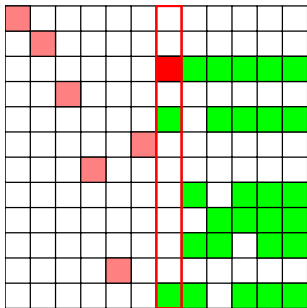
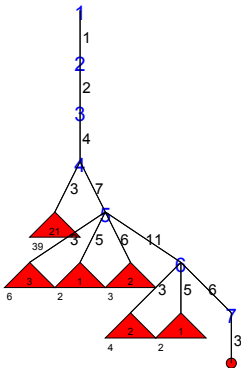
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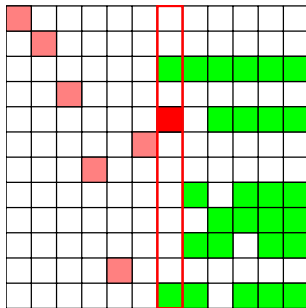
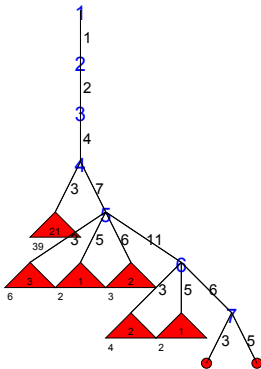
◀ Back to Start

▶▶ Skip Animation

Improved Model



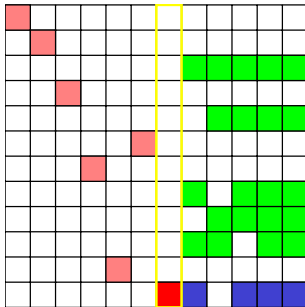
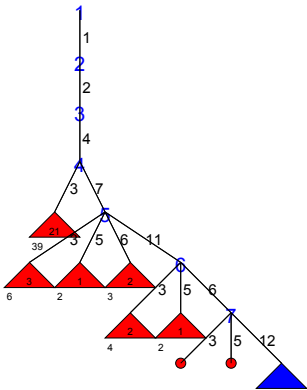
Improved Model



◀ Back to Start

▶▶ Skip Animation

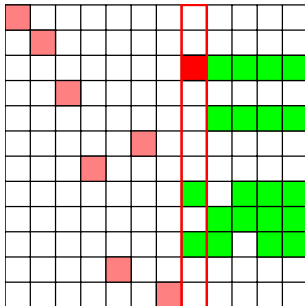
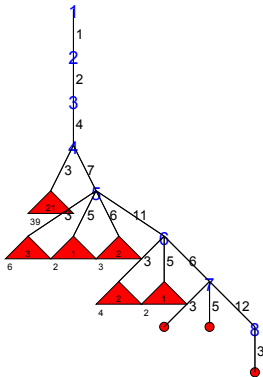
Improved Model



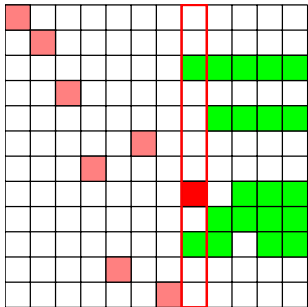
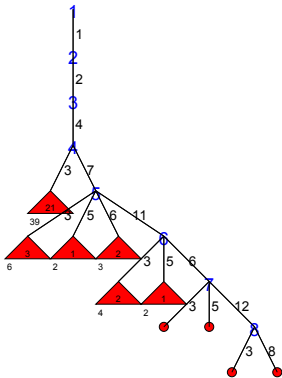
◀ Back to Start

▶▶ Skip Animation

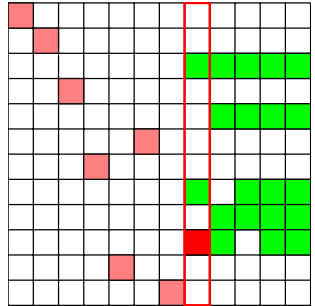
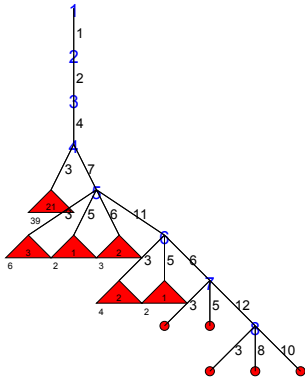
Improved Model



Improved Model



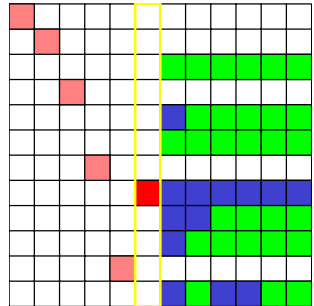
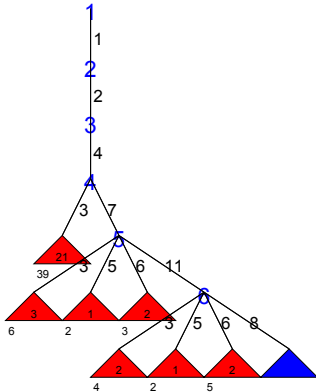
Improved Model



◀ Back to Start

▶▶ Skip Animation

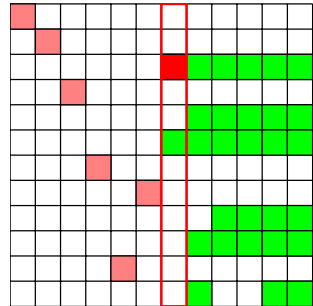
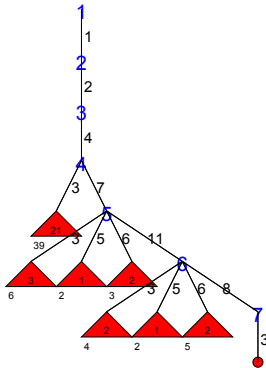
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◀ Back to Start

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Improved Model

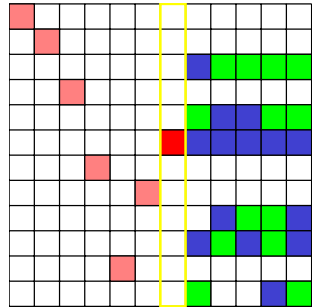
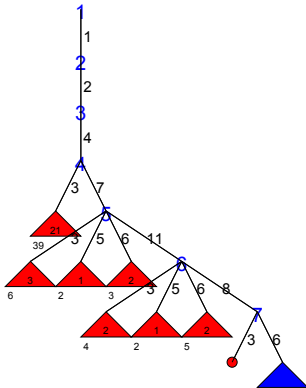


◀ Back to Start

▶▶ Skip Animation



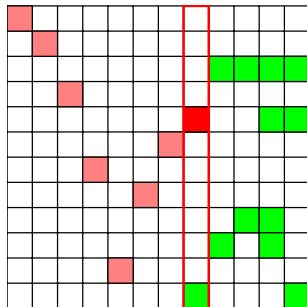
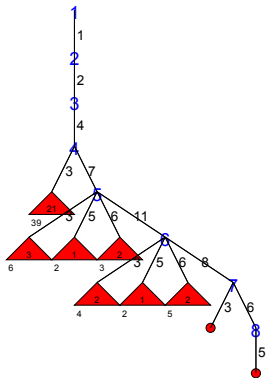
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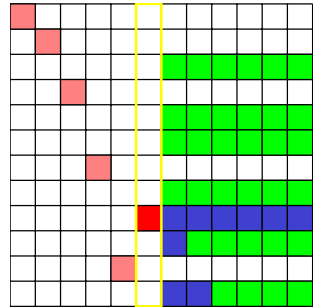
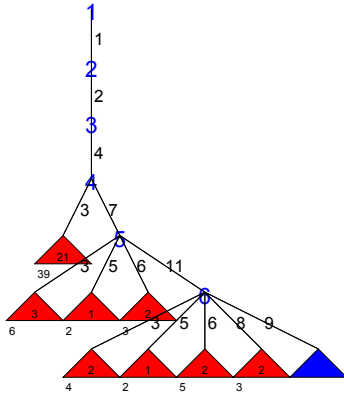
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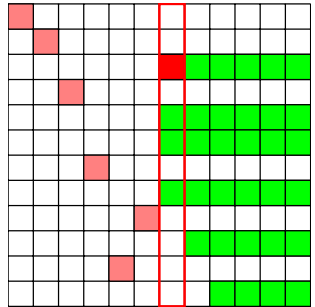
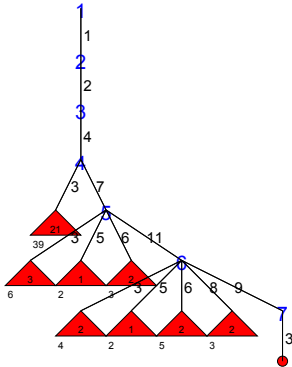
Improved Model



Improved Model



Improved Model

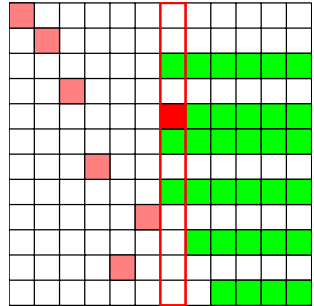
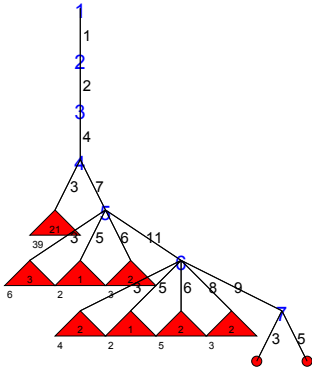


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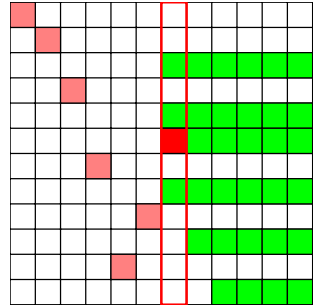
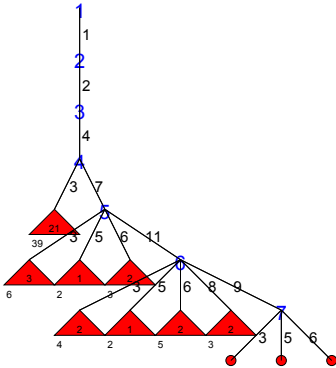
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Improved Model



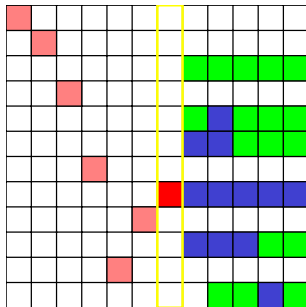
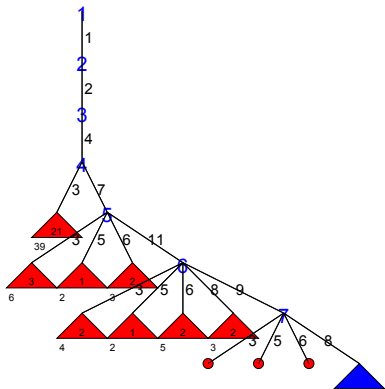
Improved Model



◀ Back to Start

▶ Skip Animation

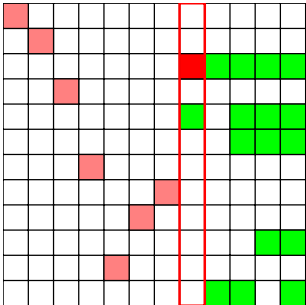
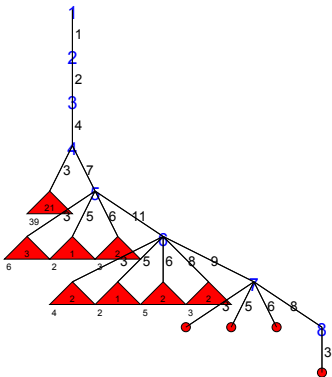
Improved Model



◀ Back to Start

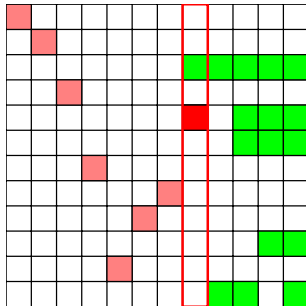
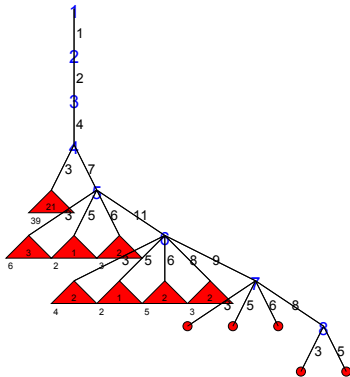
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Improved Model



◀ Back to Start ▶ Skip Animation

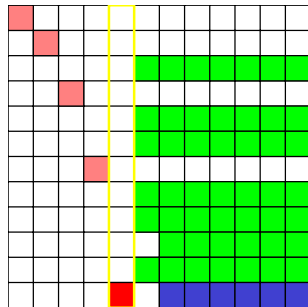
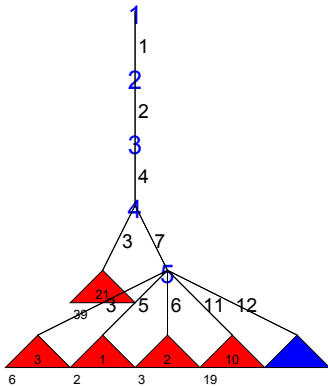
Improved Model



◀ Back to Start ▶ Skip Animation

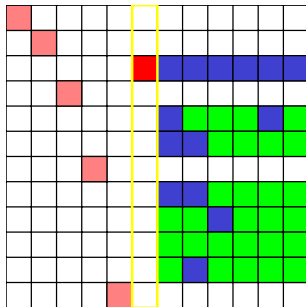
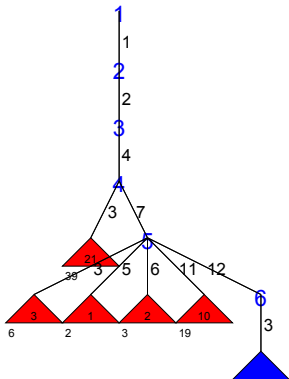


Improved Model



◀ Back to Start ▶ Skip Animation

Improved Model

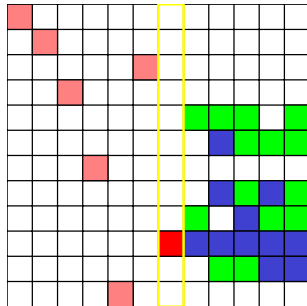
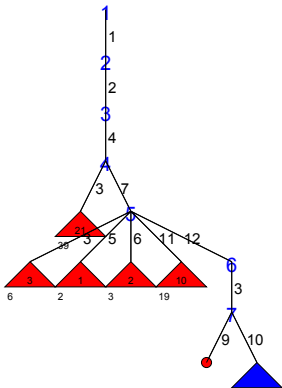


◀ Back to Start

▶ Skip Animation



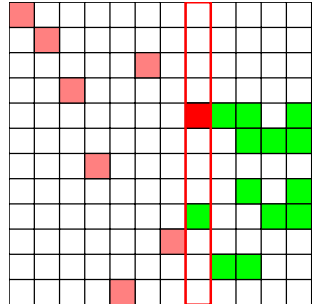
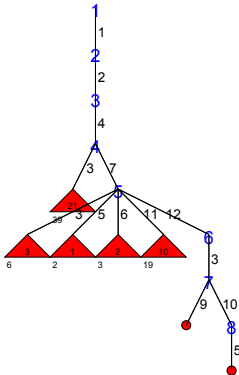
Improved Model



◀ Back to Start

▶ Skip Animation

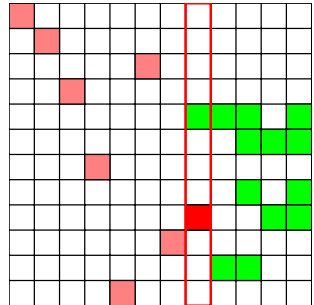
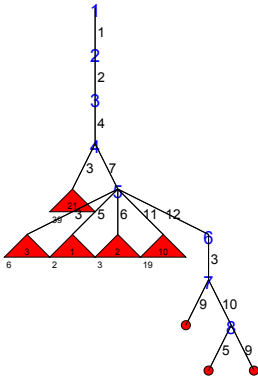
Improved Model



◀ Back to Start

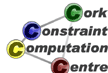
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Improved Model

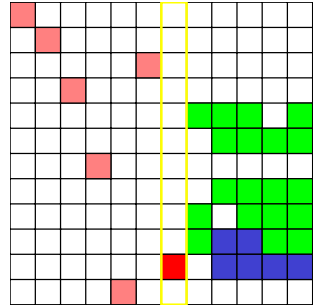
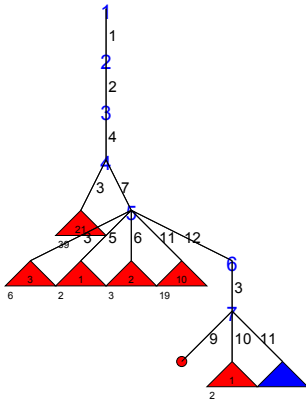


◀ Back to Start

▶ Skip Animation



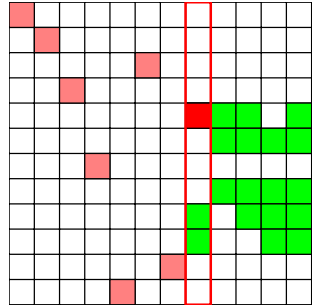
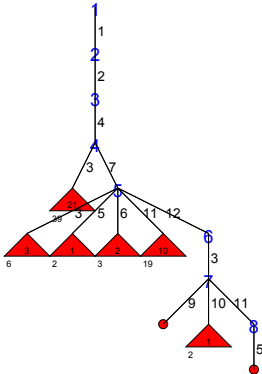
Improved Model



◀ Back to Start

▶ Skip Animation

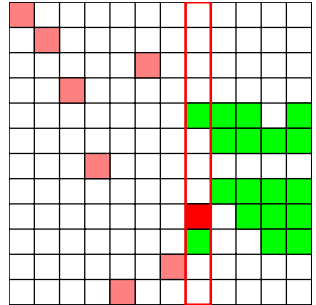
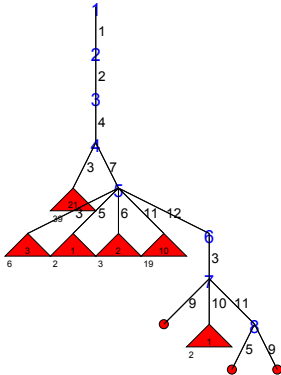
Improved Model



◀ Back to Start

▶ Skip Animation

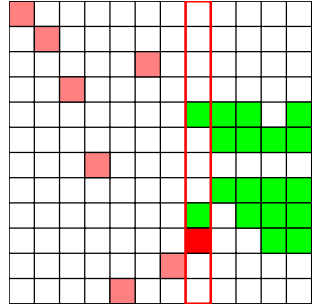
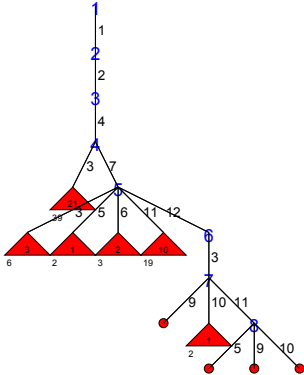
Improved Model



◀ Back to Start

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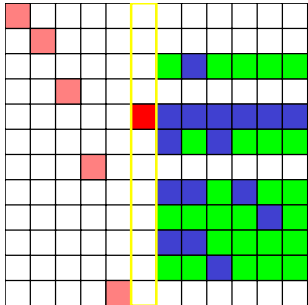
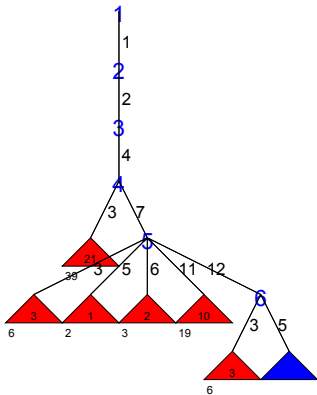
Improved Model



◀ Back to Start

▶ Skip Animation

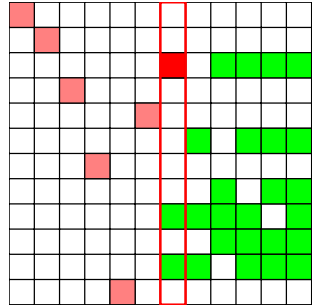
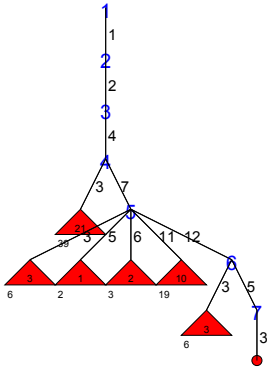
Improved Model



◀ Back to Start

▶ Skip Animation

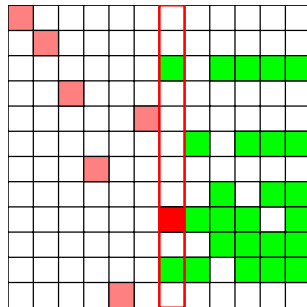
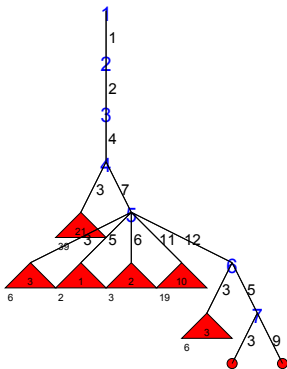
Improved Model



◀ Back to Start

▶ Skip Animation

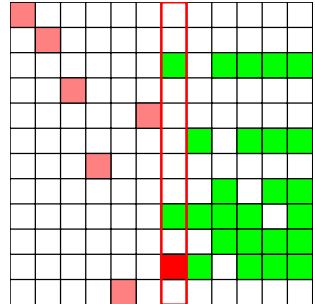
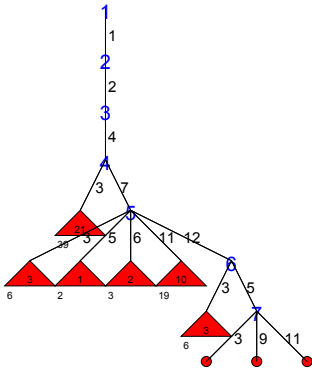
Improved Model



◀ Back to Start

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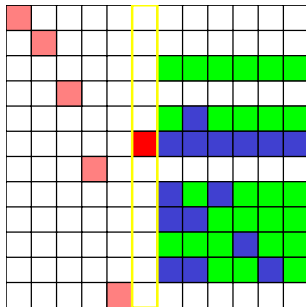
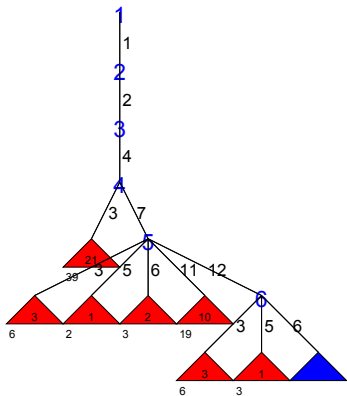
Improved Model



◀ Back to Start

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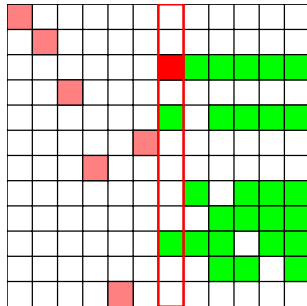
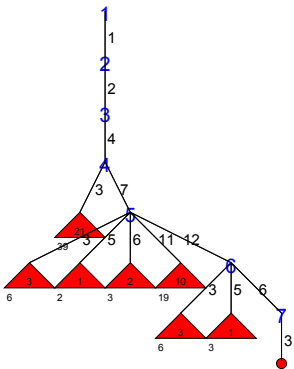
Improved Model



◀ Back to Start

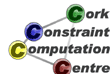
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Improved Model

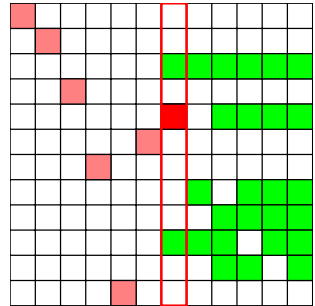
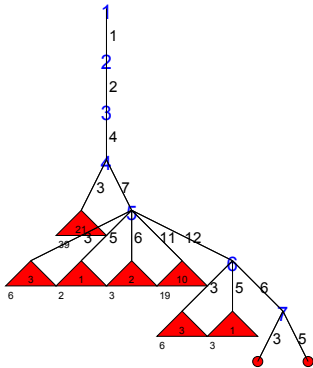


◀ Back to Start

▶ Skip Animation



Improved Model

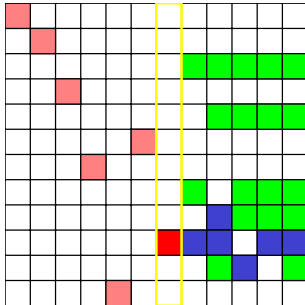
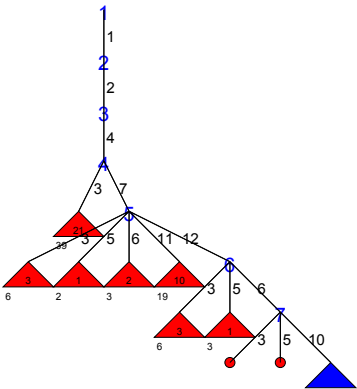


◀ Back to Start

▶ Skip Animation



Improved Model

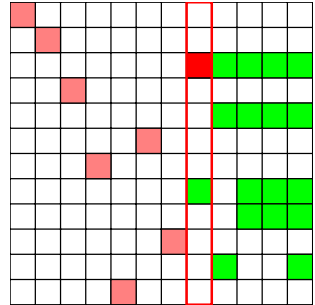
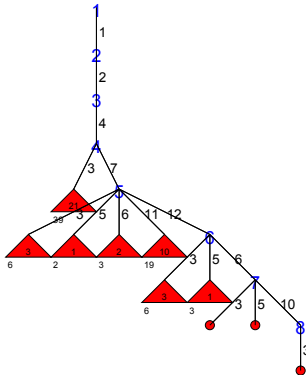


◀ Back to Start

▶ Skip Animation



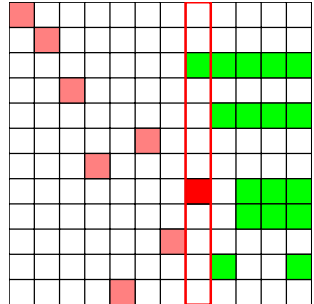
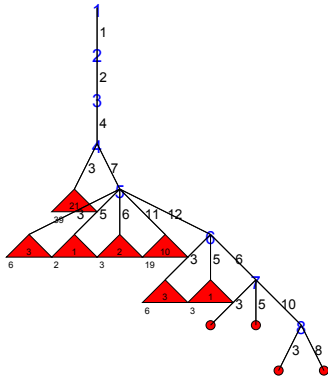
Improved Model



◀ Back to Start

▶ Skip Animation

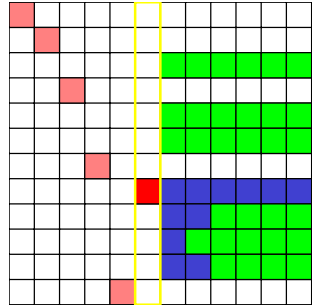
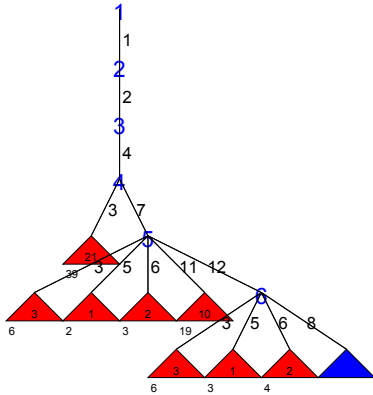
Improved Model



◀ Back to Start

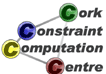
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Improved Model

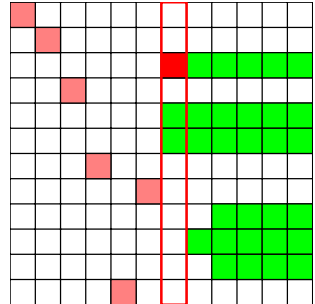
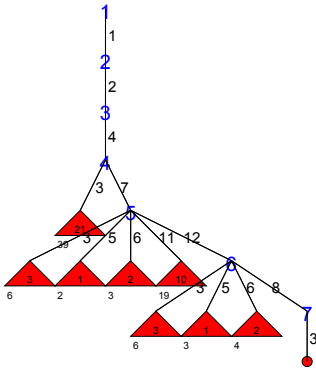


◀ Back to Start

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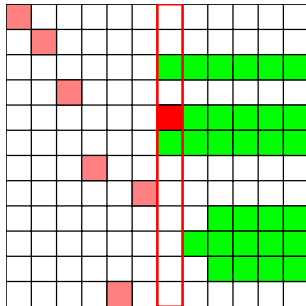
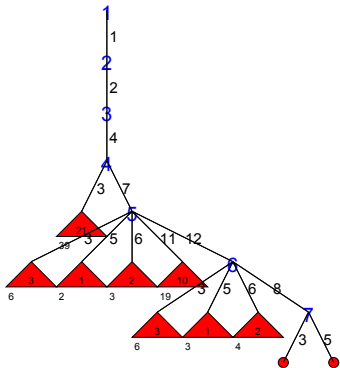
Improved Model



◀ Back to Start

▶ Skip Animation

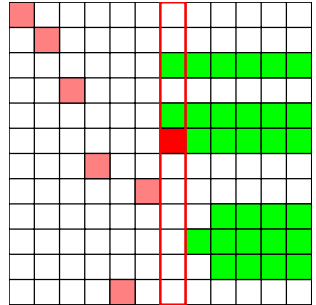
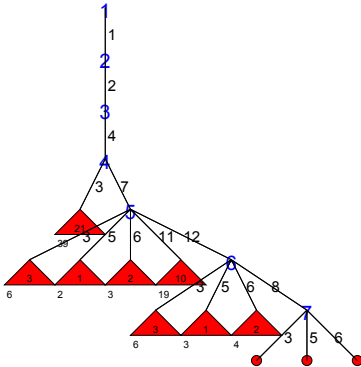
Improved Model



◀ Back to Start

▶ Skip Animation

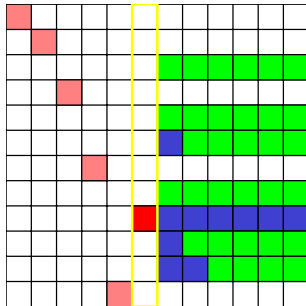
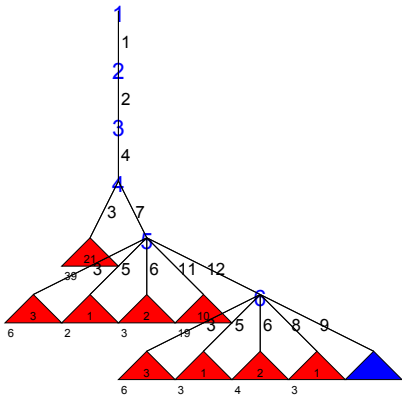
Improved Model



◀ Back to Start

▶ Skip Animation

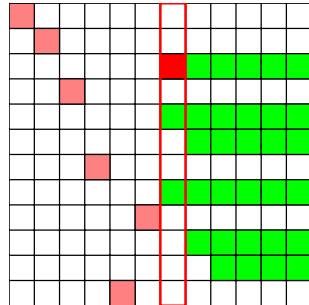
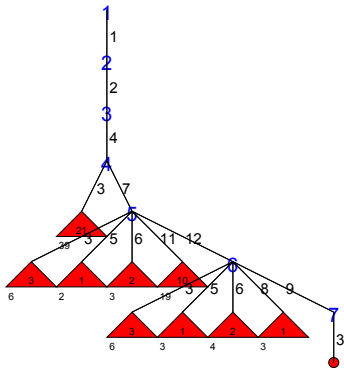
Improved Model



◀ Back to Start

▶ Skip Animation

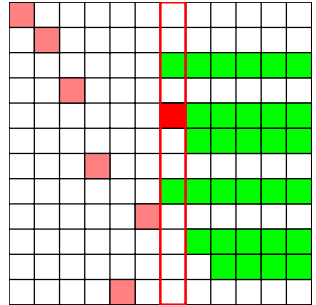
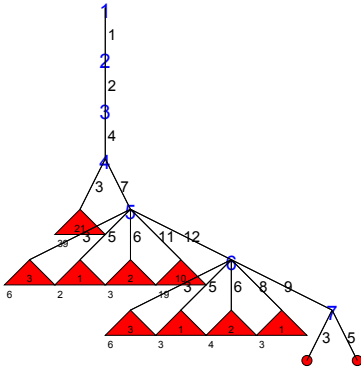
Improved Model



◀ Back to Start

▶ Skip Animation

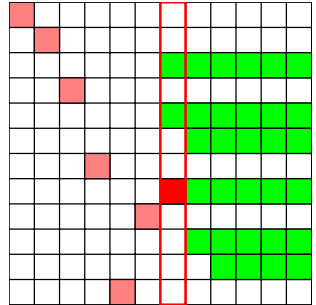
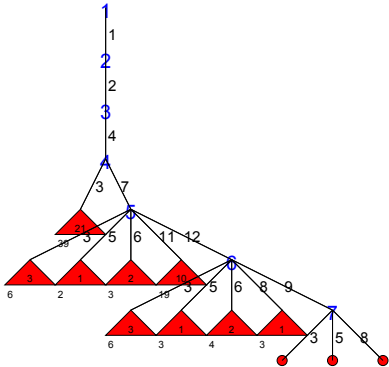
Improved Model



◀ Back to Start

▶ Skip Animation

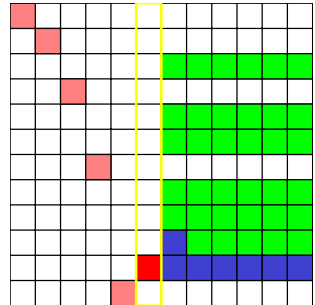
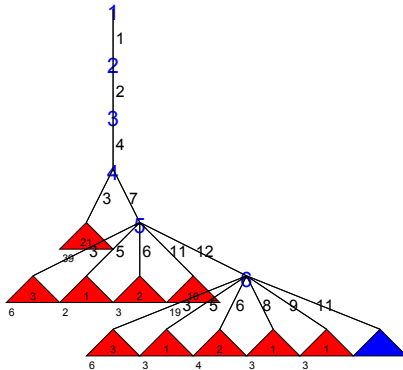
Improved Model



◀ Back to Start

▶ Skip Animation

Improved Model

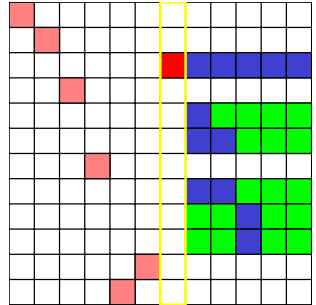
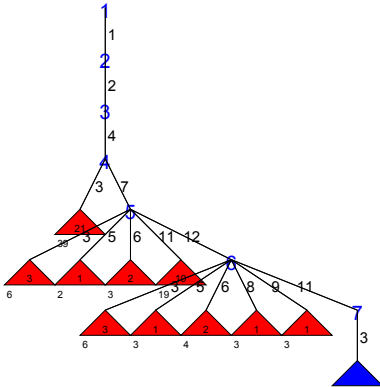


◀ Back to Start

▶ Skip Animation



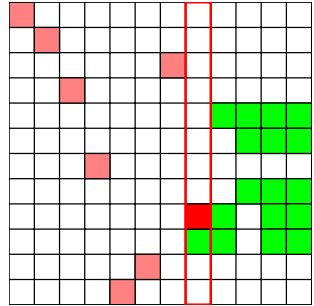
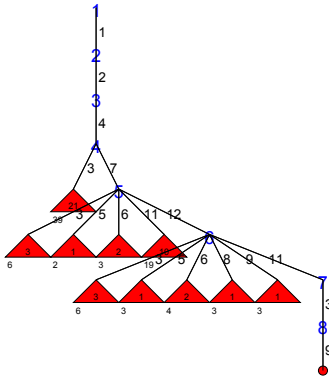
Improved Model



◀ Back to Start

▶ Skip Animation

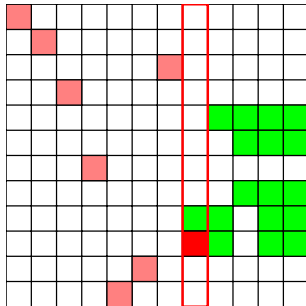
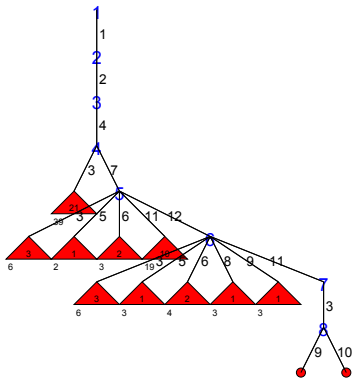
Improved Model



◀ Back to Start

▶▶ Skip Animation

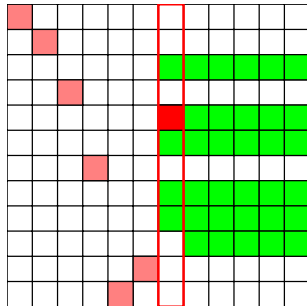
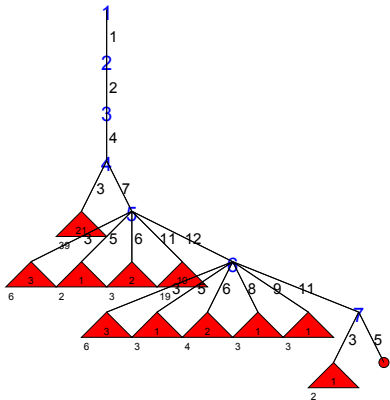
Improved Model



◀ Back to Start

▶ Skip Animation

Improved Model

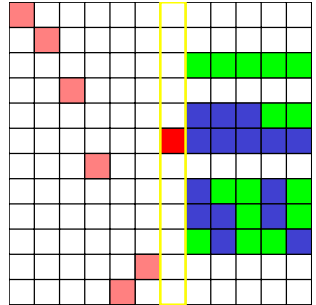
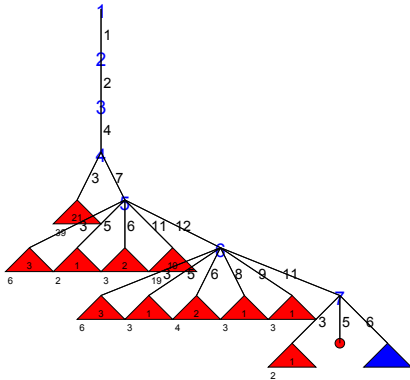


◀ Back to Start

▶ Skip Animation



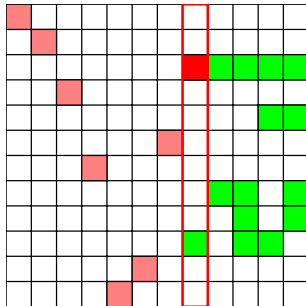
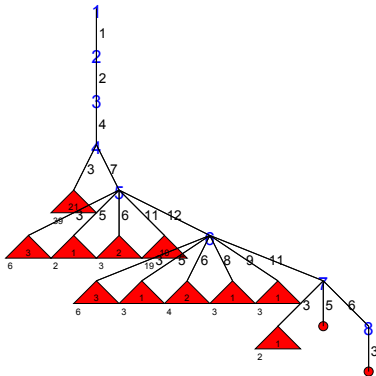
Improved Model



◀ Back to Start

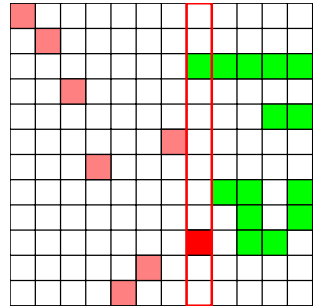
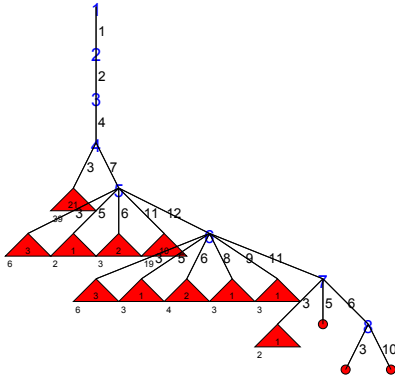
▶ Skip Animation

Improved Model



◀ Back to Start ▶ Skip Animation

Improved Model

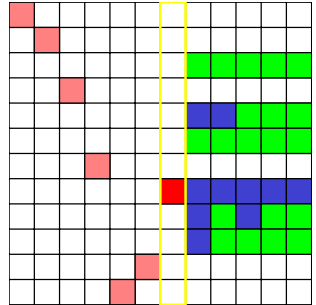
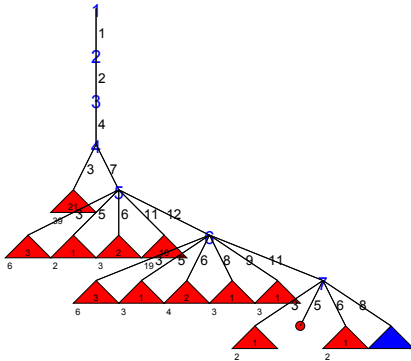


◀ Back to Start

▶▶ Skip Animation



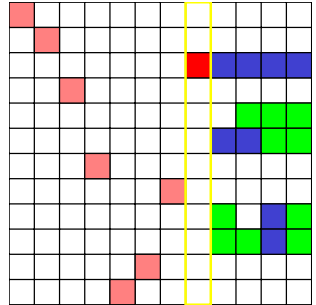
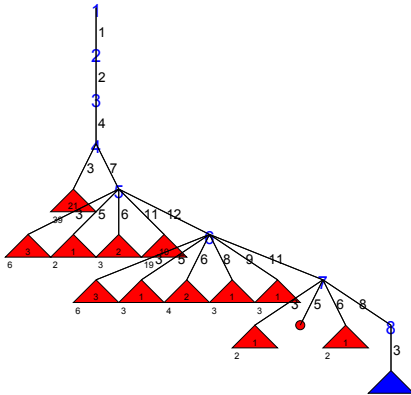
Improved Model



◀ Back to Start

▶ Skip Animation

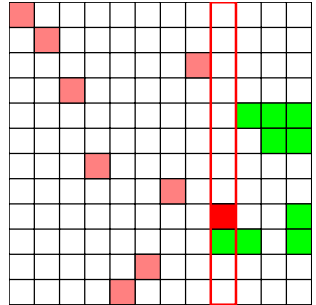
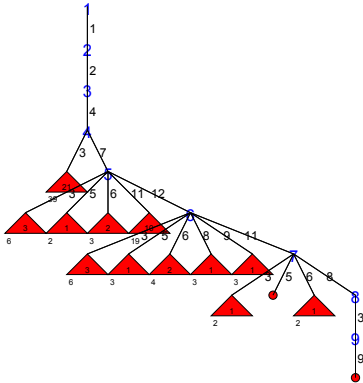
Improved Model



◀ Back to Start

▶ Skip Animation

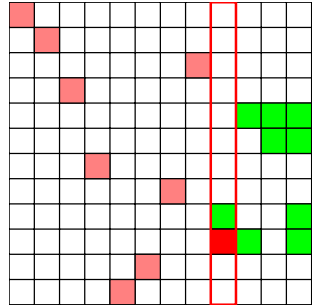
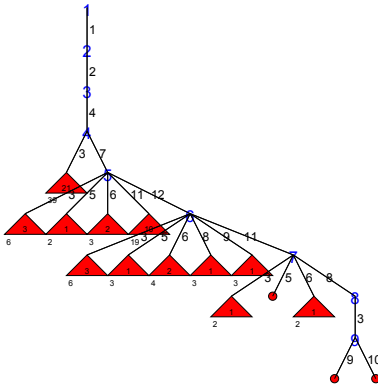
Improved Model



◀ Back to Start

▶ Skip Animation

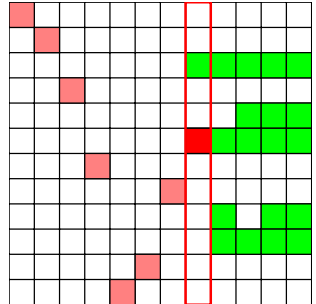
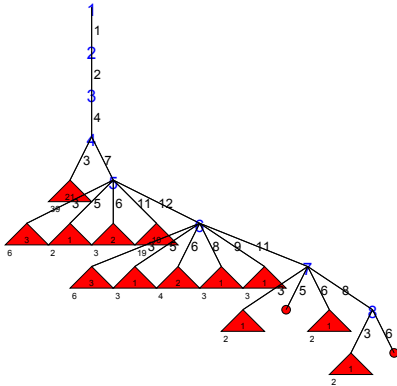
Improved Model



◀ Back to Start

▶ Skip Animation

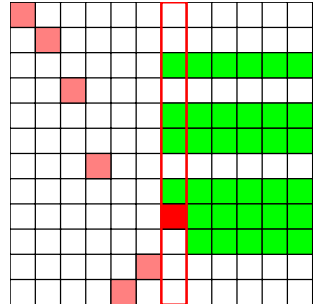
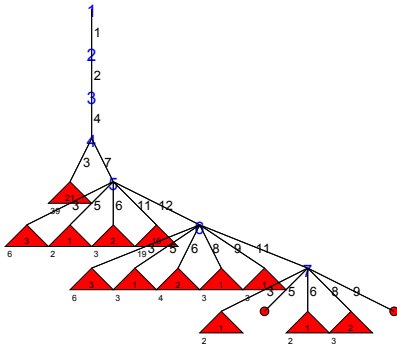
Improved Model



◀ Back to Start

▶▶ Skip Animation

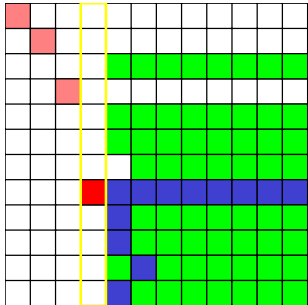
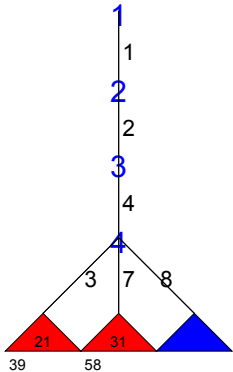
Improved Model



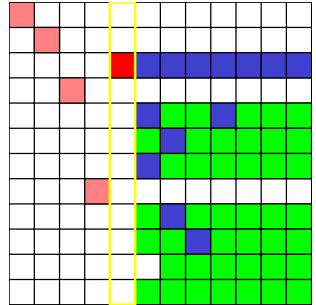
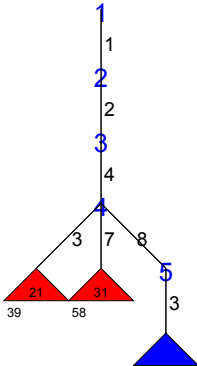
◀ Back to Start

▶ Skip Animation

Improved Model



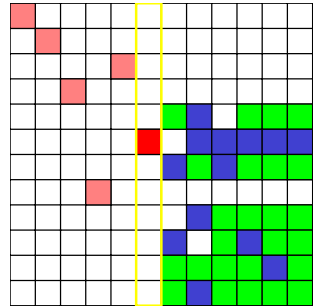
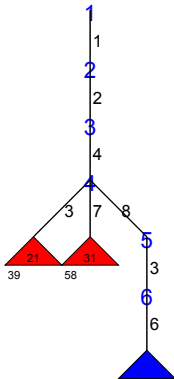
Improved Model



◀ Back to Start

▶ Skip Animation

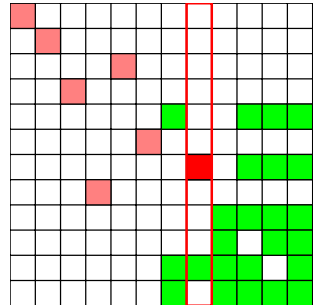
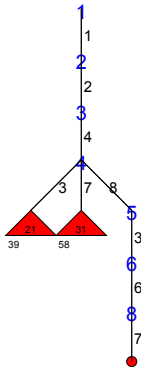
Improved Model



◀ Back to Start

▶ Skip Animation

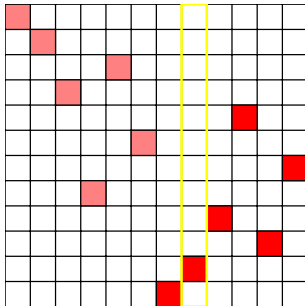
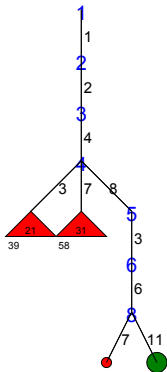
Improved Model



◀ Back to Start

▶ Skip Animation

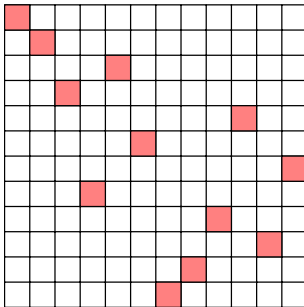
Improved Model



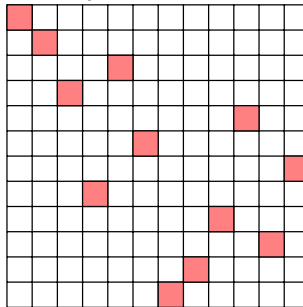
◀ Back to Start

Comparison (Solutions)

Initial Model

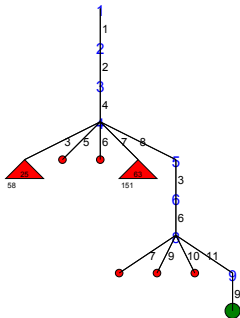


Improved Model

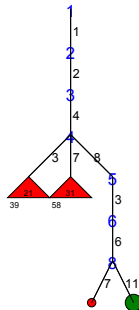


Comparison (Search Trees)

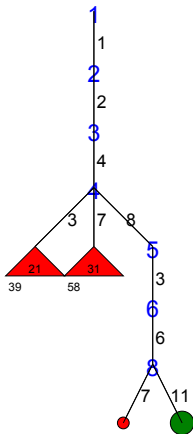
Initial Model



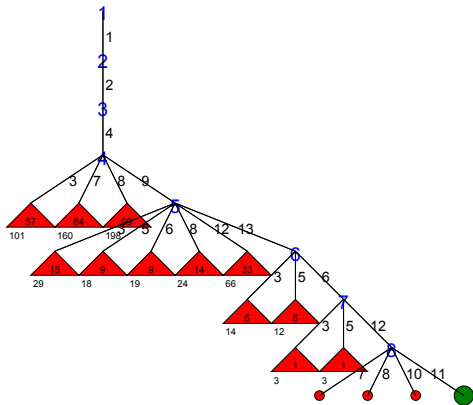
Improved Model



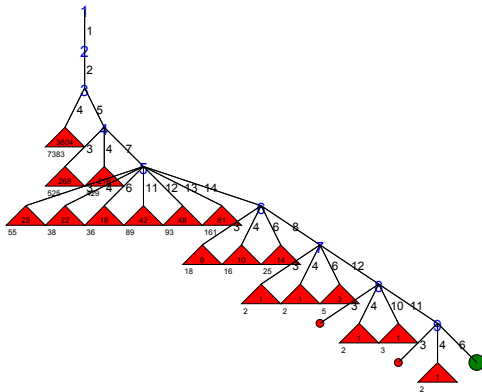
Search tree (Size 12)



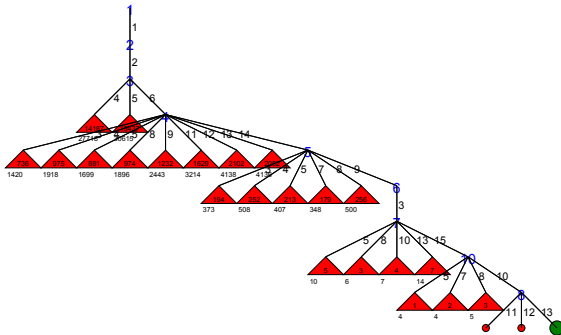
Search tree (Size 13)



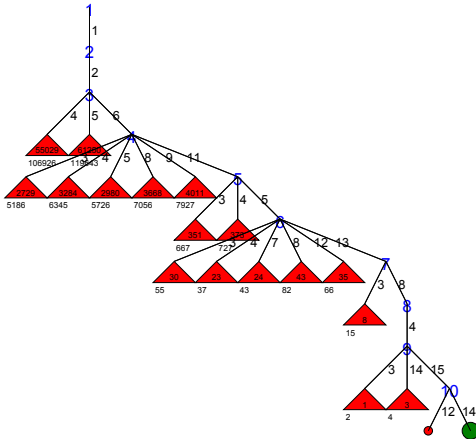
Search tree (Size 14)



Search tree (Size 15)

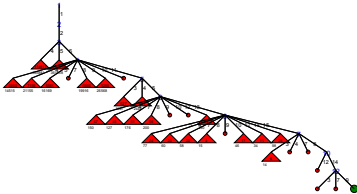


Search tree (Size 16)

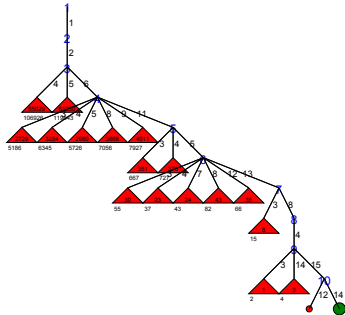


Comparison (Search Tree, size 16)

Initial Model



Improved Model



Other Problem Sizes

Size	Basic Model		Improved Model	
	Backtrack	Time	Backtrack	Time
10	4	0.00	4	0.16
11	118	0.08	77	1.44
12	50	0.05	31	0.94
13	335	0.36	216	6.22
14	5008	6.23	2875	95.94
15	47332	68.92	25820	1046.75
16	157773	271.22	84161	4099.52
17	1641685	3278.19	825590	49371.02
18	115745	283.97	55102	4530.83

Observation

- Changes reduce backtracks by 50%
- But, run times explode
- Being clever does not always pay
- Or, perhaps, we did not make the right improvements?

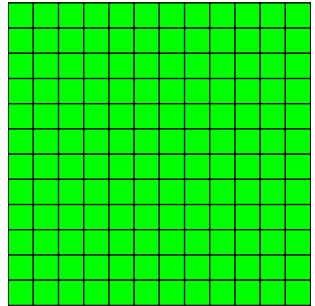


Change of Search Strategy

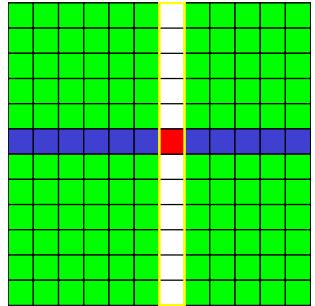
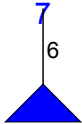
- Idea: Make more difficult choices first
- Reorder variables to start from middle
- Assign values starting in middle



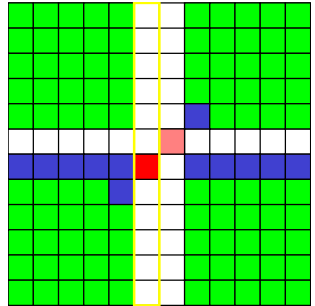
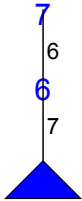
Labeling From Middle



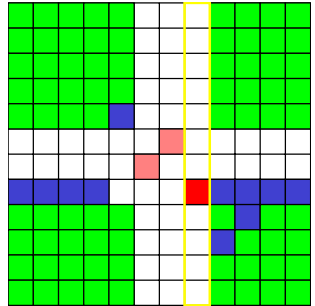
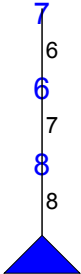
Labeling From Middle



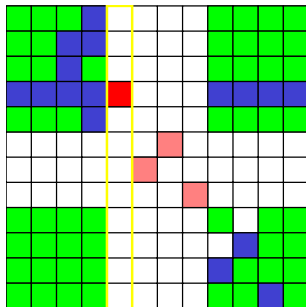
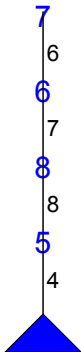
Labeling From Middle



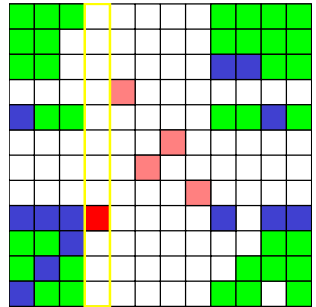
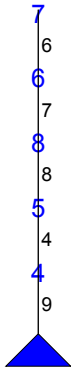
Labeling From Middle



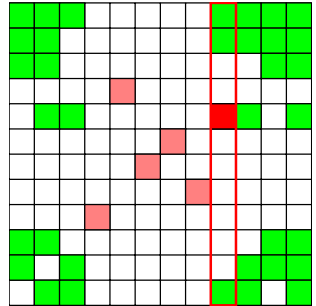
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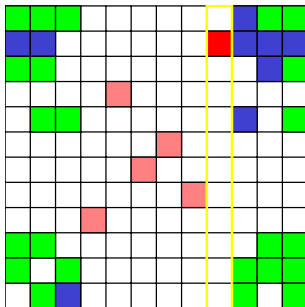
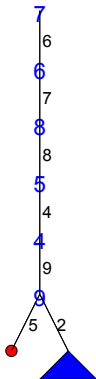
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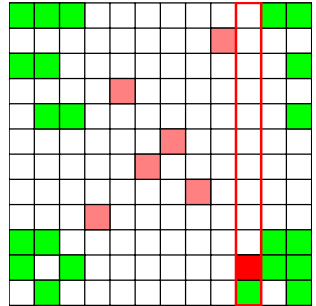
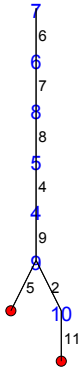
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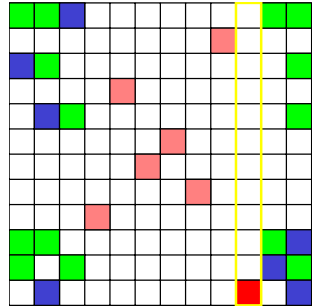
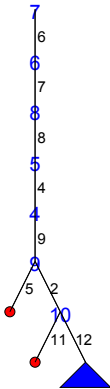
Labeling From Middle



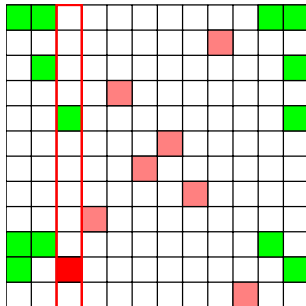
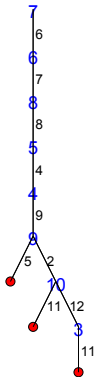
Labeling From Middle



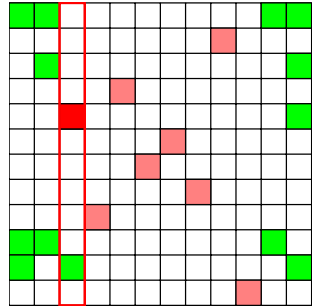
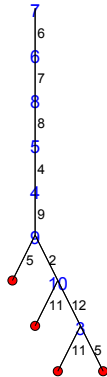
Labeling From Middle



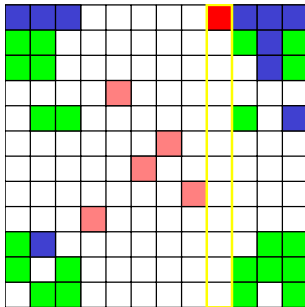
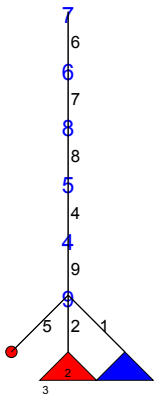
Labeling From Middle



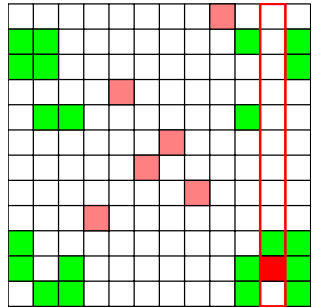
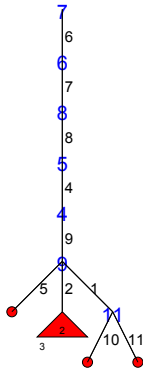
Labeling From Middle



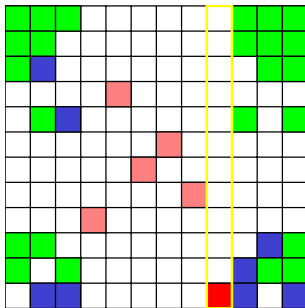
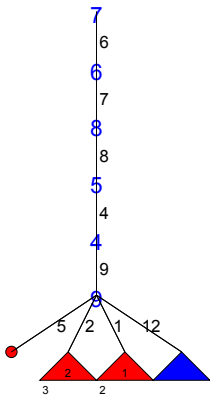
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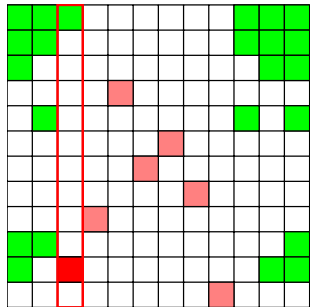
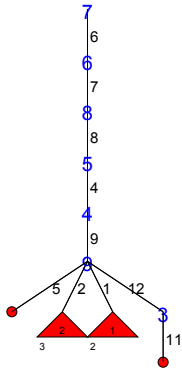
Labeling From Middle



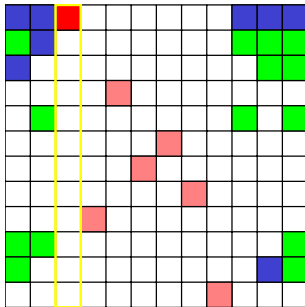
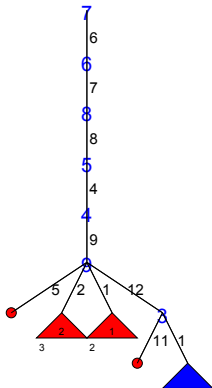
Labeling From Middle



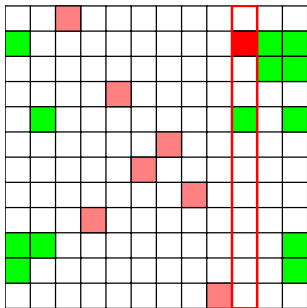
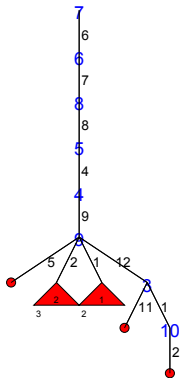
Labeling From Middle



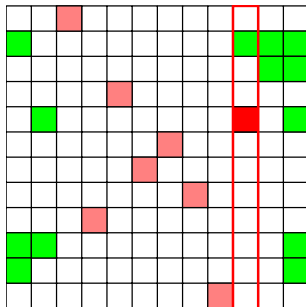
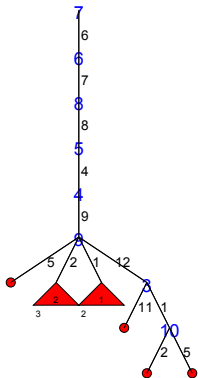
Labeling From Middle



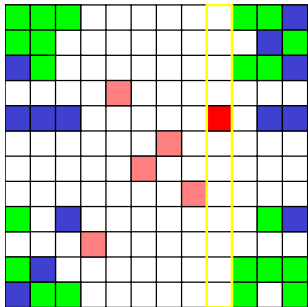
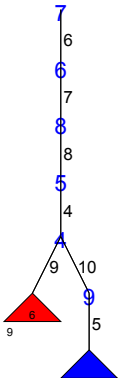
Labeling From Middle



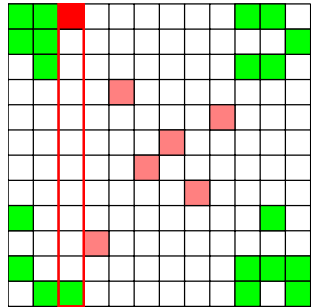
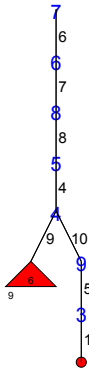
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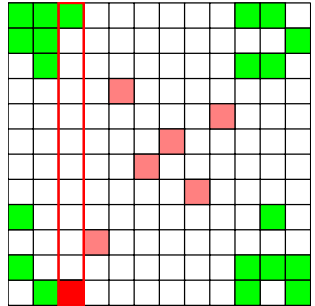
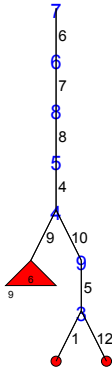
Labeling From Middle



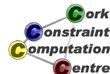
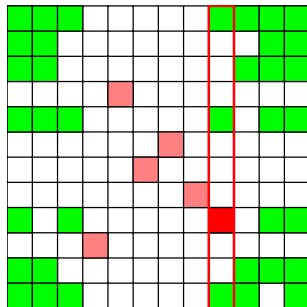
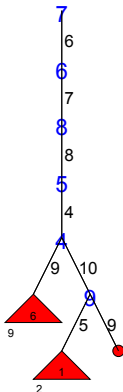
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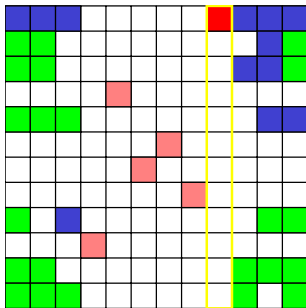
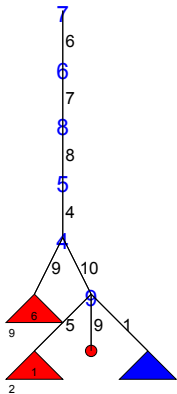
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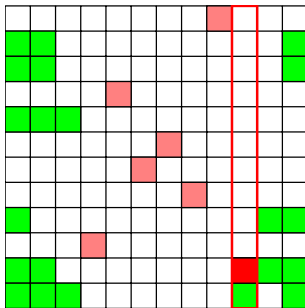
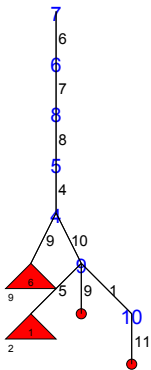
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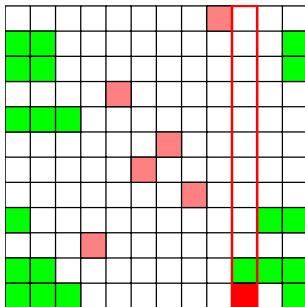
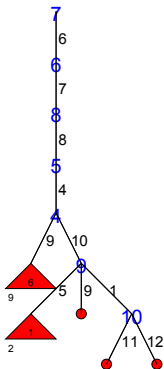
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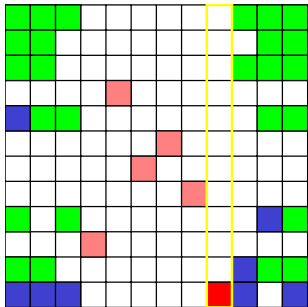
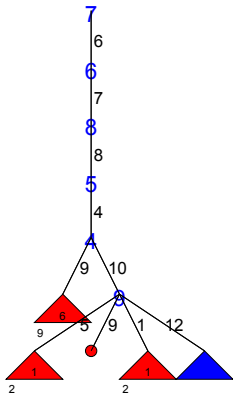
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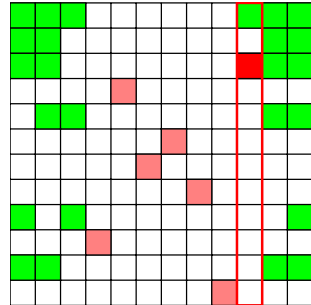
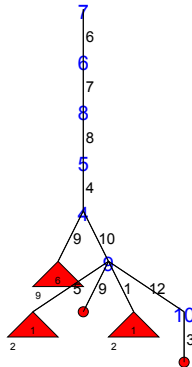
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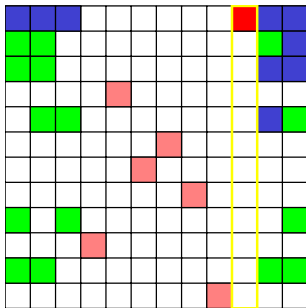
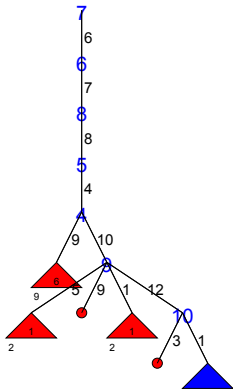
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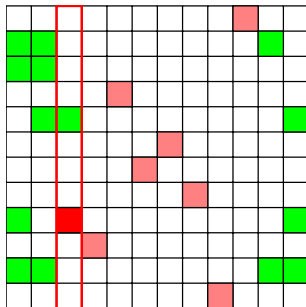
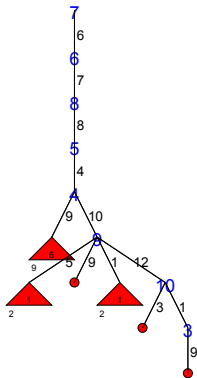
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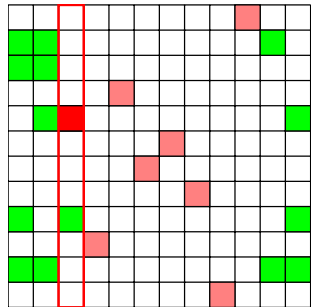
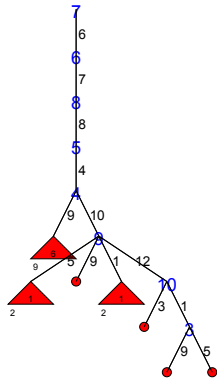
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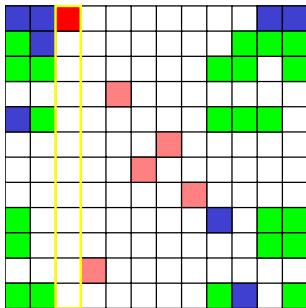
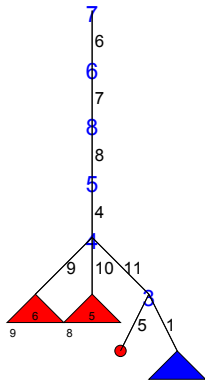
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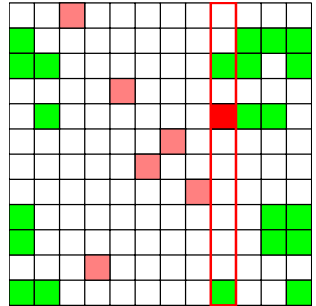
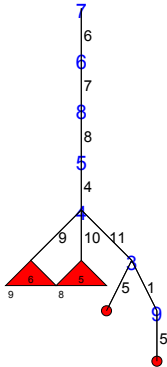
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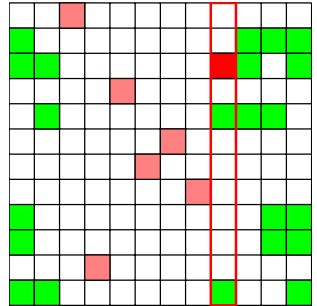
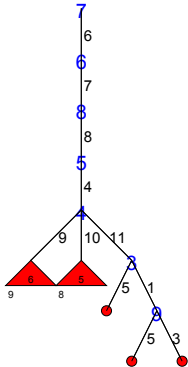
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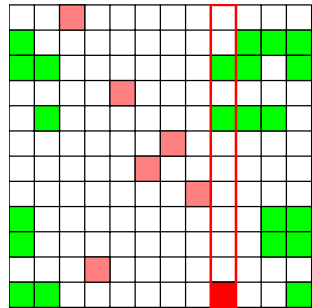
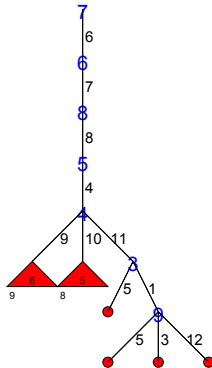
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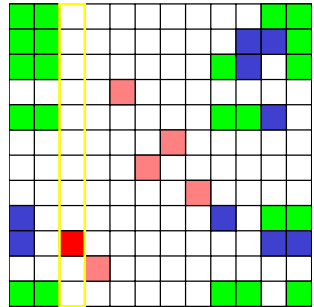
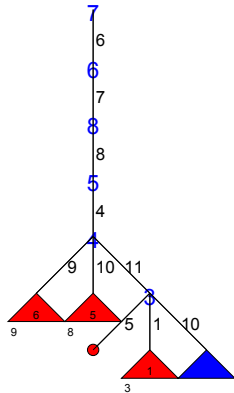
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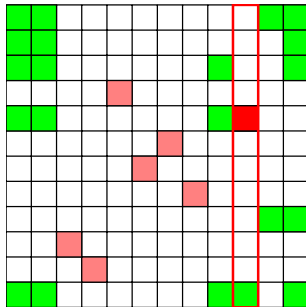
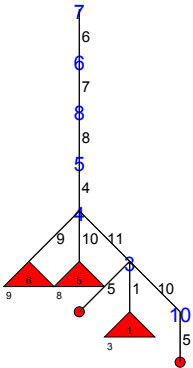
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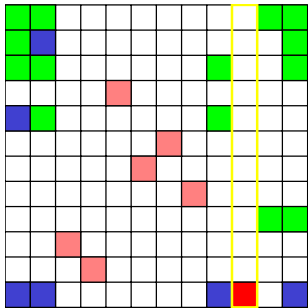
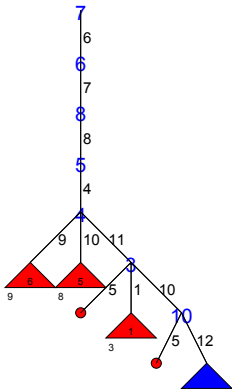
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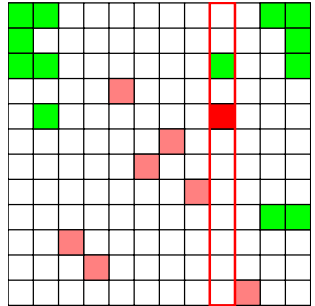
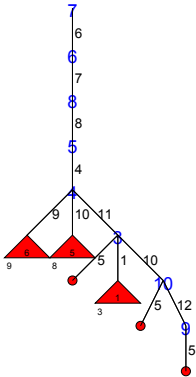
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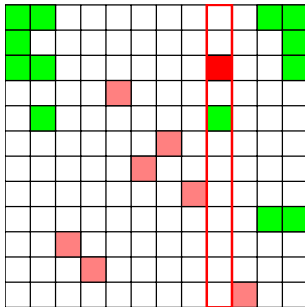
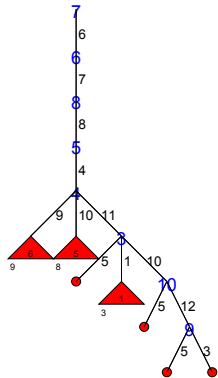
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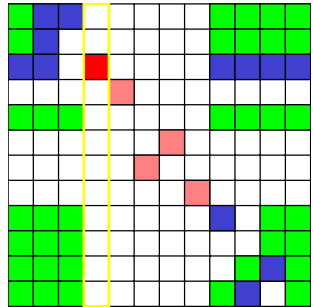
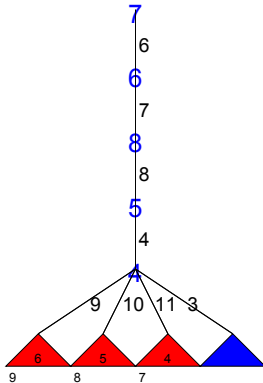
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Labeling From Middle



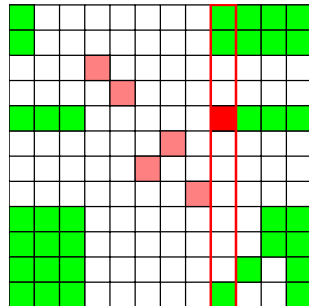
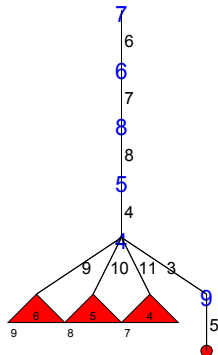
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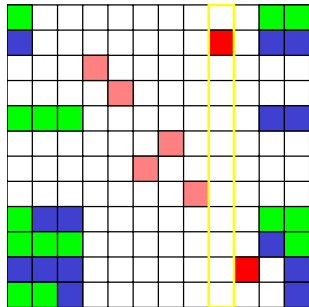
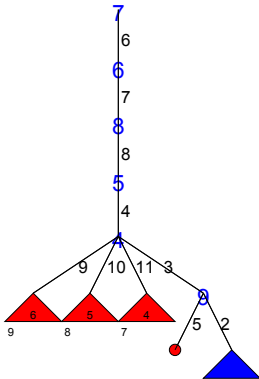
◀ Back to Start

▶▶ Skip Animation

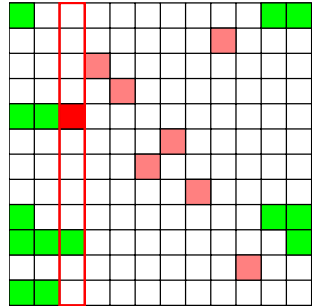
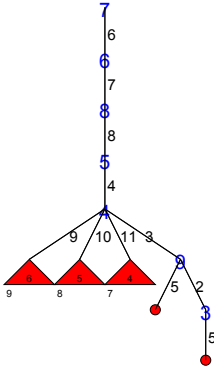
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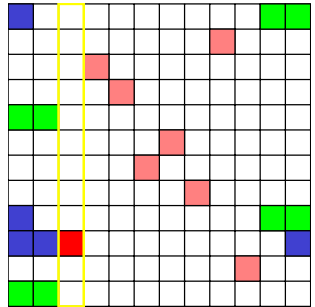
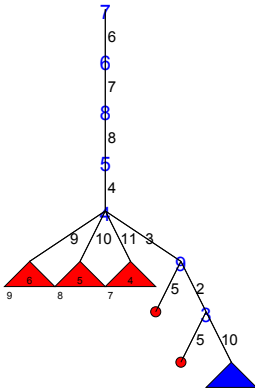
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Labeling From Middle



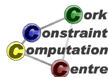
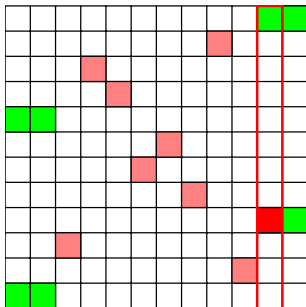
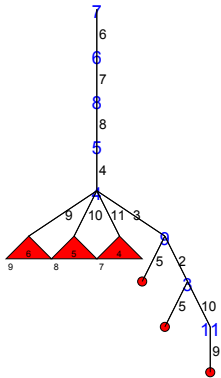
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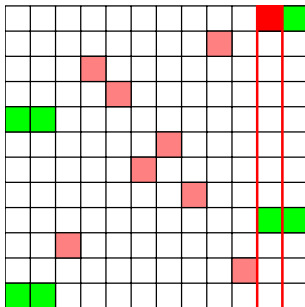
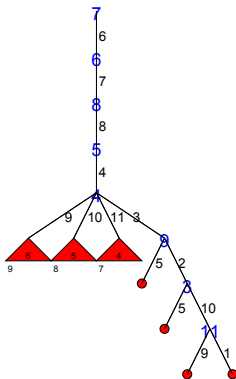
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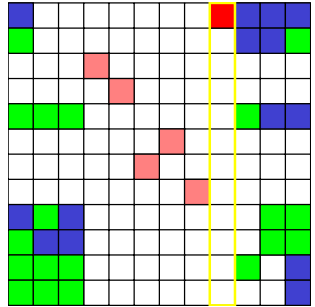
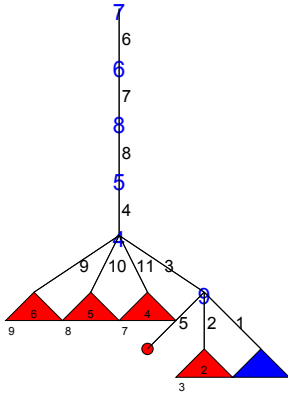
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Labeling From Middle



Labeling From Middle

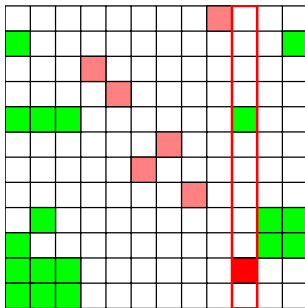
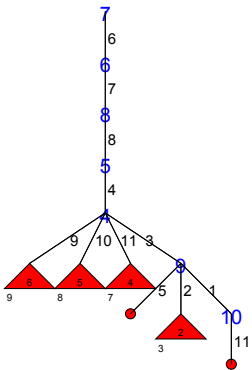


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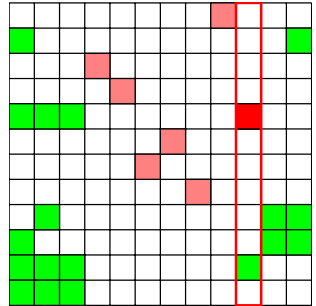
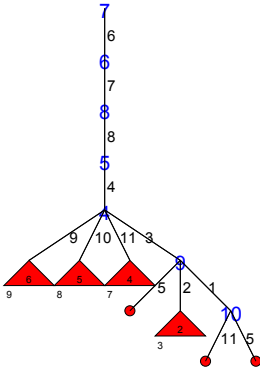
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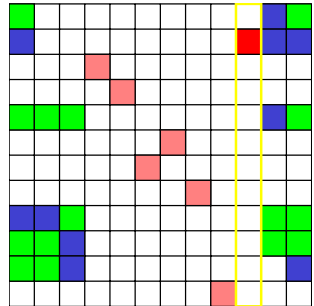
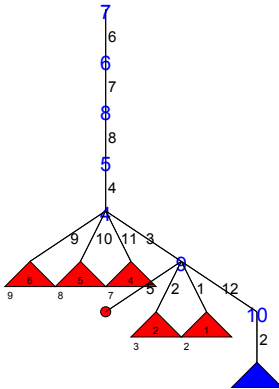
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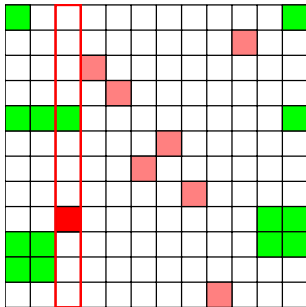
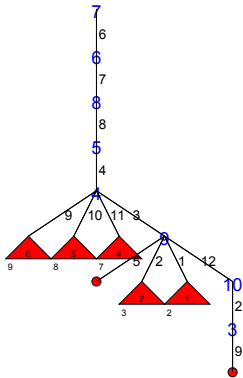
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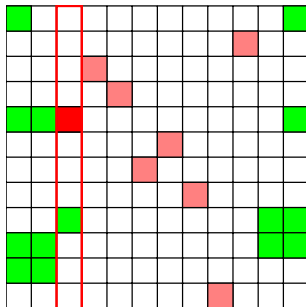
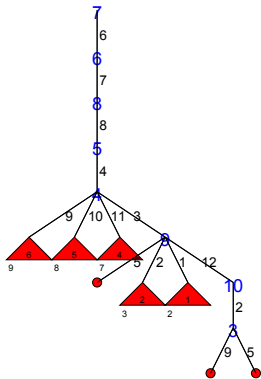
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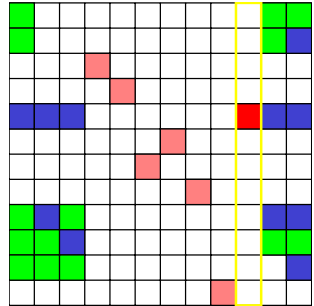
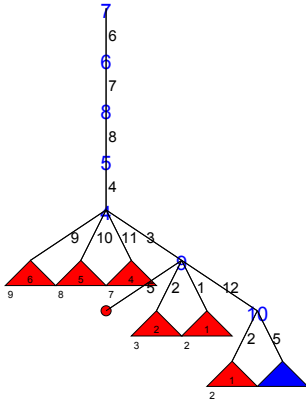
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Labeling From Middle



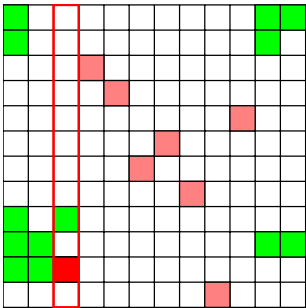
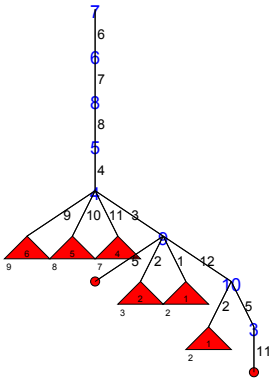
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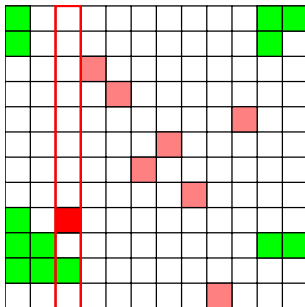
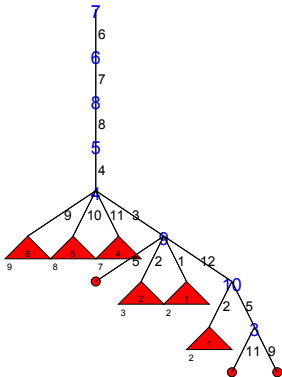
◀ Back to Start

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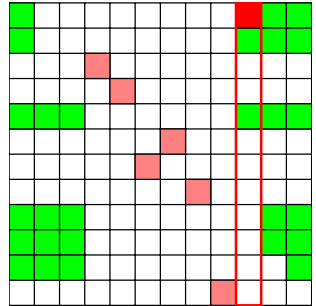
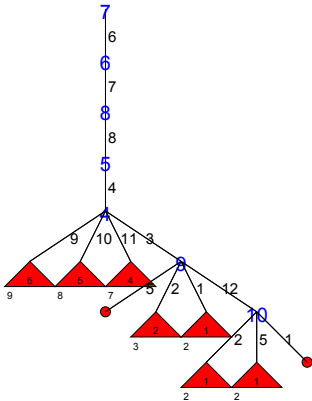
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Labeling From Middle



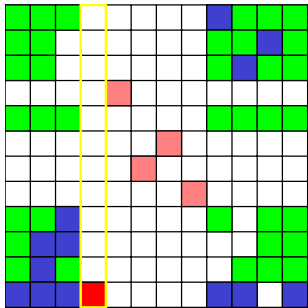
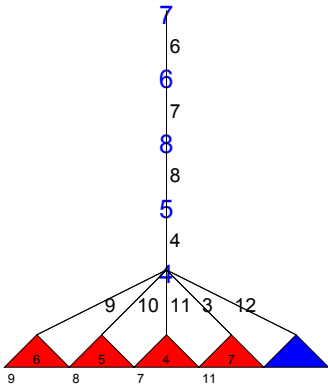
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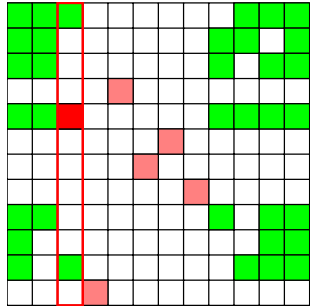
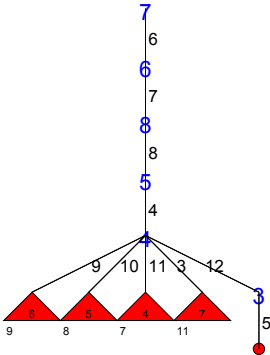
◀ Back to Start

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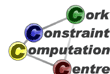
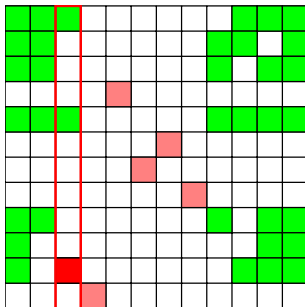
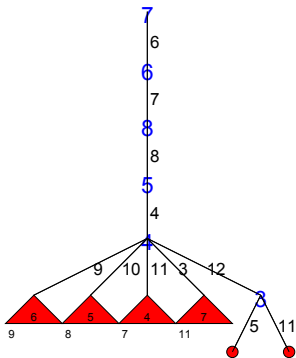
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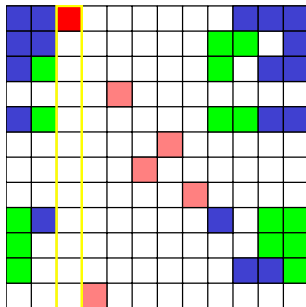
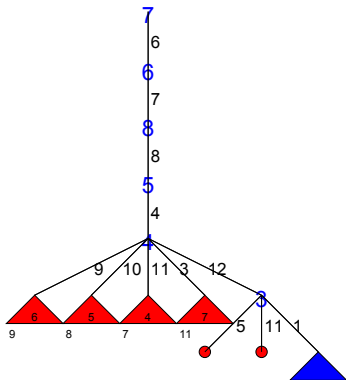
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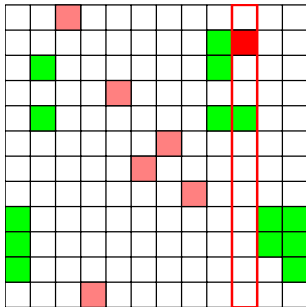
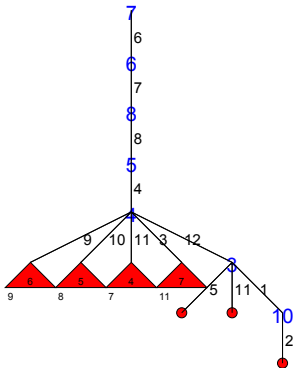
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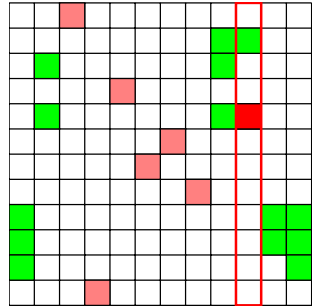
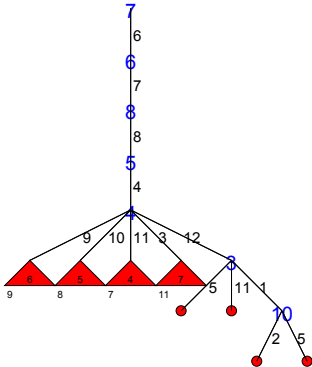
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Labeling From Middle



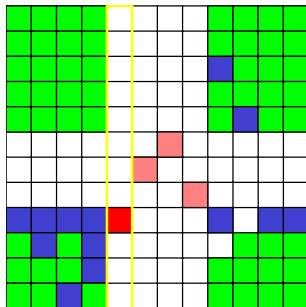
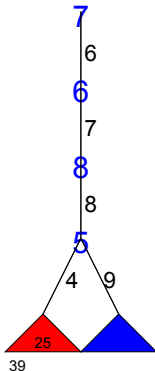
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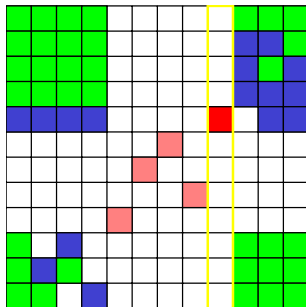
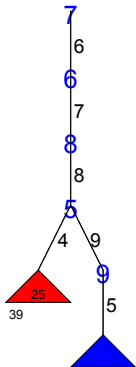
◀ Back to Start

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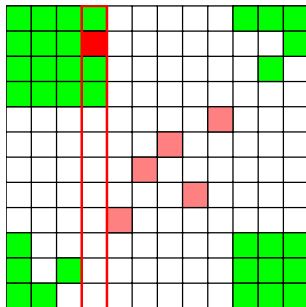
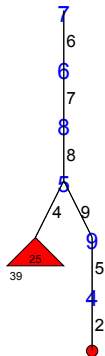
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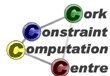
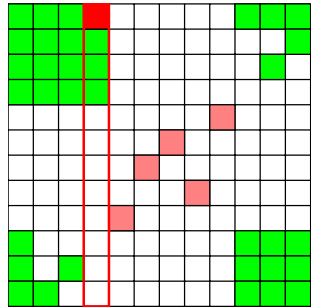
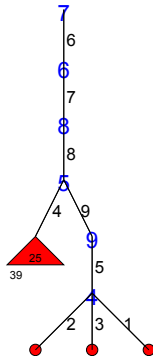
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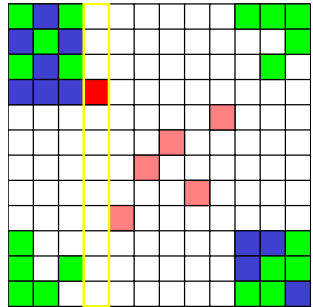
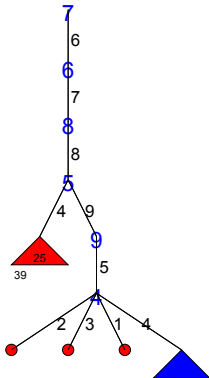
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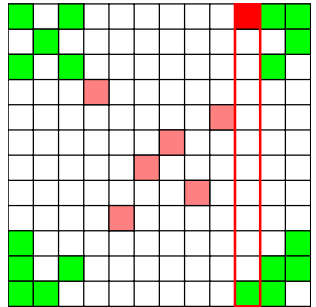
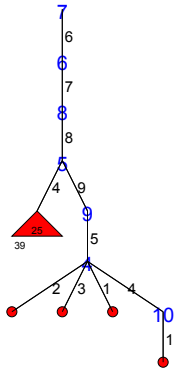
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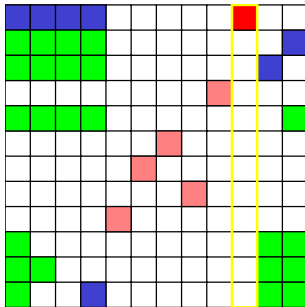
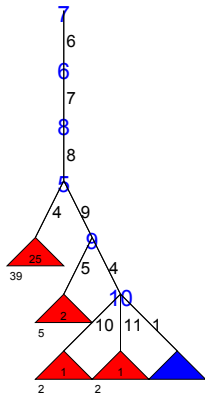
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Labeling From Middle



Labeling From Middle

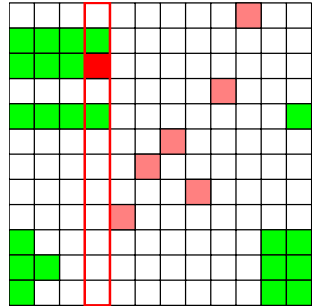
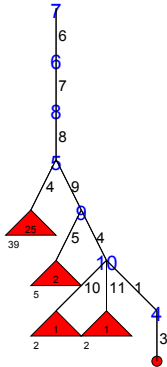


◀ Back to Start

▶▶ Skip Animation



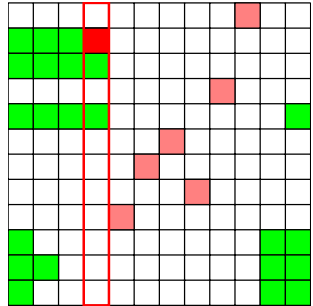
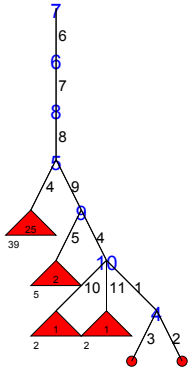
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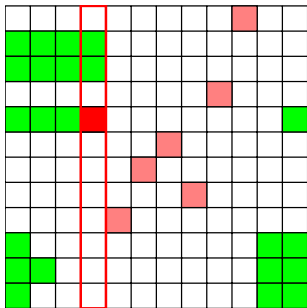
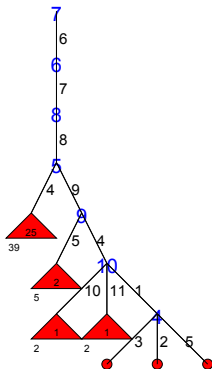
◀ Back to Start

▶▶ Skip Animation

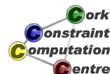
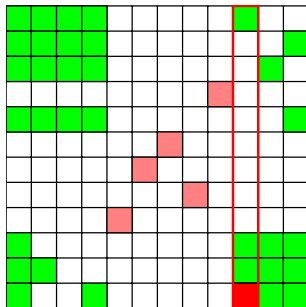
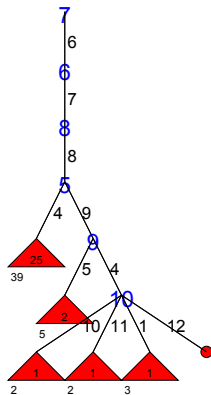
Labeling From Middle



Labeling From Middle



Labeling From Middle

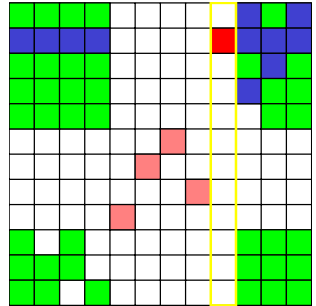
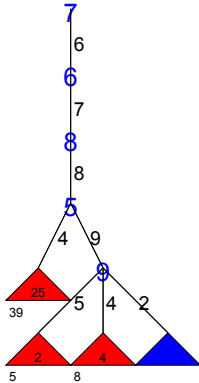


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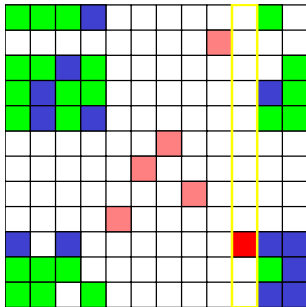
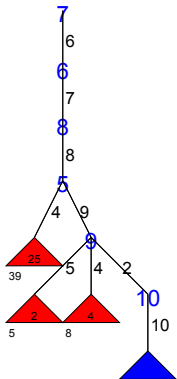
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◀ Back to Start

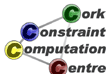
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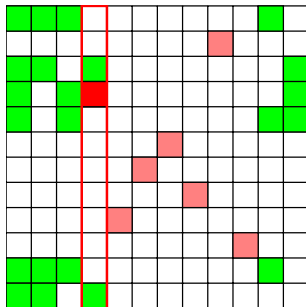
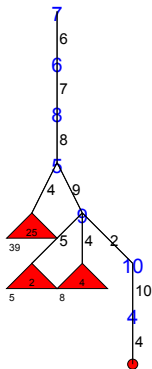


◀ Back to Start

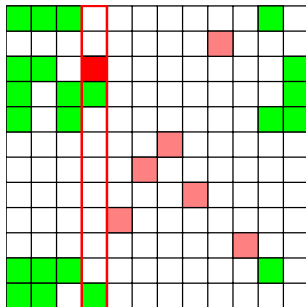
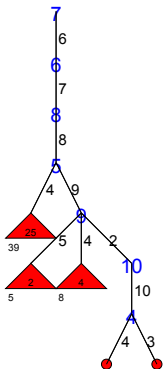
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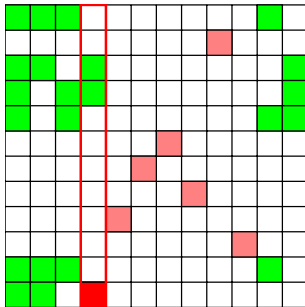
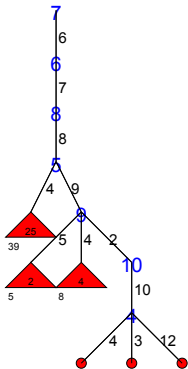
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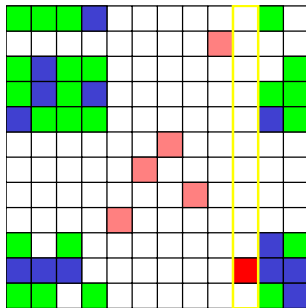
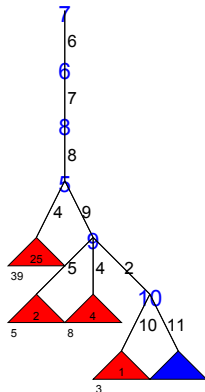
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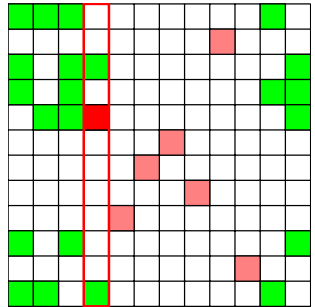
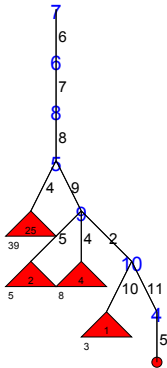
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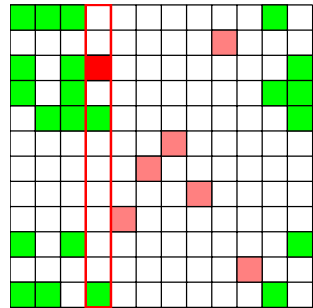
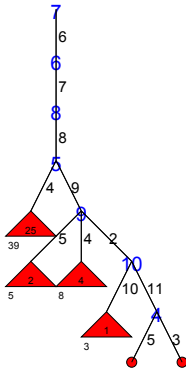
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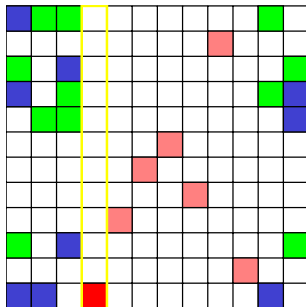
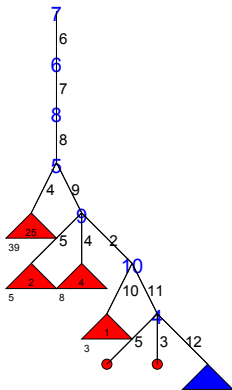
Labeling From Middle



Labeling From Middle



Labeling From Middle

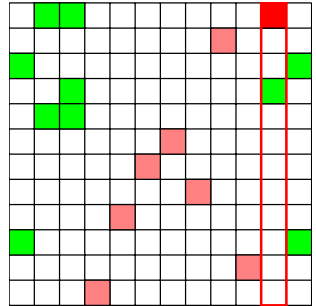
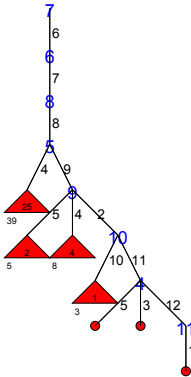


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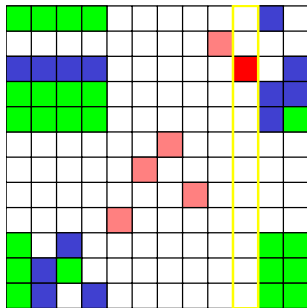
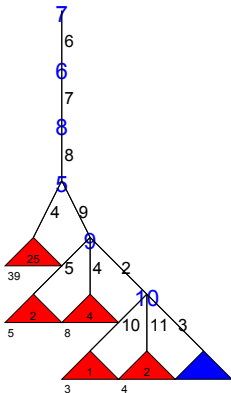
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Labeling From Middle



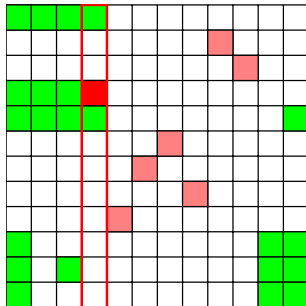
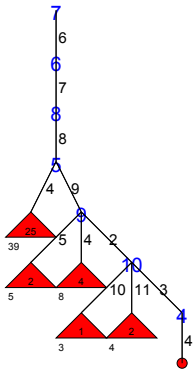
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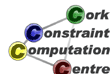
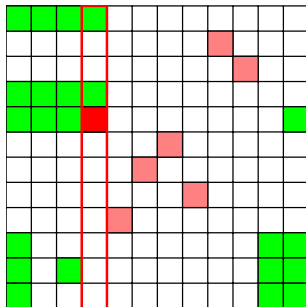
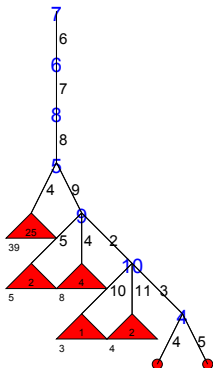
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



Labeling From Middle

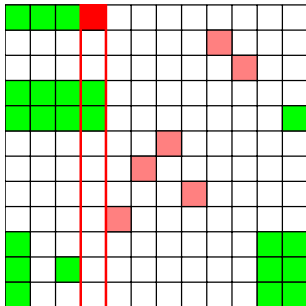
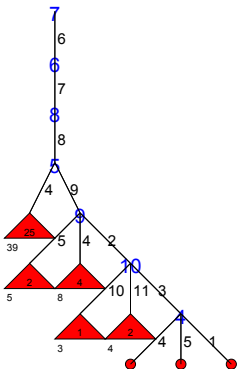


◀ Back to Start

▶▶ Skip Animation



Labeling From Middle

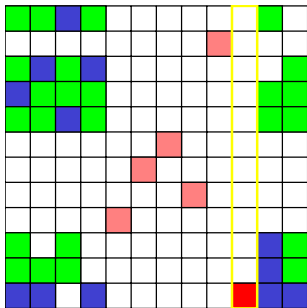
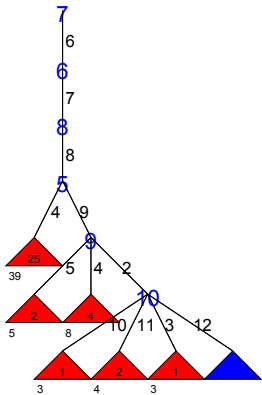


◀ Back to Start

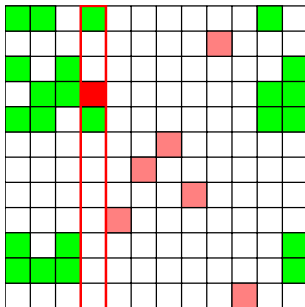
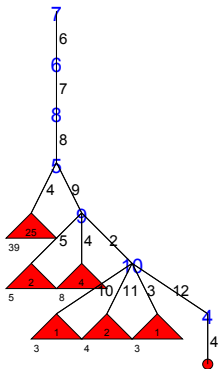
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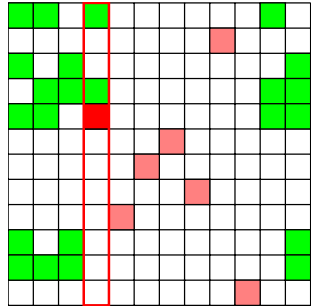
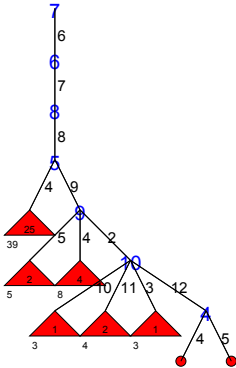
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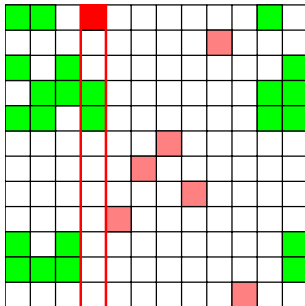
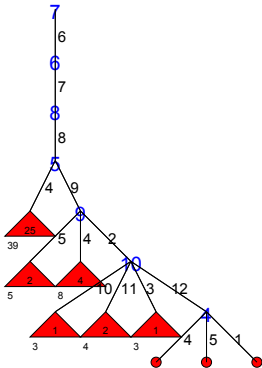
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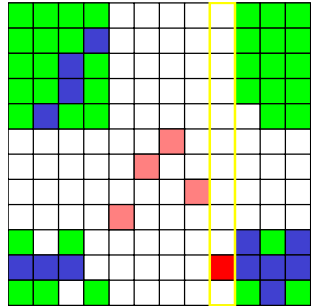
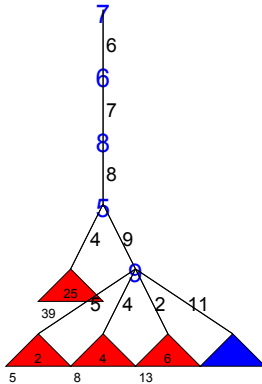
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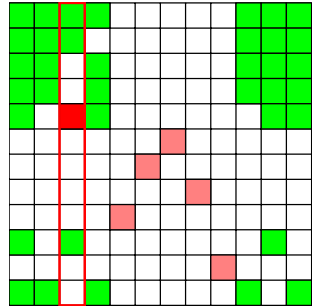
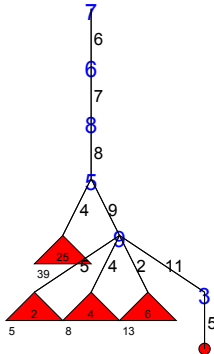
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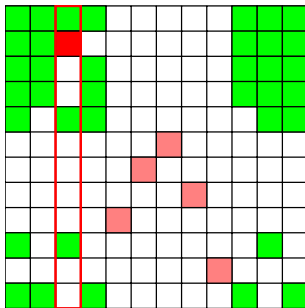
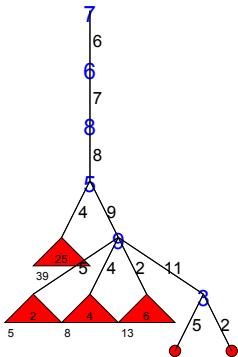
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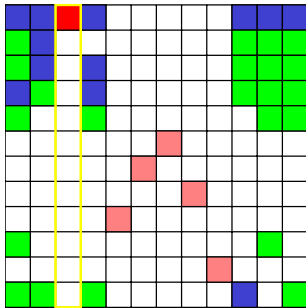
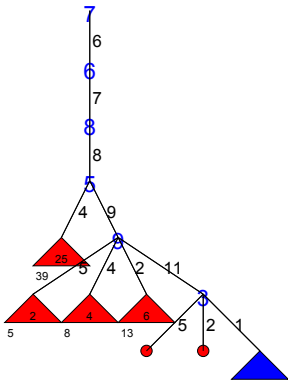
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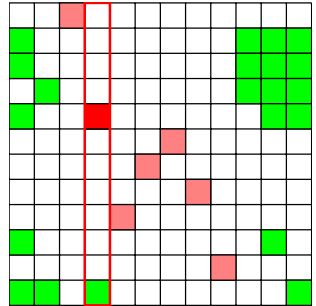
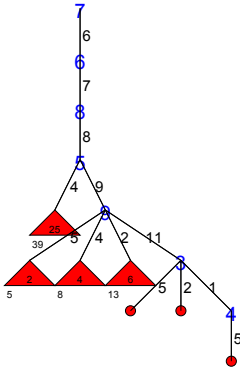
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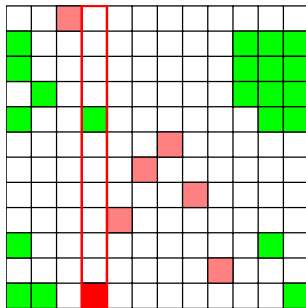
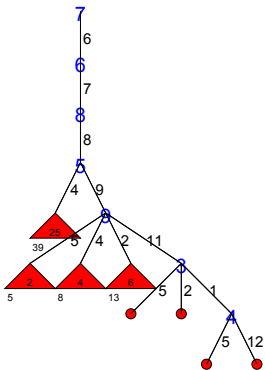
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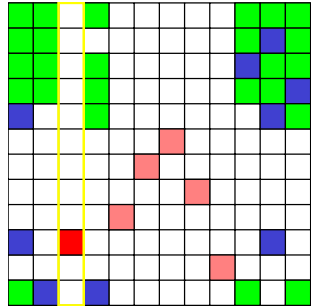
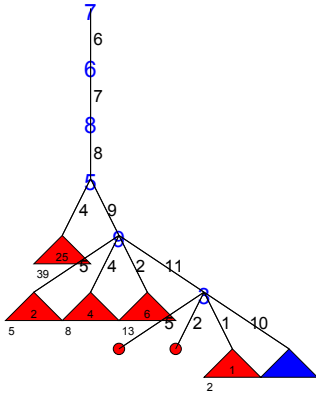
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Labeling From Middle



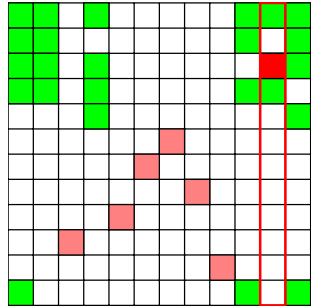
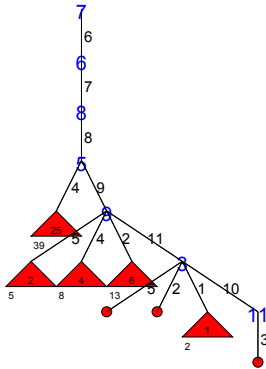
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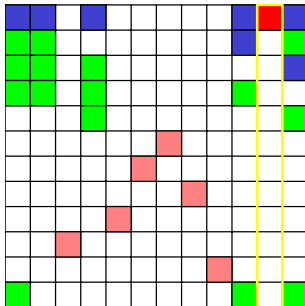
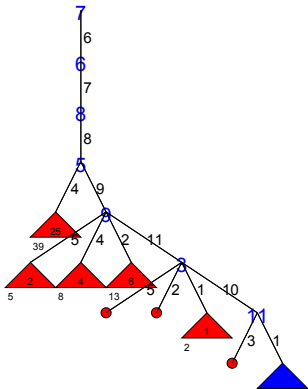
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



Labeling From Middle

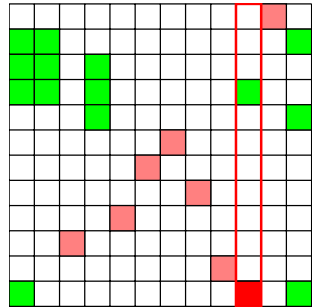
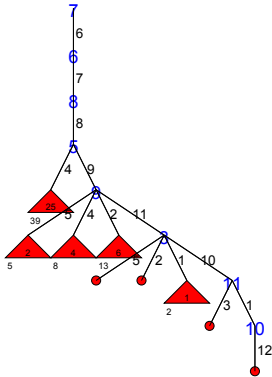


◀ Back to Start

▶ Skip Animation



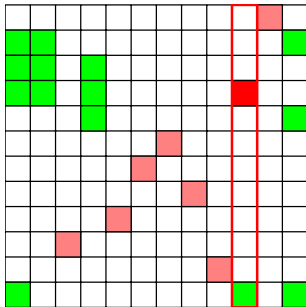
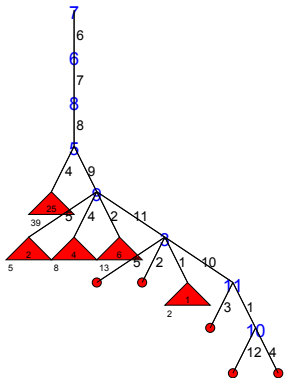
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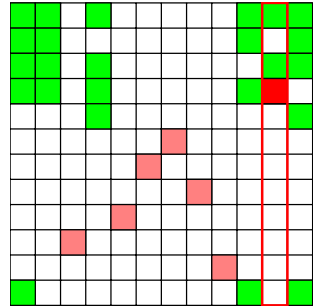
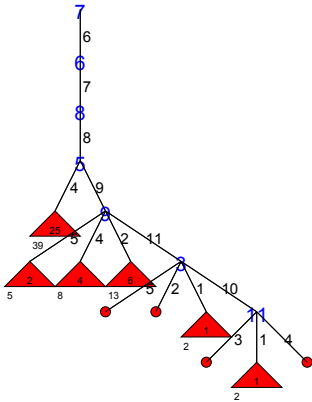
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



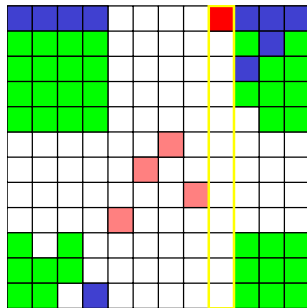
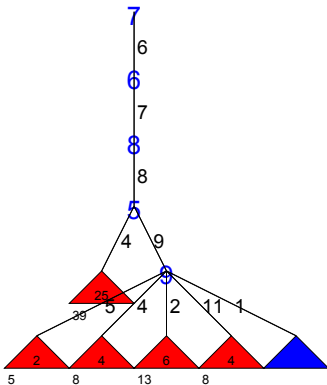
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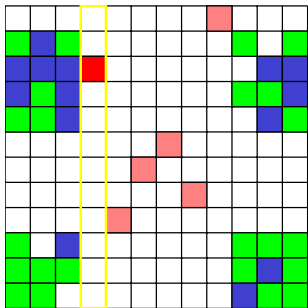
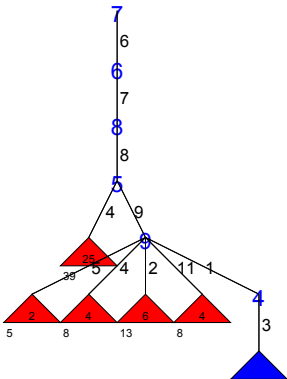
◀ Back to Start

▶▶ Skip Animation

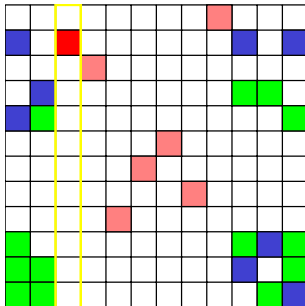
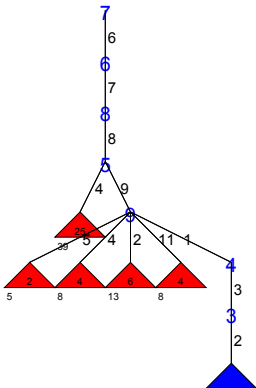
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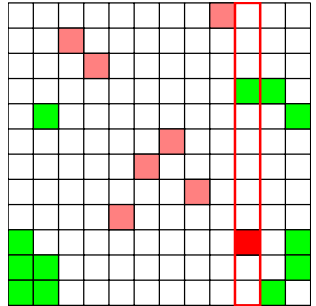
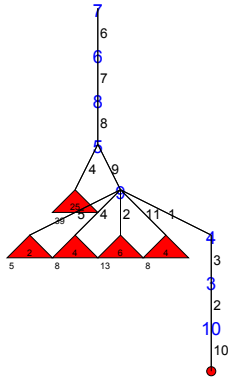
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Labeling From Middle



Labeling From Middle

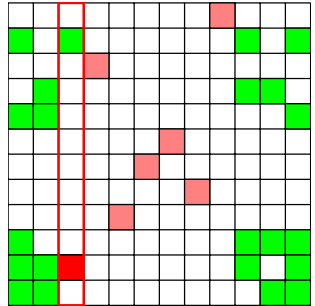
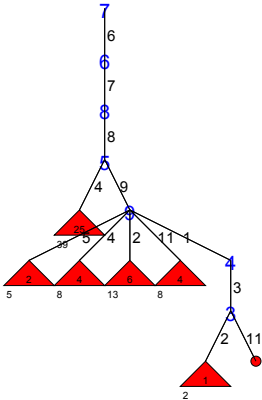


◀ Back to Start

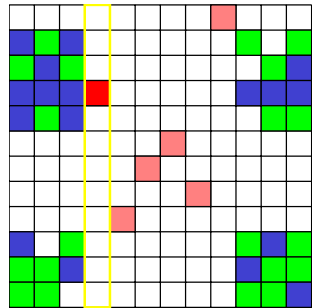
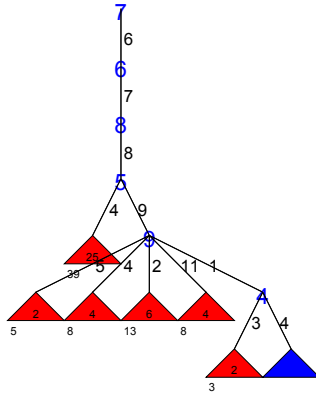
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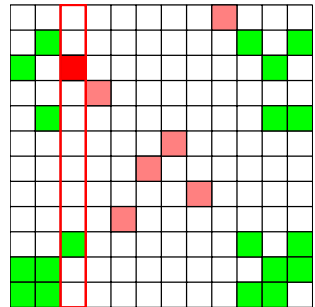
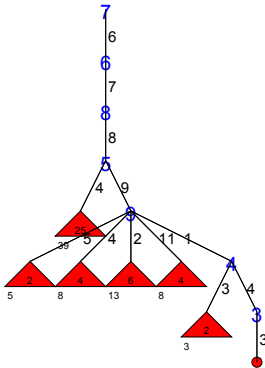
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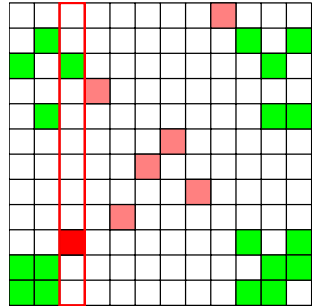
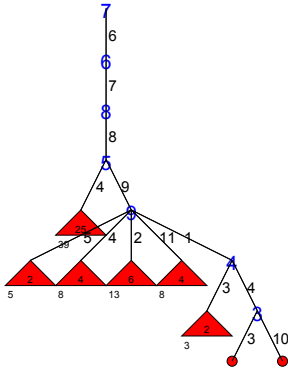
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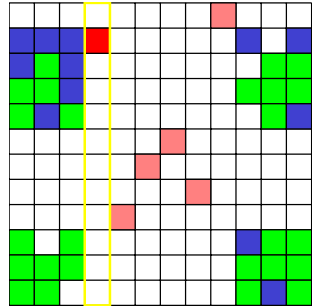
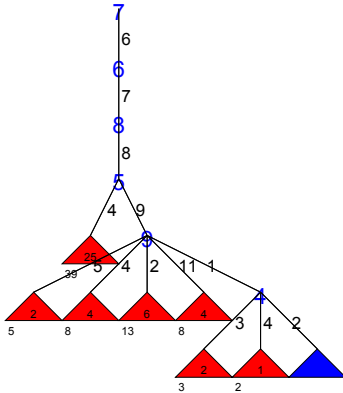
Labeling From Middle



Labeling From Middle



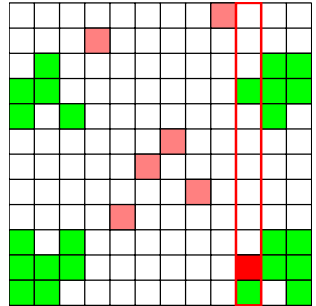
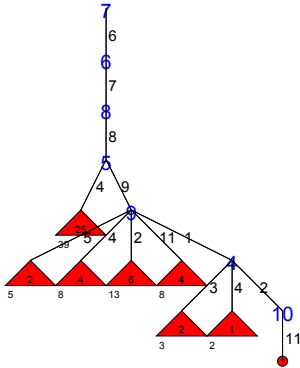
Labeling From Middle



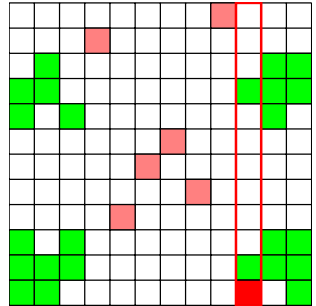
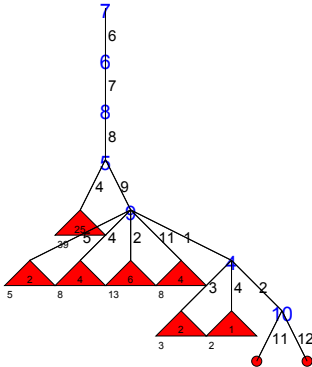
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



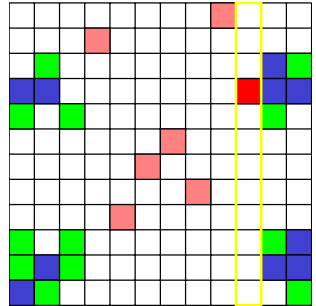
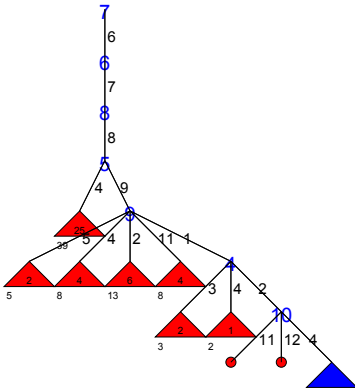
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

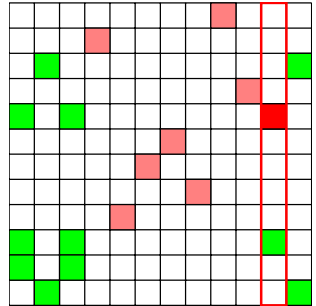
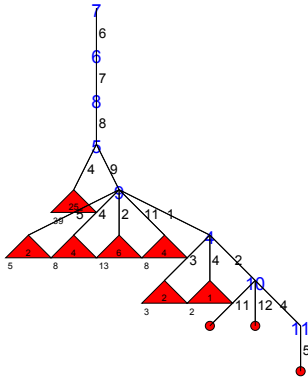
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

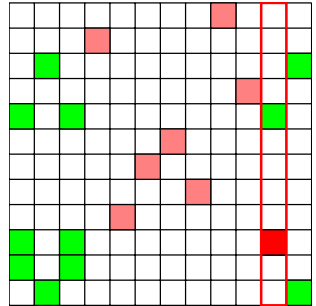
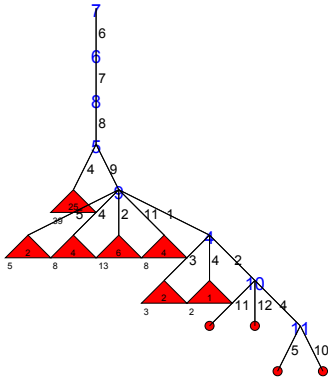
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◀ Back to Start

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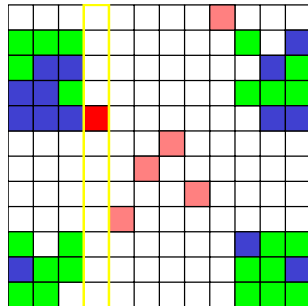
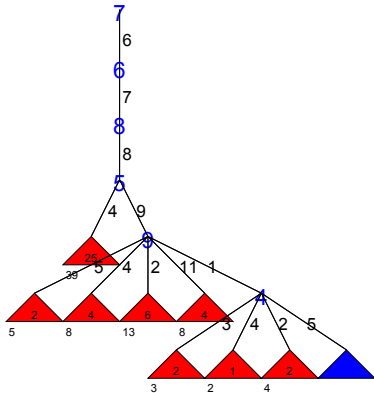
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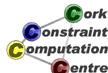
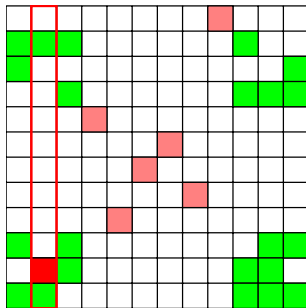
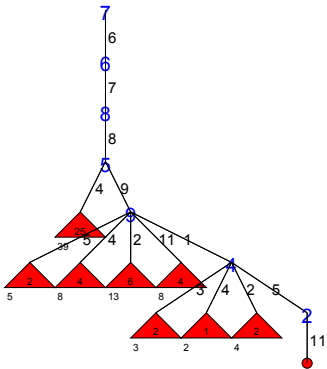
◀ Back to Start

▶▶ Skip Animation

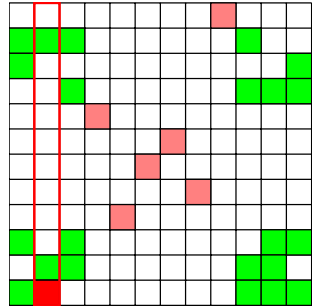
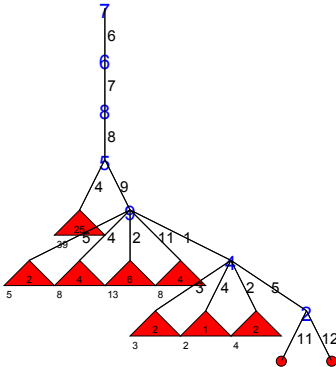
Labeling From Middle



Labeling From Middle



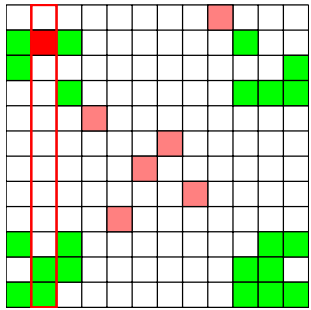
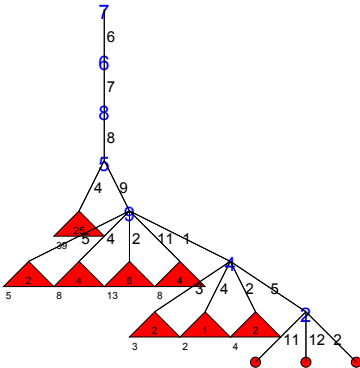
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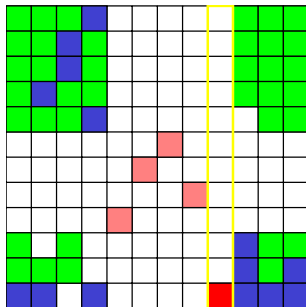
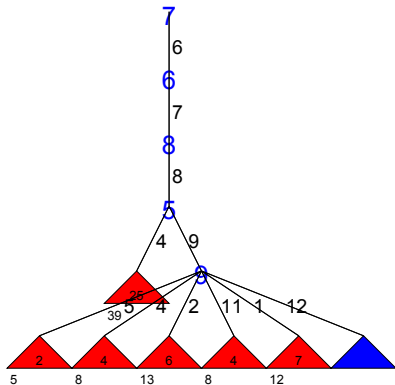
◀ Back to Start

▶ Skip Animation

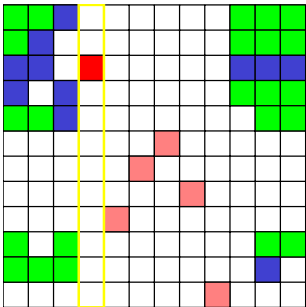
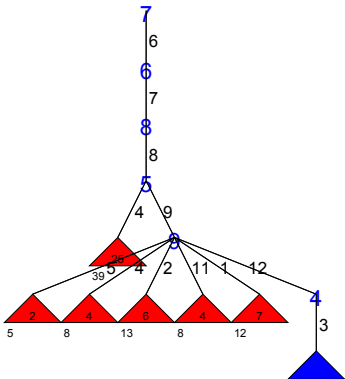
Labeling From Middle



Labeling From Middle



Labeling From Middle

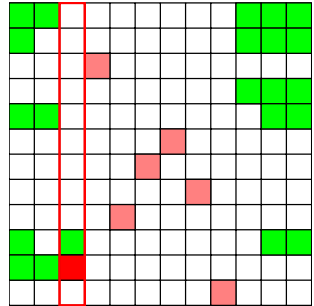
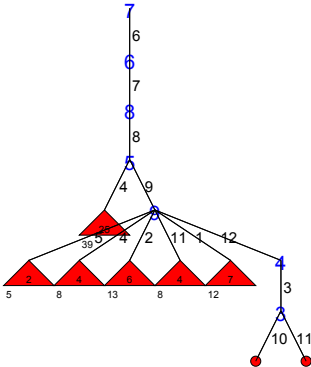


◀ Back to Start

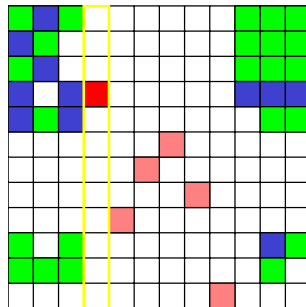
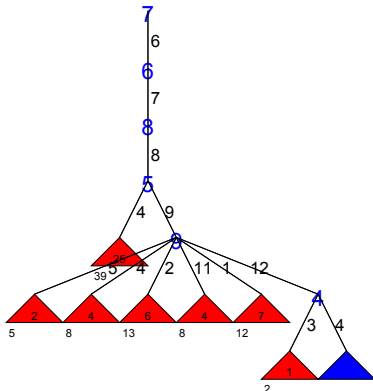
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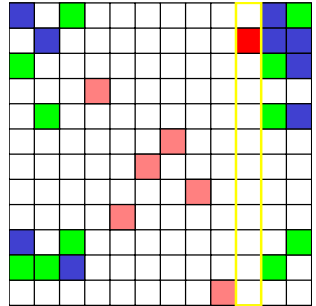
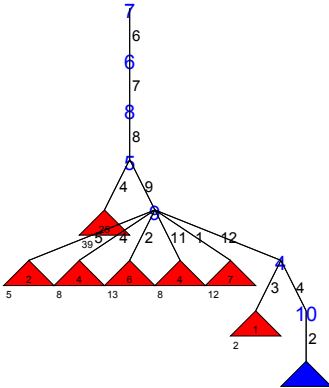
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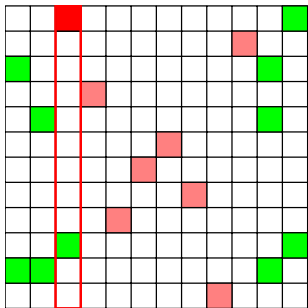
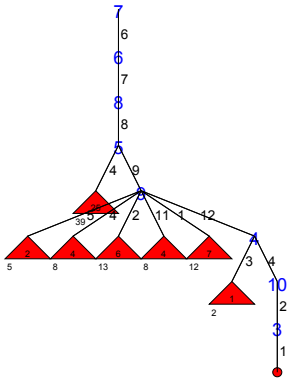
Labeling From Middle



Labeling From Middle



Labeling From Middle

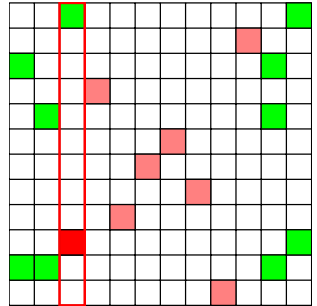
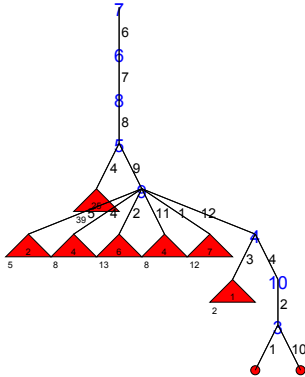


◀ Back to Start

▶▶ Skip Animation



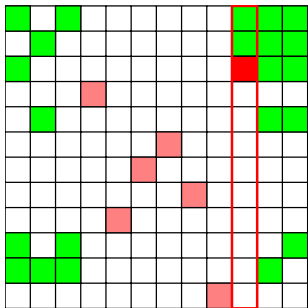
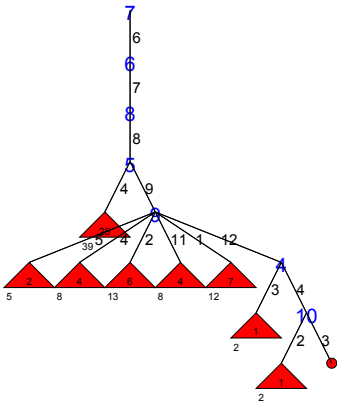
Labeling From Middle



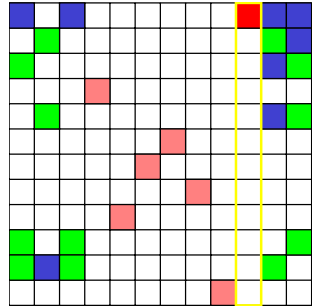
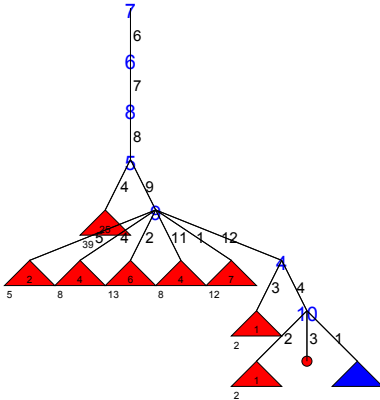
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



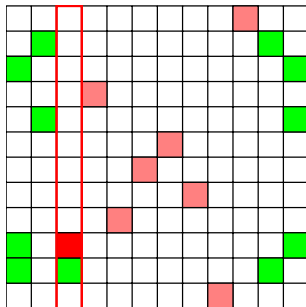
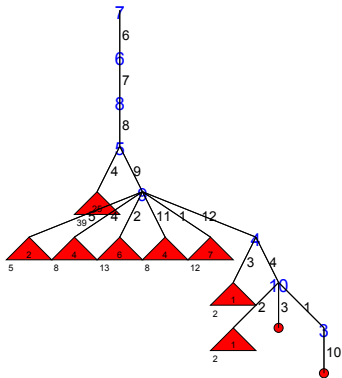
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

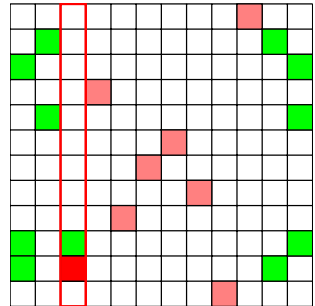
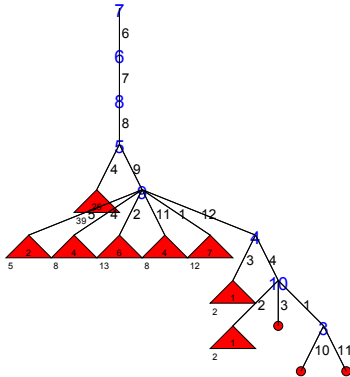
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

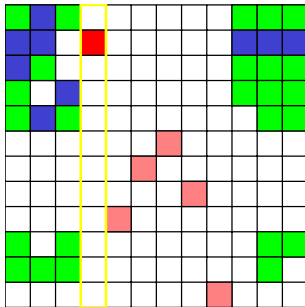
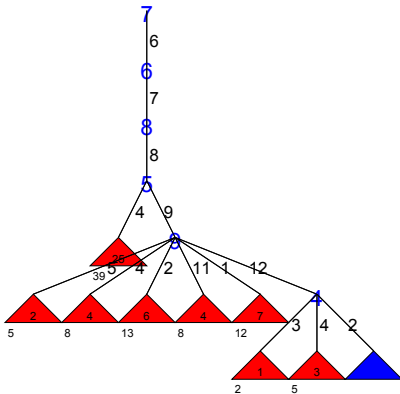


◀ Back to Start

▶▶ Skip Animation



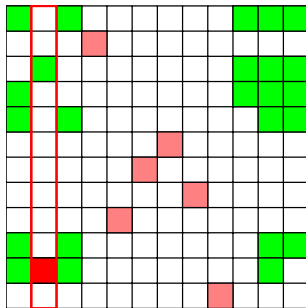
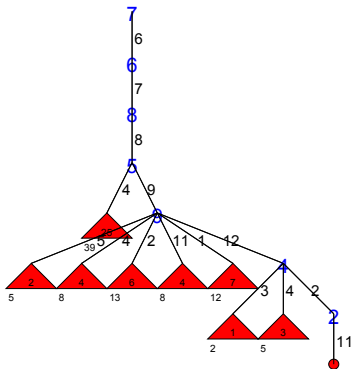
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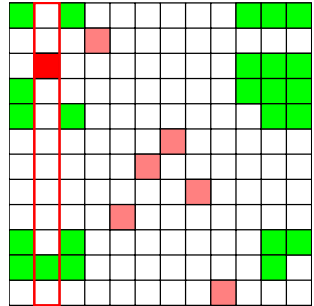
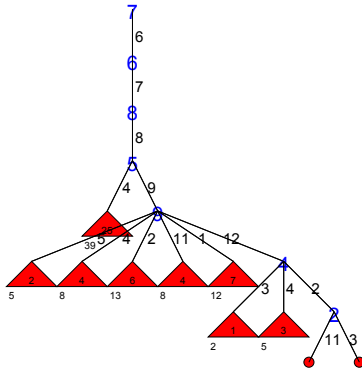
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



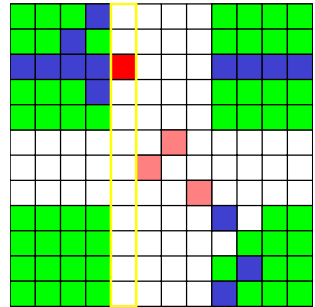
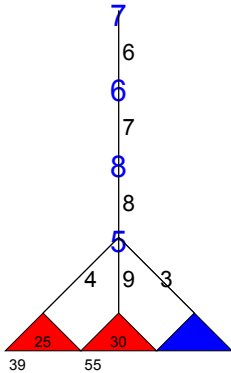
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

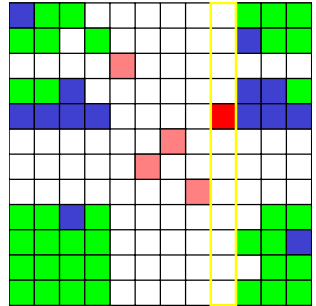
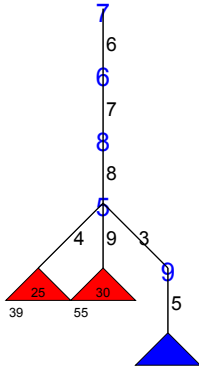
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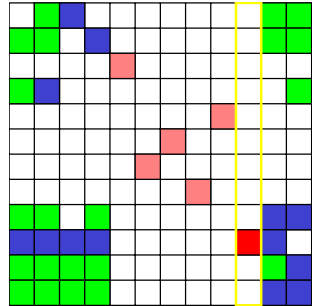
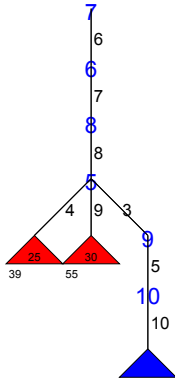
◀ Back to Start

▶▶ Skip Animation

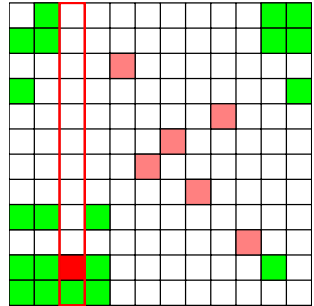
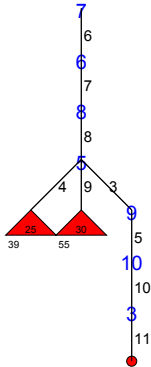
Labeling From Middle



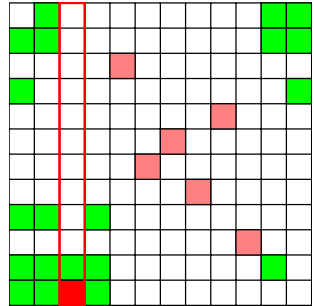
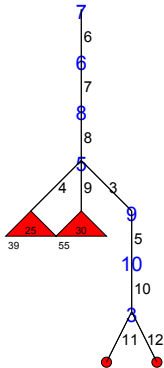
Labeling From Middle



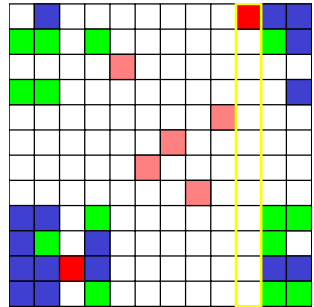
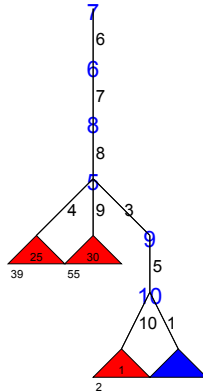
Labeling From Middle



Labeling From Middle



Labeling From Middle

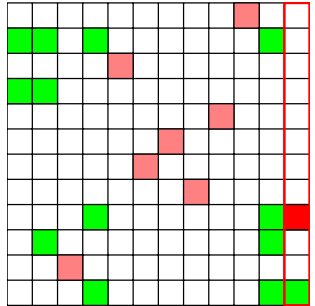
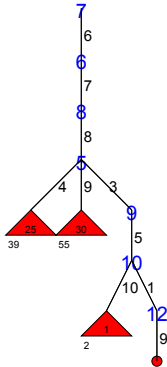


◀ Back to Start

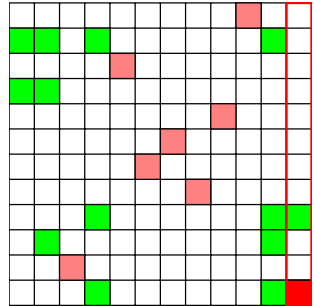
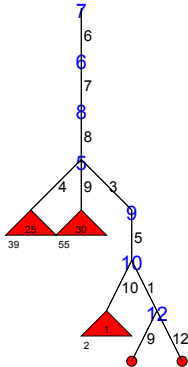
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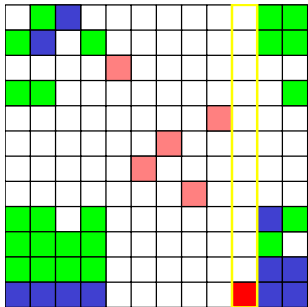
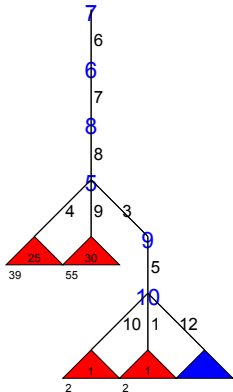
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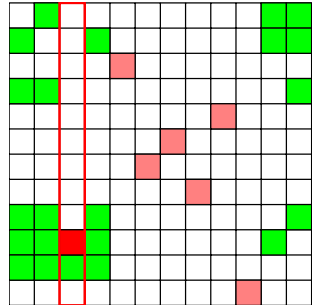
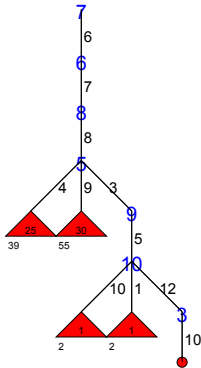
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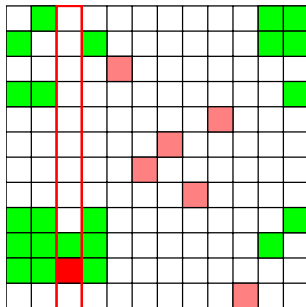
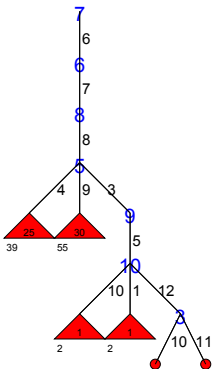
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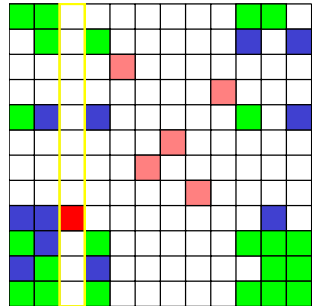
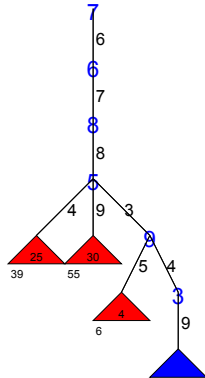
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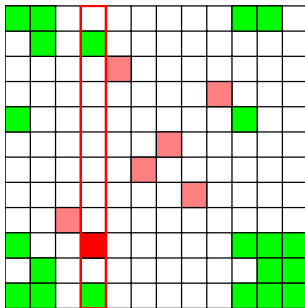
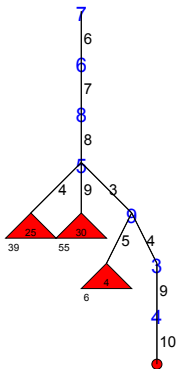
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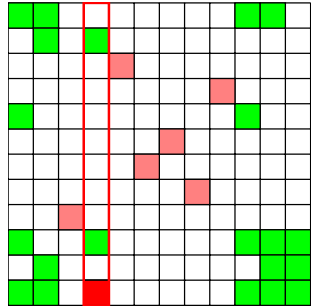
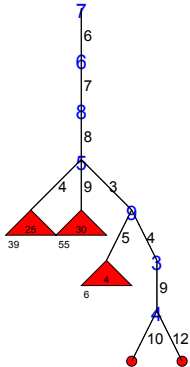
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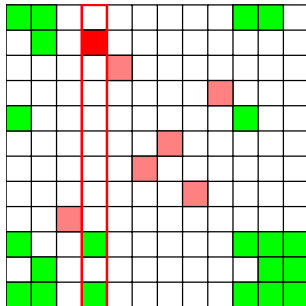
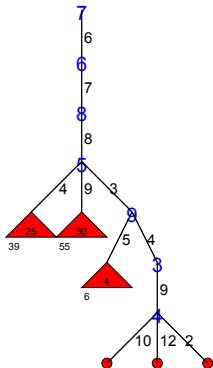
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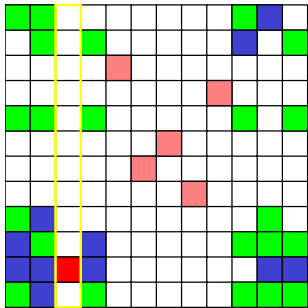
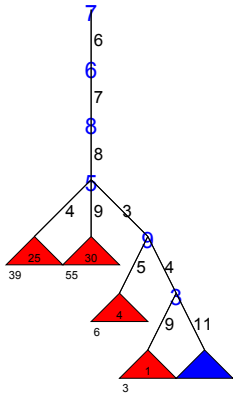
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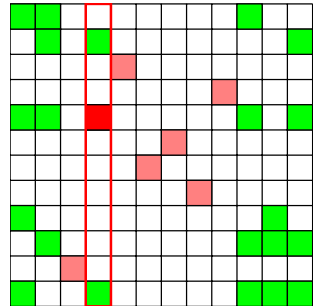
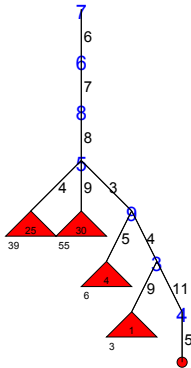
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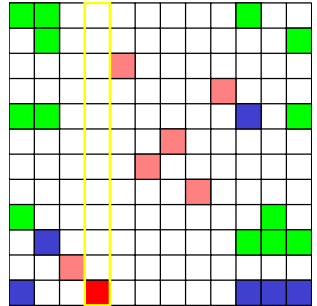
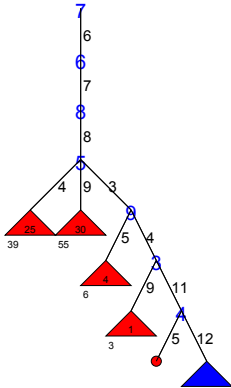
Labeling From Middle



Labeling From Middle



Labeling From Middle

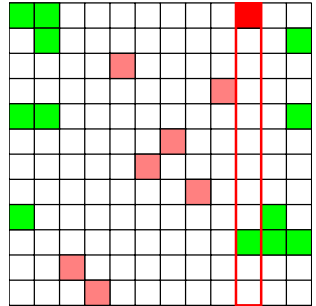
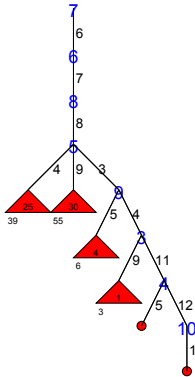


◀ Back to Start

▶ Skip Animation



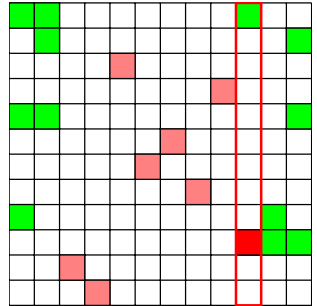
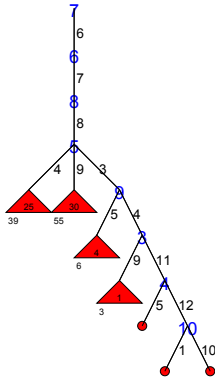
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

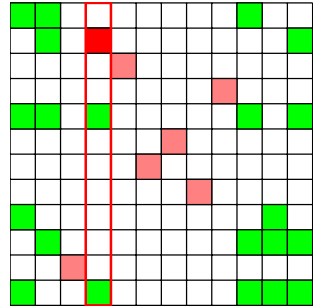
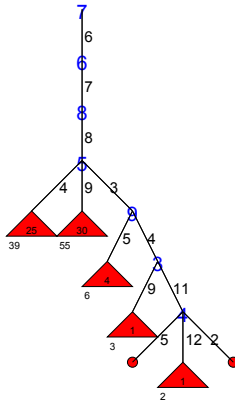


◀ Back to Start

▶▶ Skip Animation



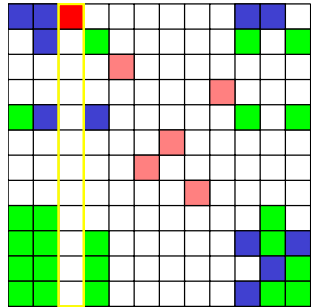
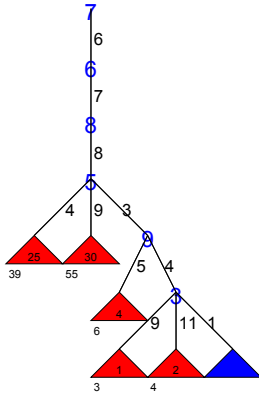
Labeling From Middle



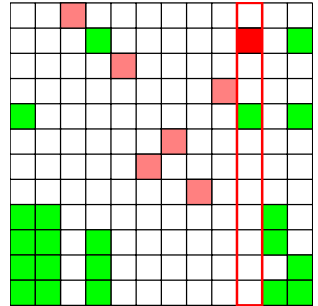
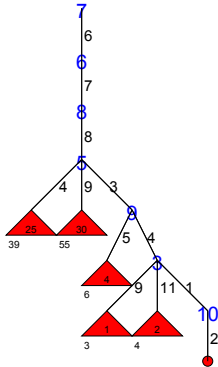
◀ Back to Start

▶▶ Skip Animation

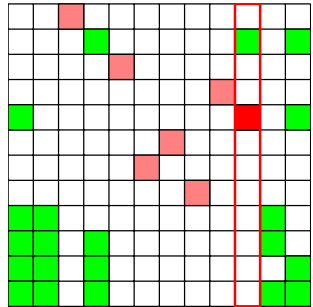
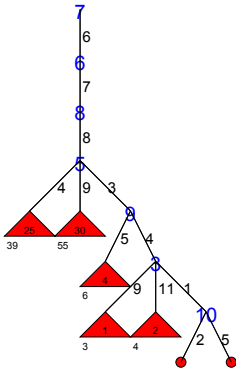
Labeling From Middle



Labeling From Middle



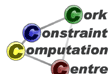
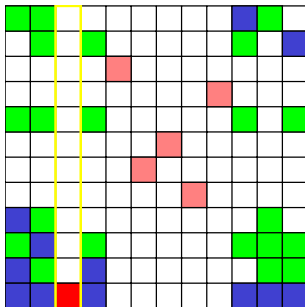
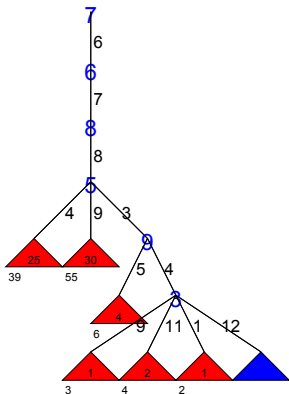
Labeling From Middle



◀ Back to Start

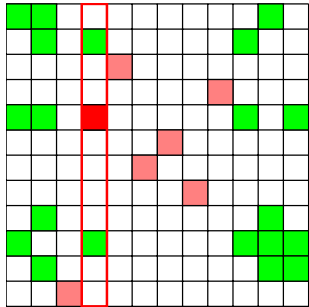
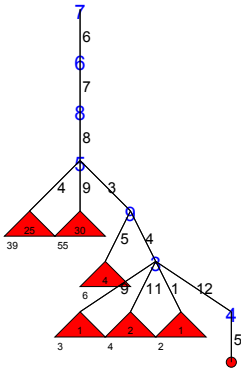
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Labeling From Middle

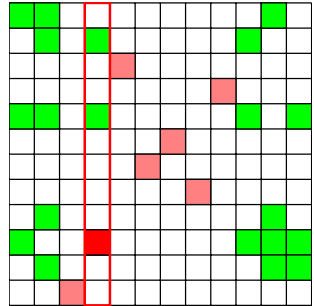
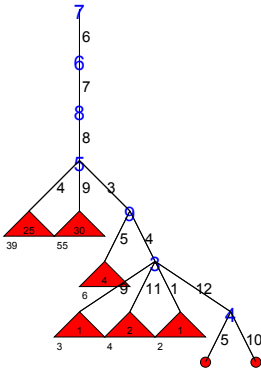


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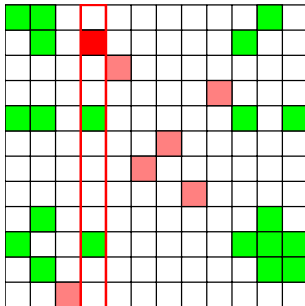
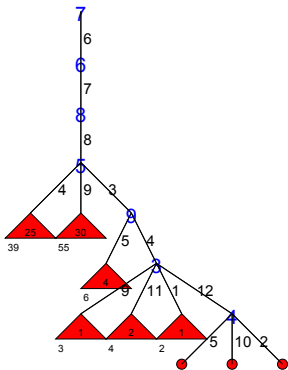
Labeling From Middle



Labeling From Middle



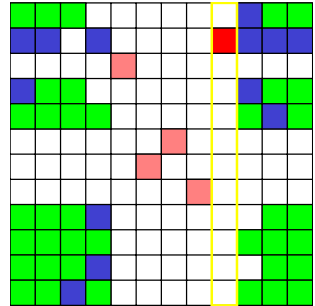
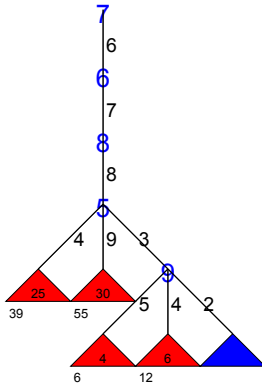
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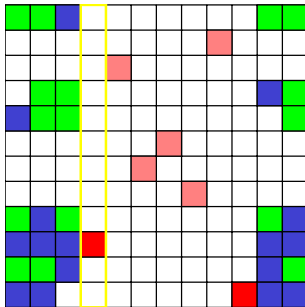
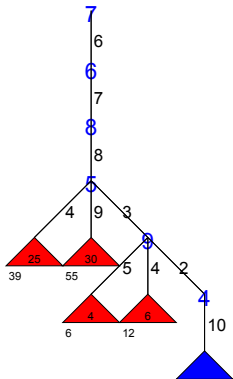
◀ Back to Start

▶ Skip Animation

Labeling From Middle



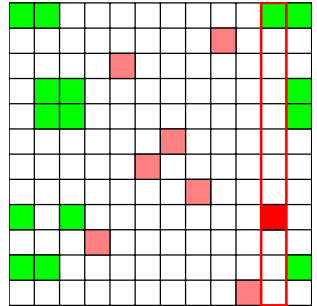
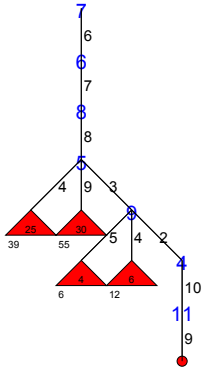
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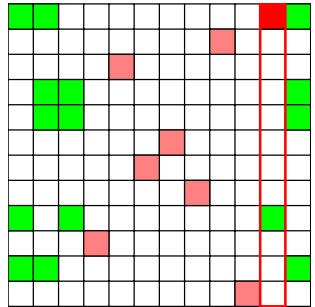
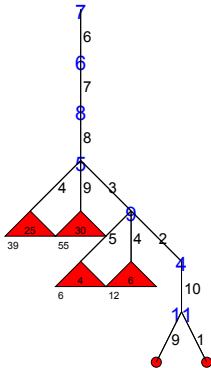
◀ Back to Start

▶▶ Skip Animation

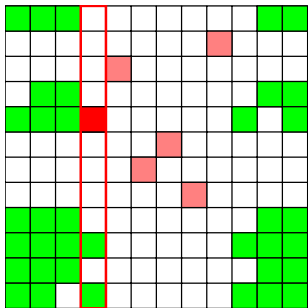
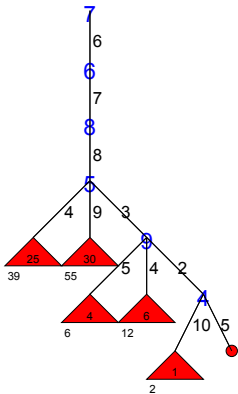
Labeling From Middle



Labeling From Middle



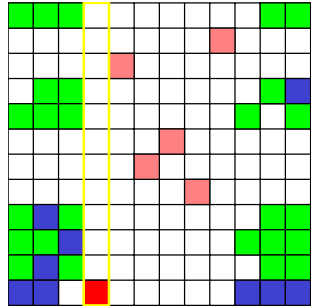
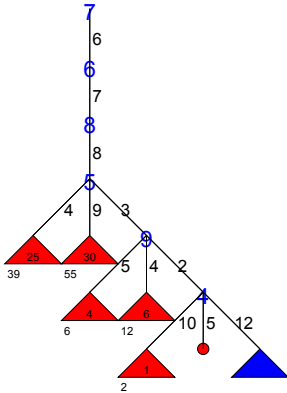
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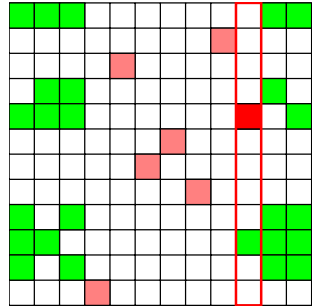
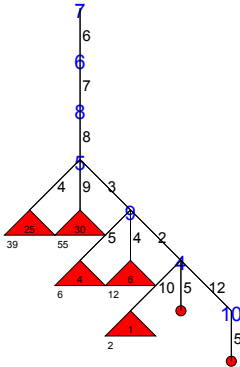
◀ Back to Start

▶▶ Skip Animation

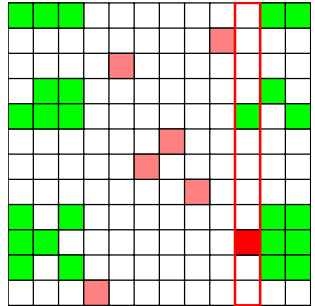
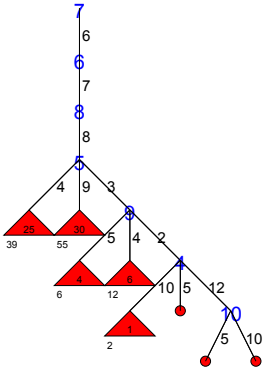
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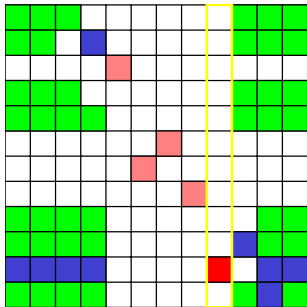
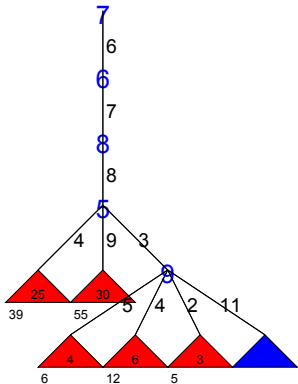
Labeling From Middle



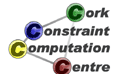
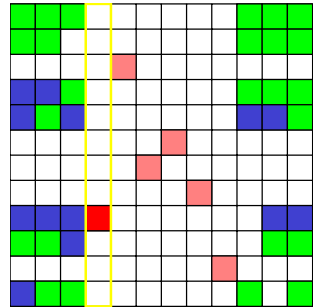
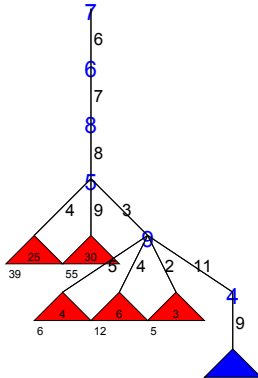
Labeling From Middle



Labeling From Middle



Labeling From Middle

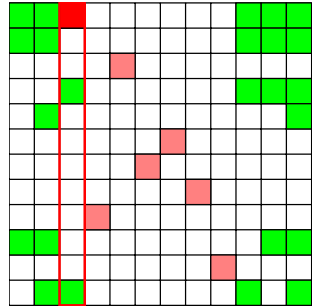
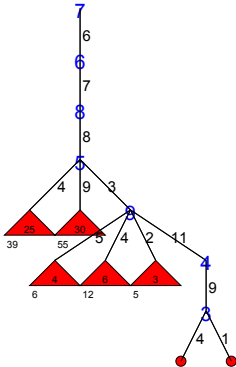


◀ Back to Start

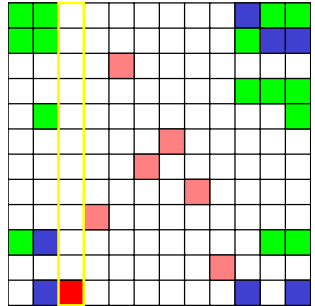
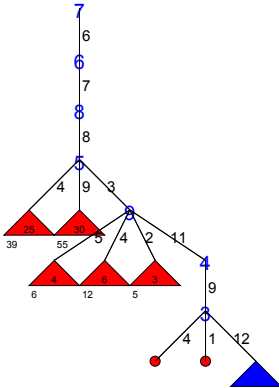
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Labeling From Middle



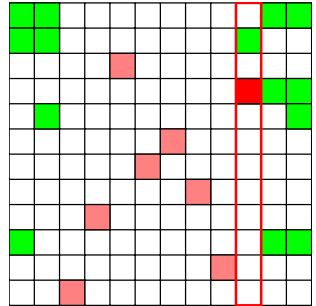
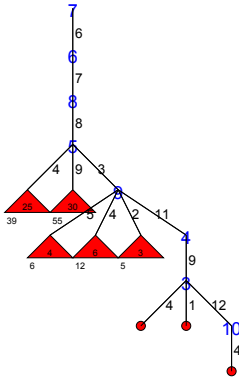
Labeling From Middle



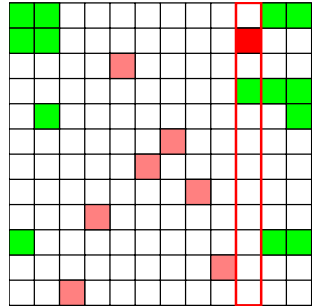
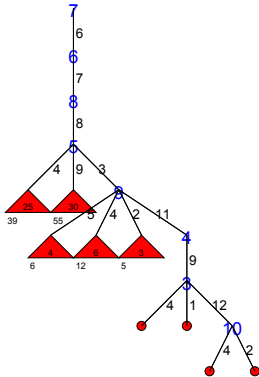
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



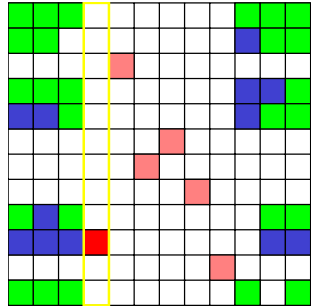
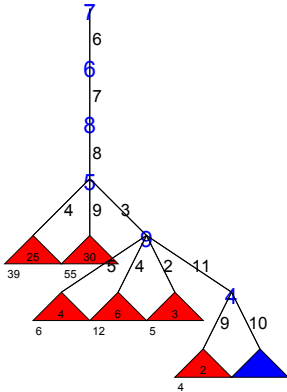
Labeling From Middle



◀ Back to Start

▶ Skip Animation

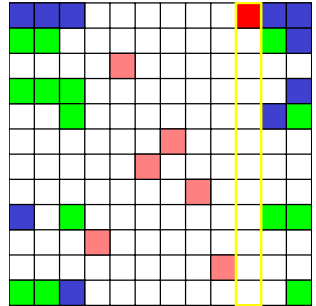
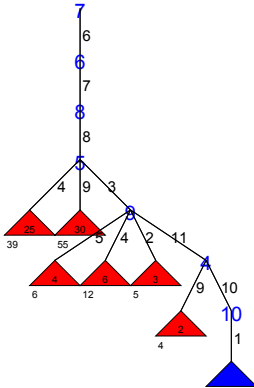
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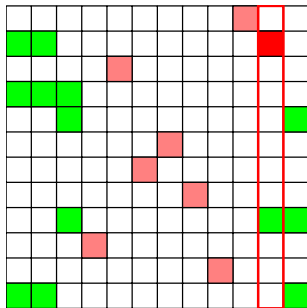
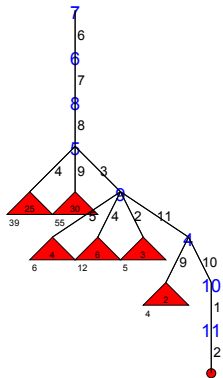
◀ Back to Start

▶ Skip Animation

Labeling From Middle



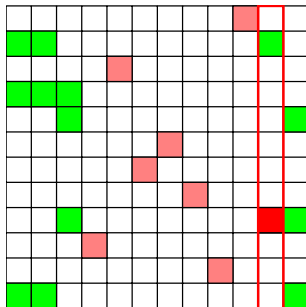
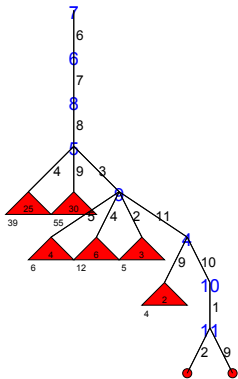
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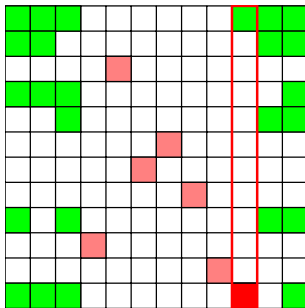
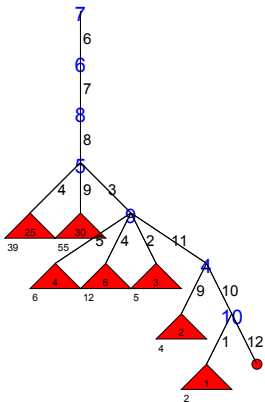
◀ Back to Start

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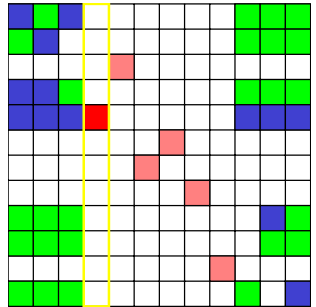
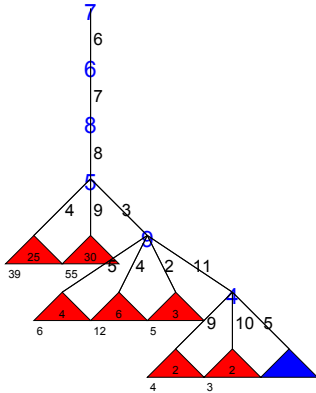
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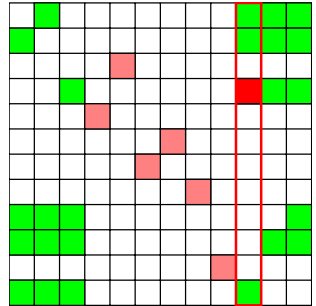
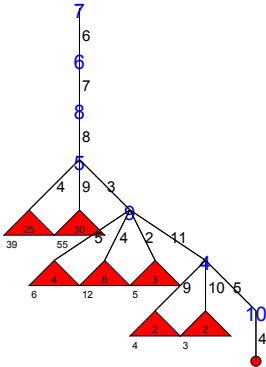
Labeling From Middle



Labeling From Middle



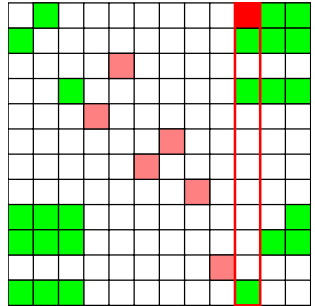
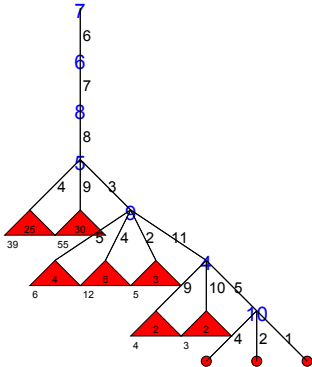
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◀ Back to Start

▶▶ Skip Animation

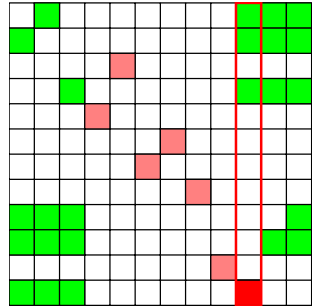
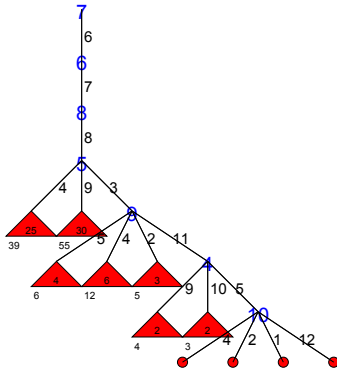
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

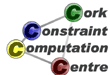
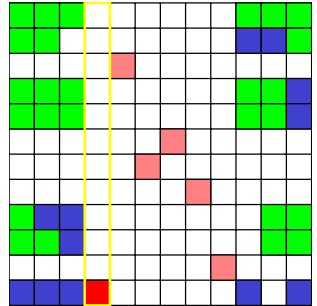
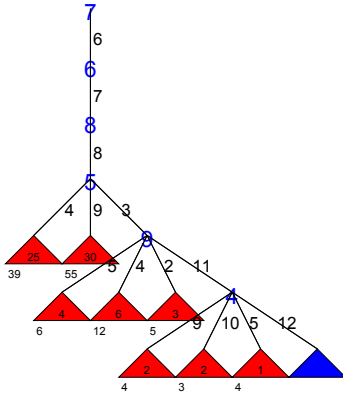
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

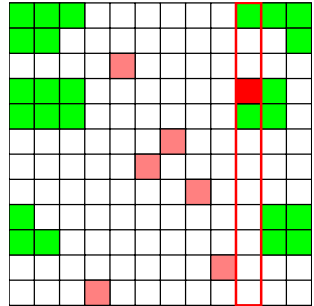
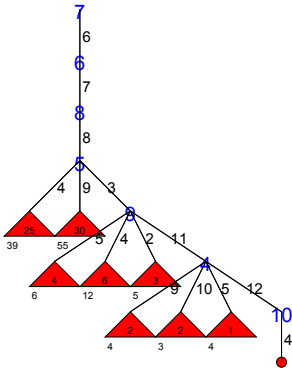


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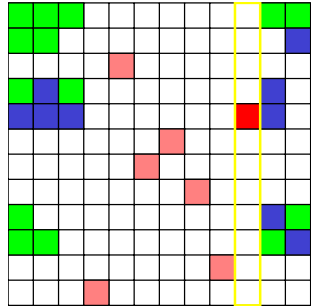
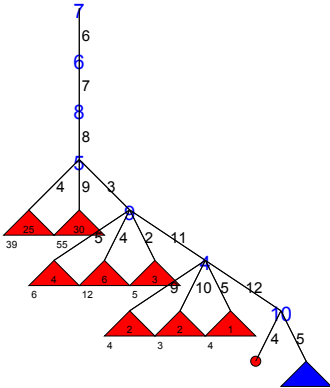
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Labeling From Middle



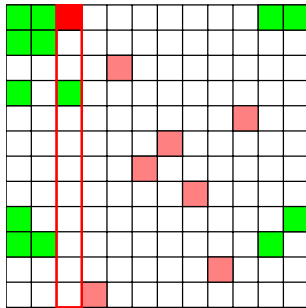
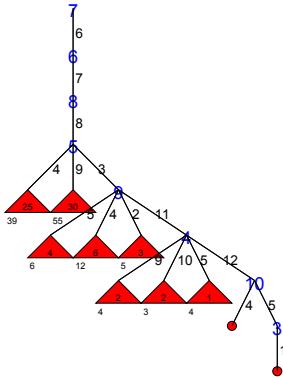
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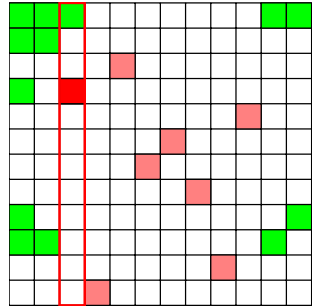
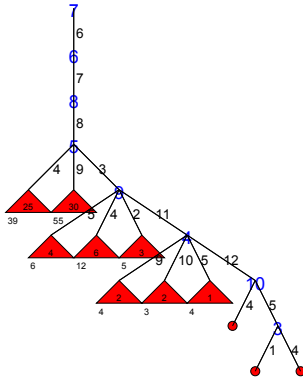
◀ Back to Start

▶▶ Skip Animation

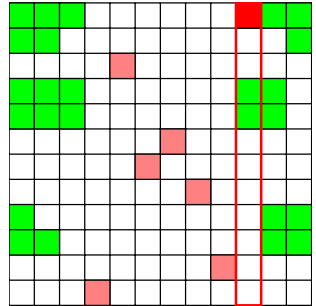
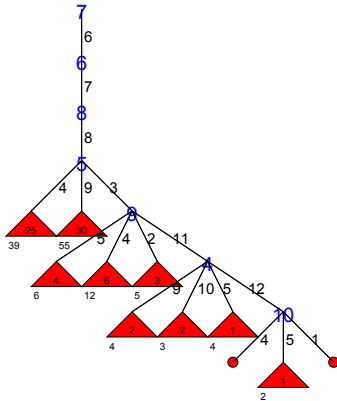
Labeling From Middle



Labeling From Middle



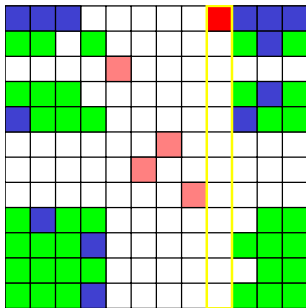
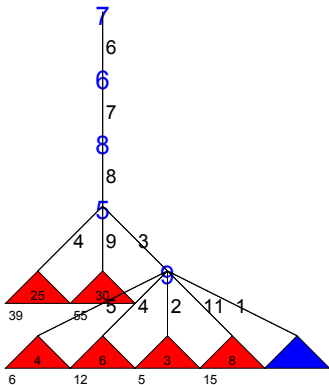
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

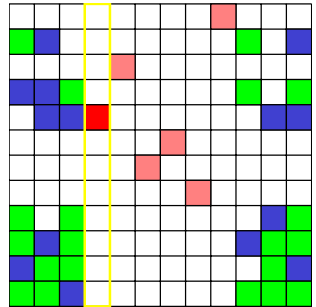
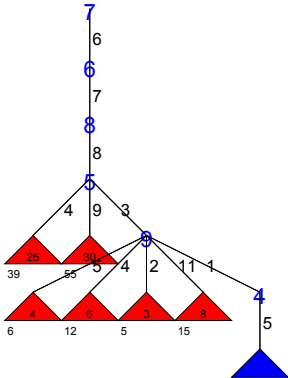
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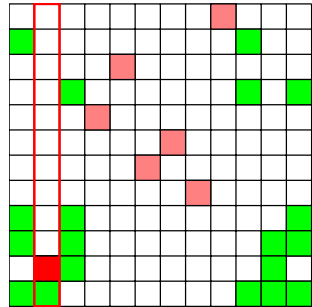
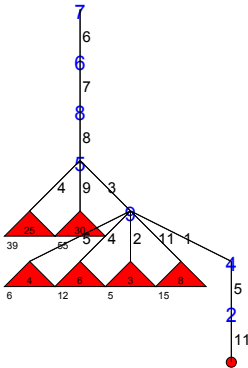
◀ Back to Start

▶ Skip Animation

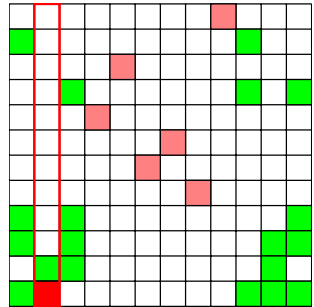
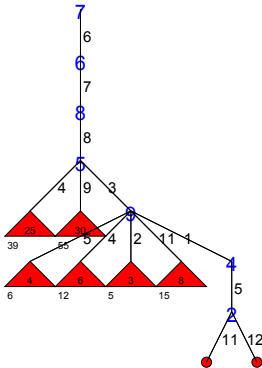
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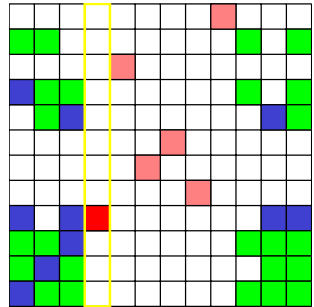
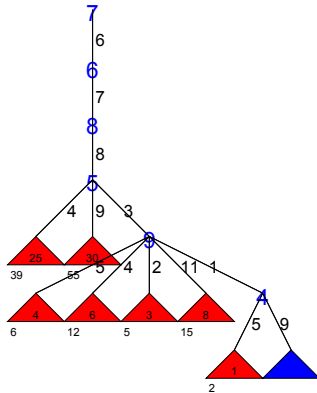
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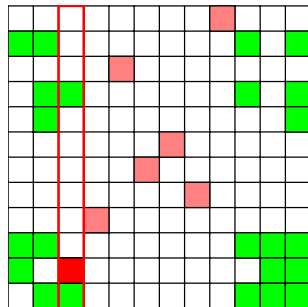
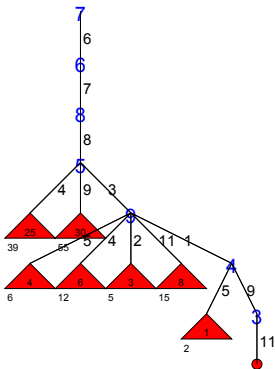
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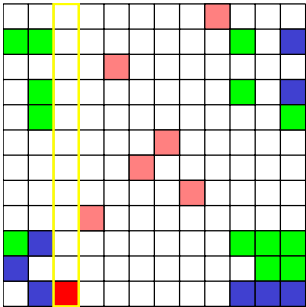
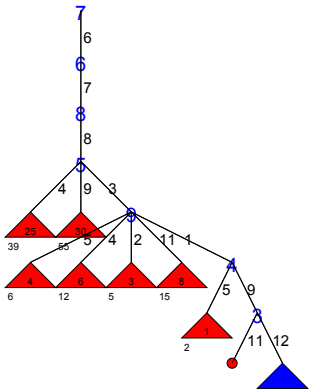
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Labeling From Middle



Labeling From Middle

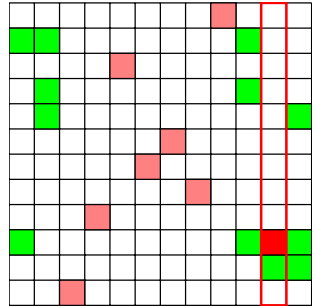
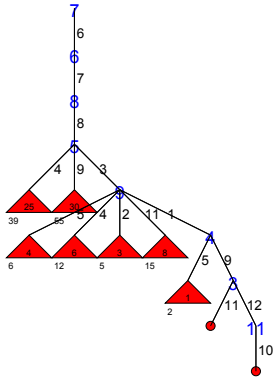


◀ Back to Start

▶▶ Skip Animation



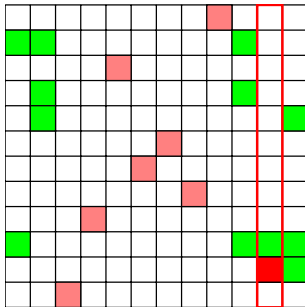
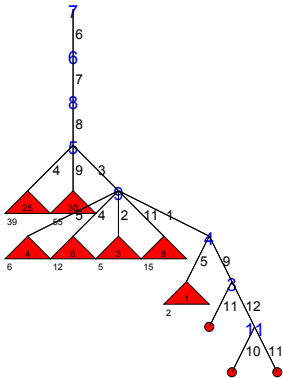
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

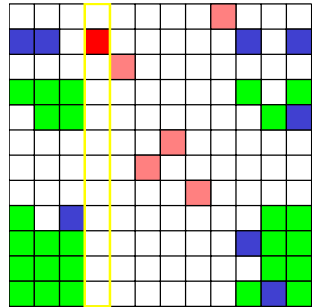
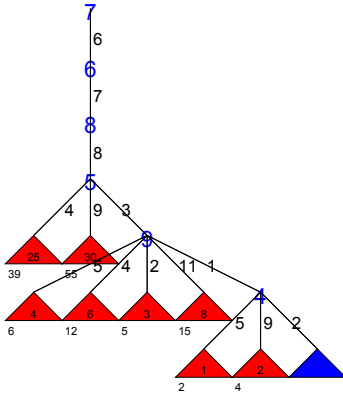
Labeling From Middle



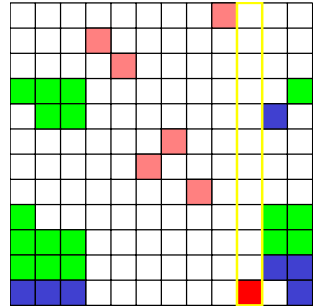
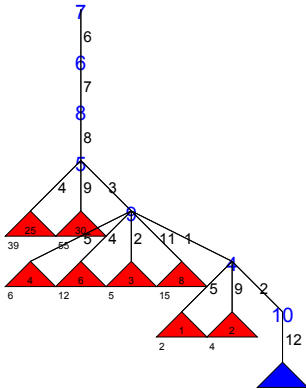
◀ Back to Start

▶ Skip Animation

Labeling From Middle



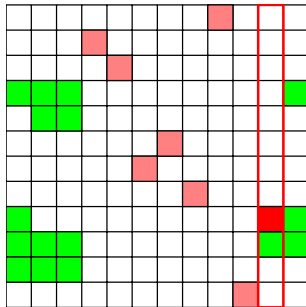
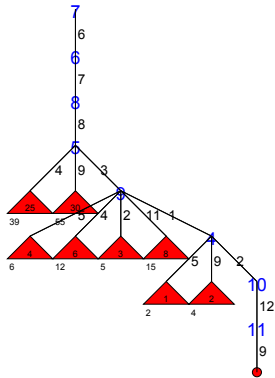
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

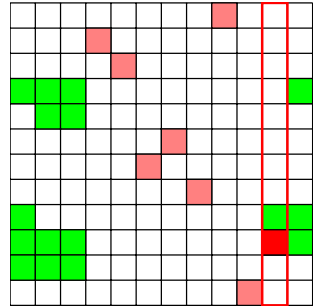
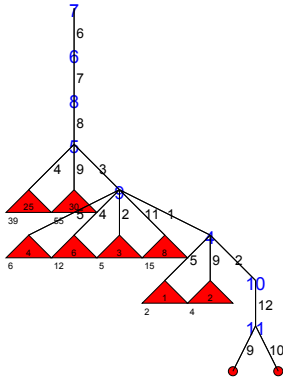


◀ Back to Start

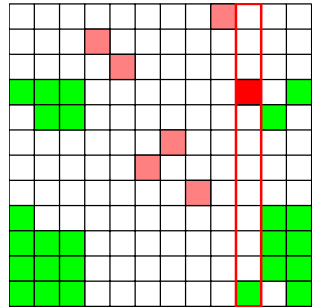
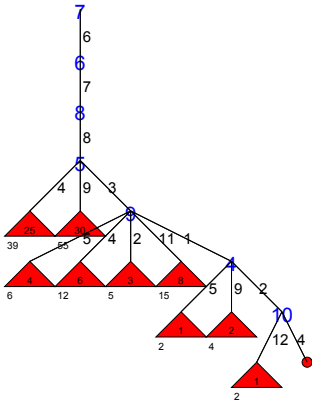
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Labeling From Middle



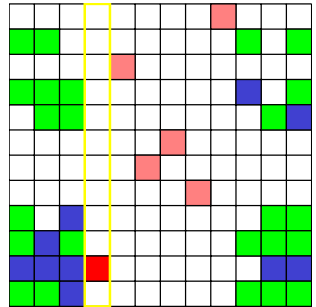
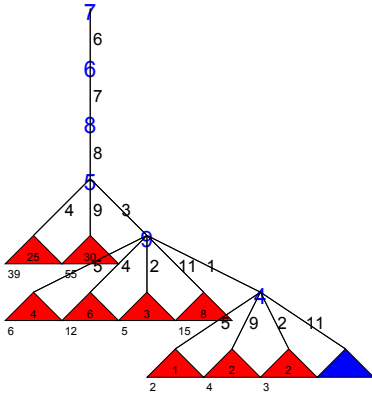
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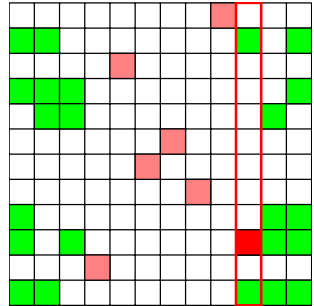
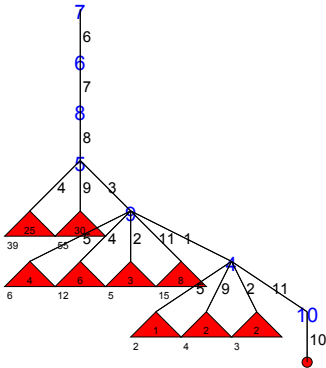
◀ Back to Start

▶ Skip Animation

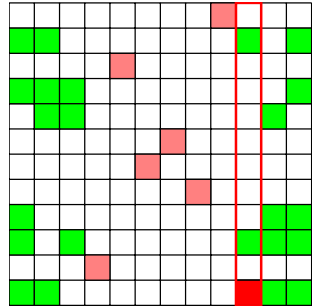
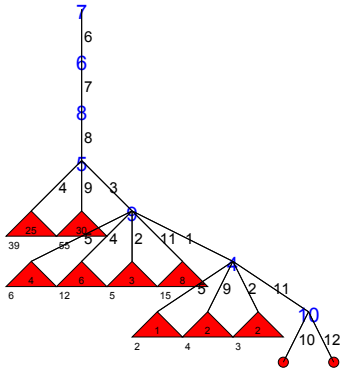
Labeling From Middle



Labeling From Middle



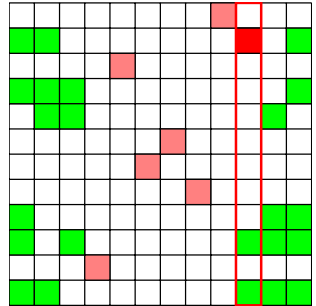
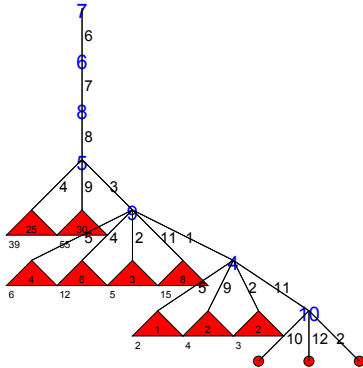
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

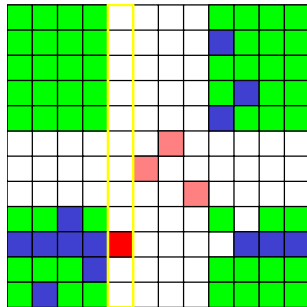
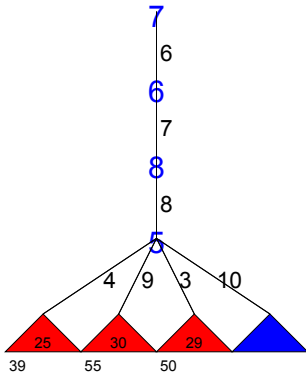
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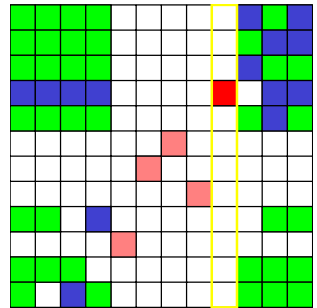
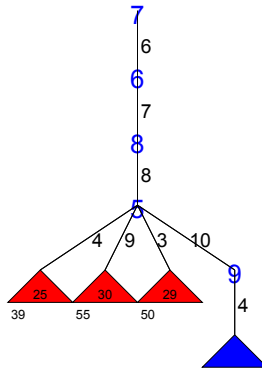
◀ Back to Start

▶▶ Skip Animation

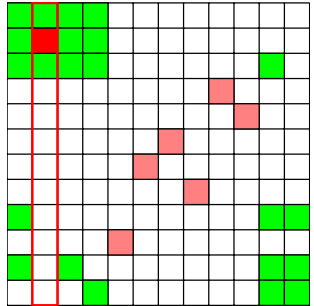
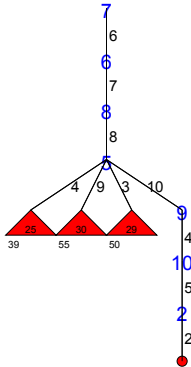
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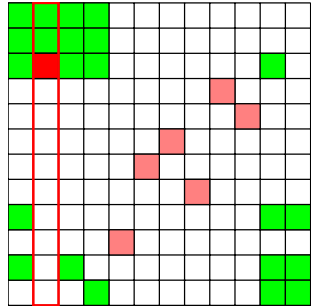
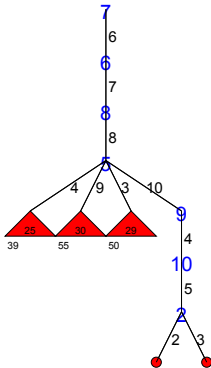
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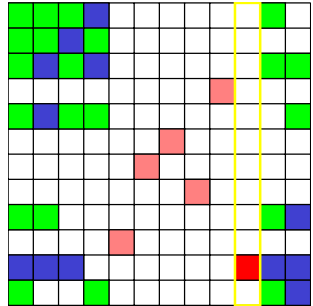
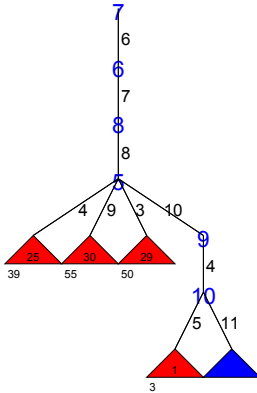
Labeling From Middle



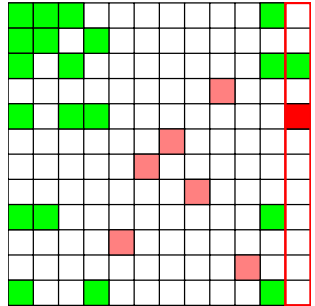
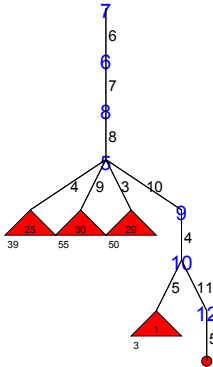
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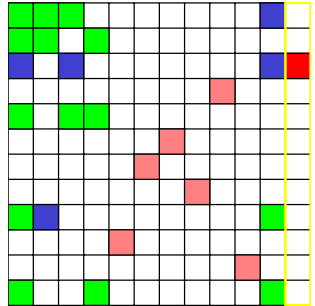
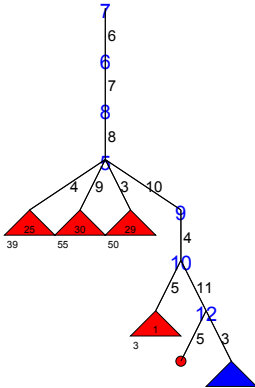
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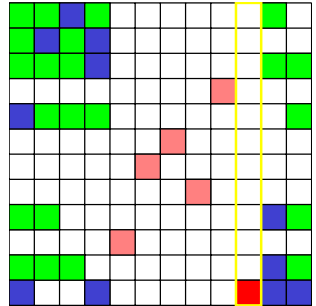
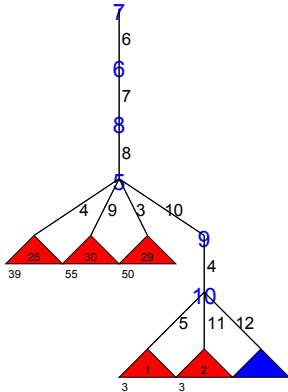
Labeling From Middle



Labeling From Middle



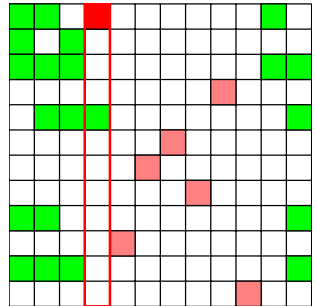
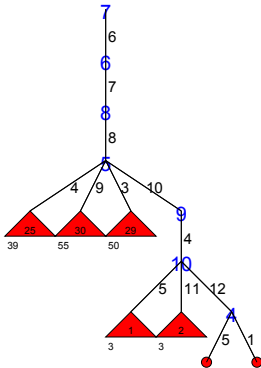
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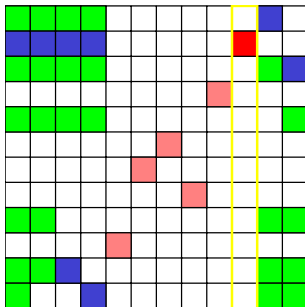
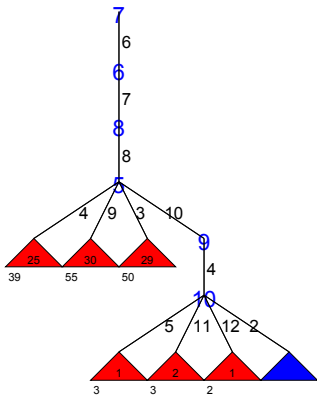
◀ Back to Start

▶ Skip Animation

Labeling From Middle



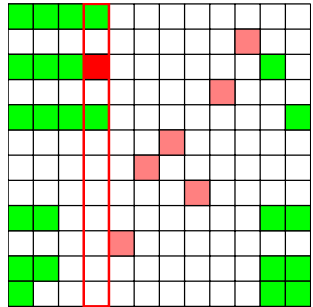
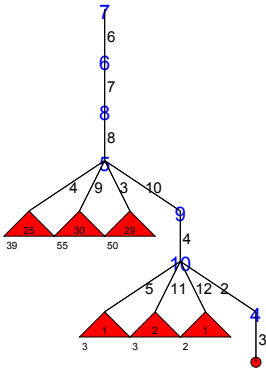
Labeling From Middle



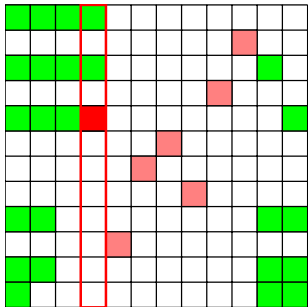
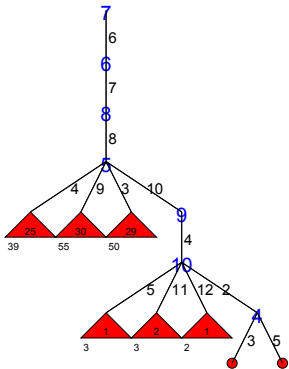
◀ Back to Start

▶ Skip Animation

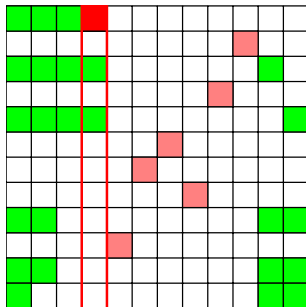
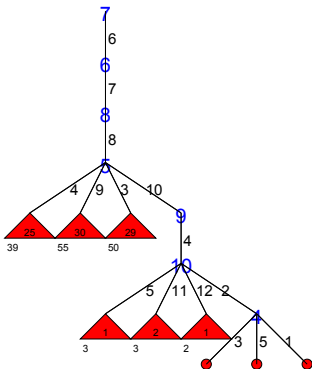
Labeling From Middle



Labeling From Middle



Labeling From Middle

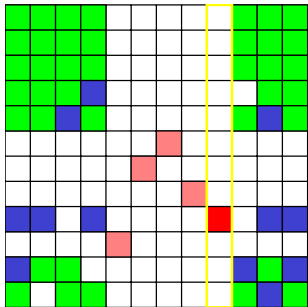
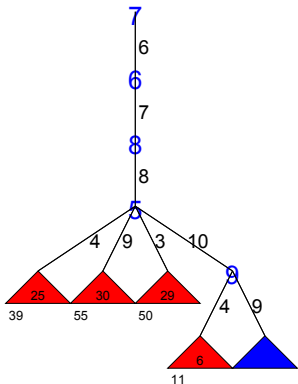


◀ Back to Start

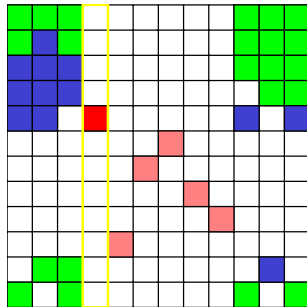
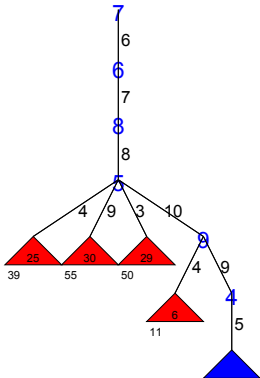
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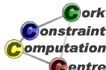
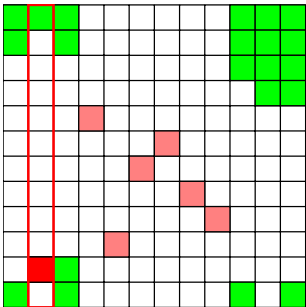
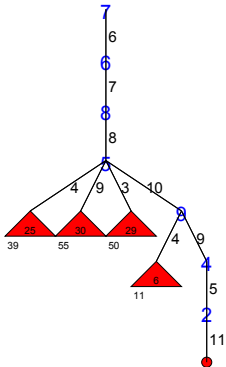
Labeling From Middle



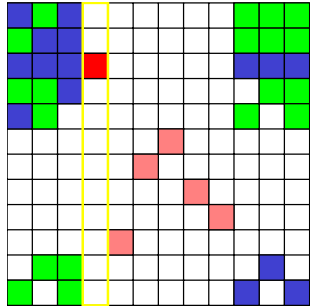
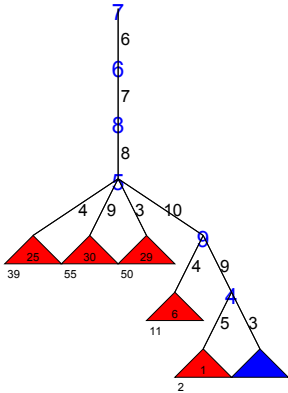
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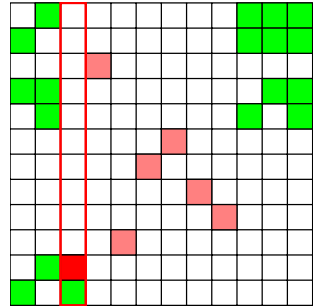
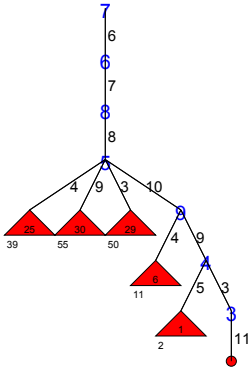
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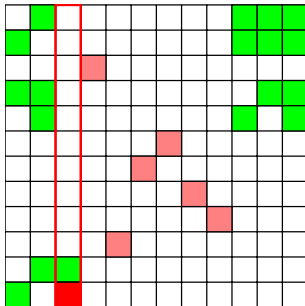
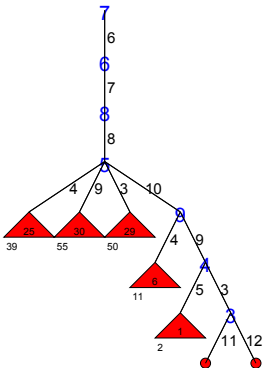
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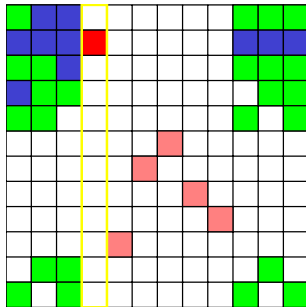
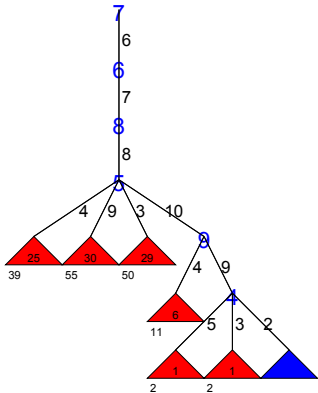
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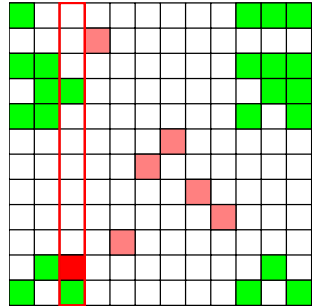
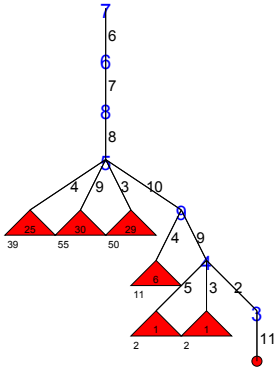
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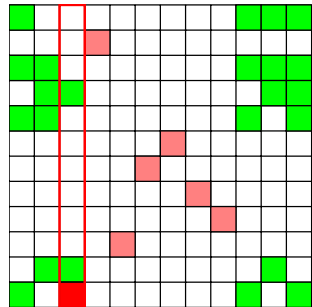
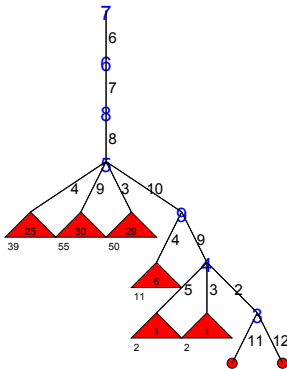
Labeling From Middle



Labeling From Middle



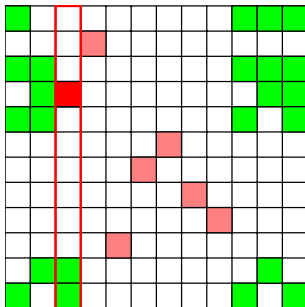
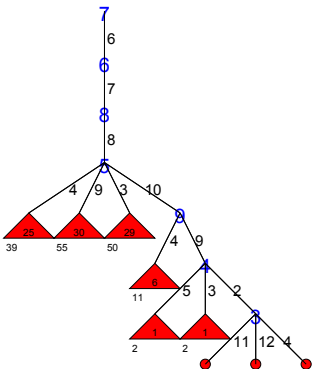
Labeling From Middle



◀ Back to Start

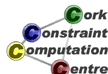
▶ Skip Animation

Labeling From Middle

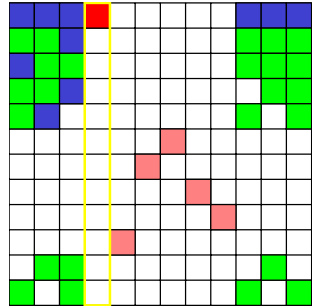
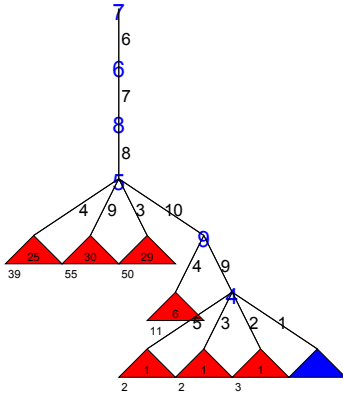


◀ Back to Start

▶▶ Skip Animation



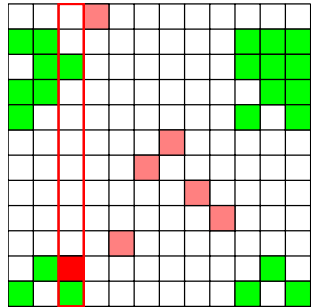
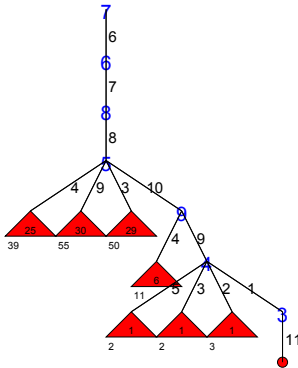
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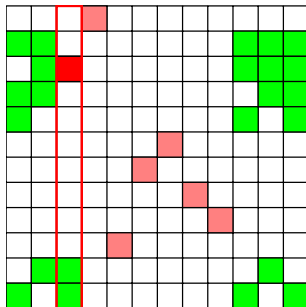
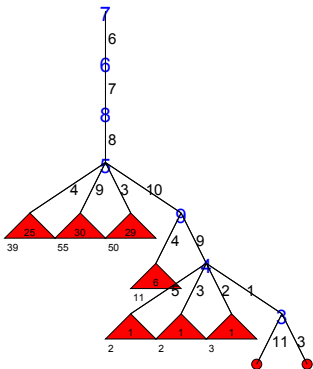
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



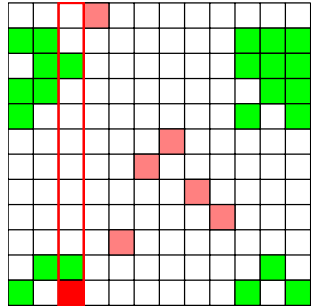
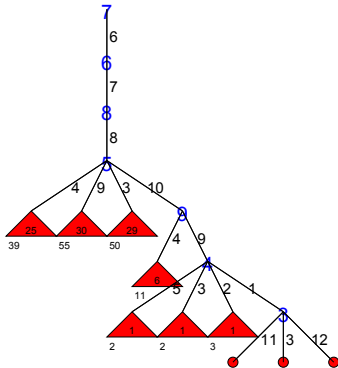
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

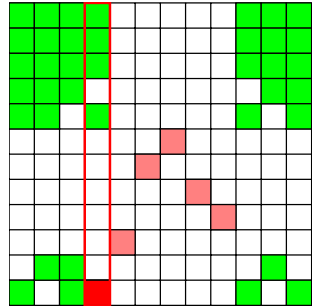
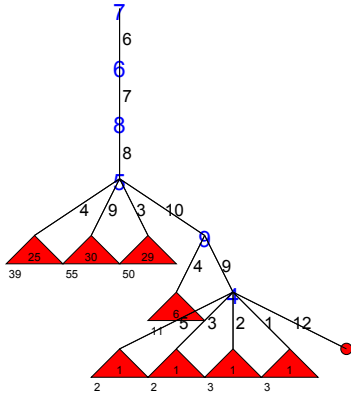
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

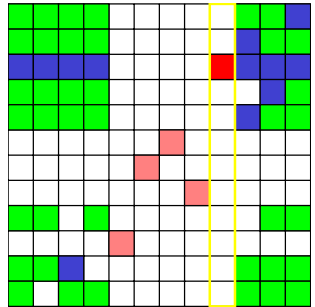
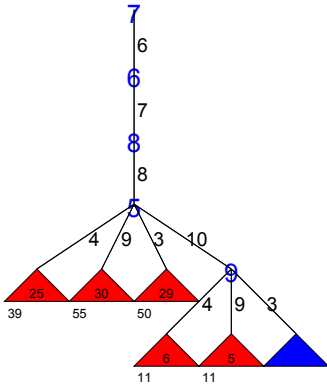
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

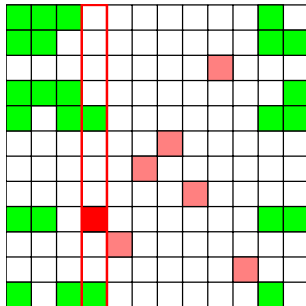
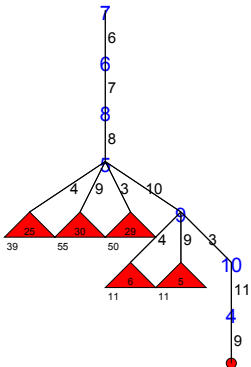
Labeling From Middle



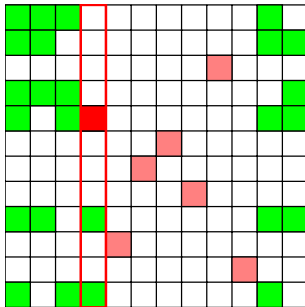
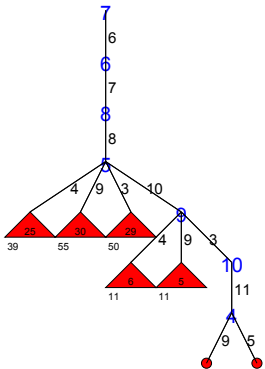
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



Labeling From Middle

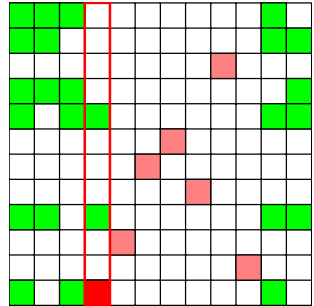
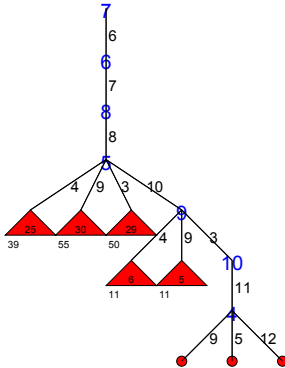


◀ Back to Start

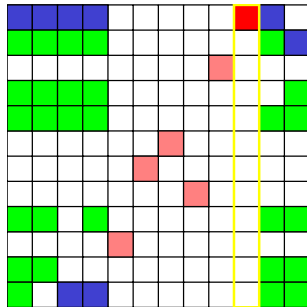
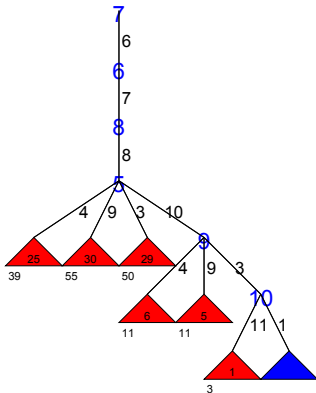
▶▶ Skip Animation



Labeling From Middle



Labeling From Middle

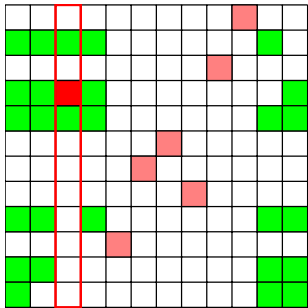
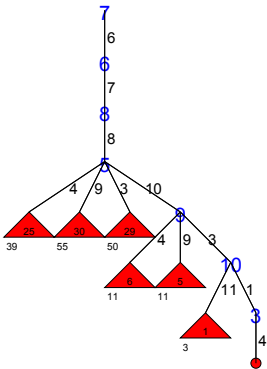


◀ Back to Start

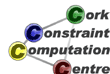
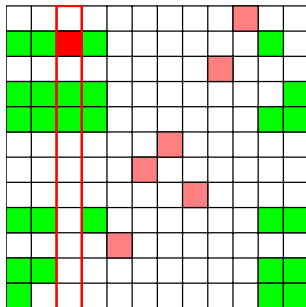
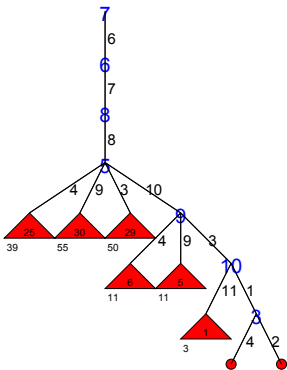
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Labeling From Middle



Labeling From Middle

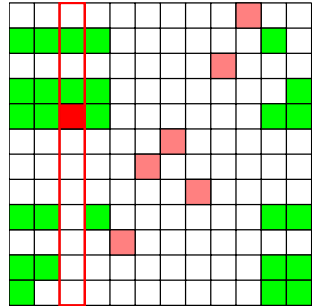
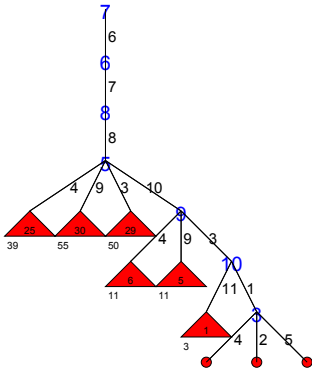


◀ Back to Start

▶▶ Skip Animation



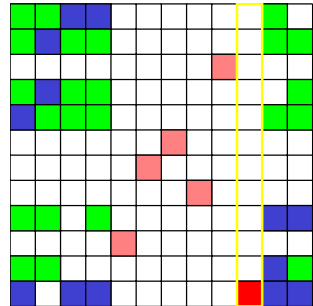
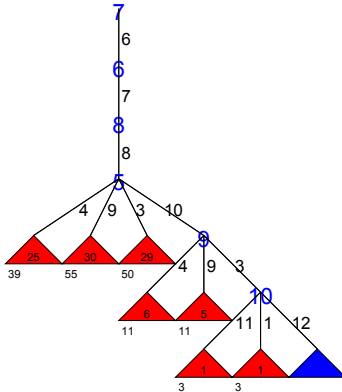
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

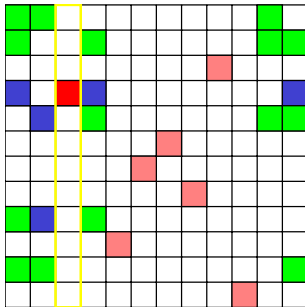
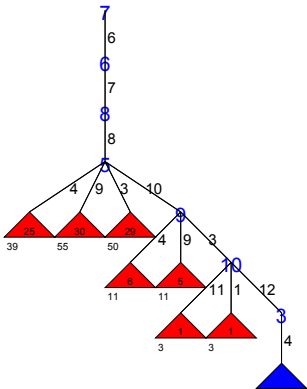


◀ Back to Start

▶▶ Skip Animation



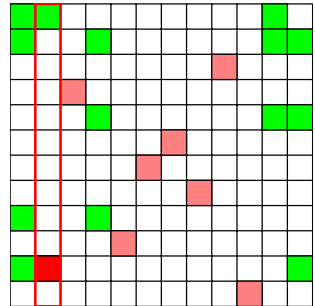
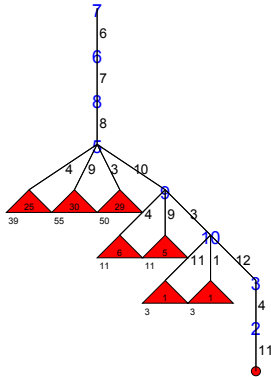
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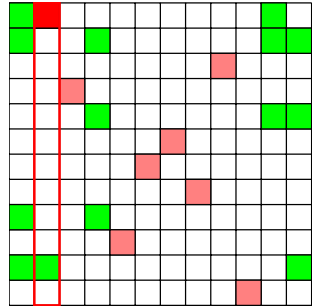
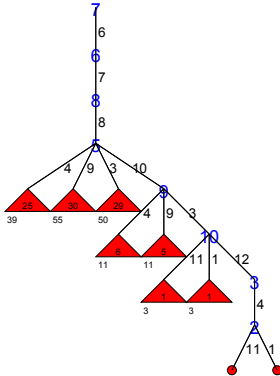
◀ Back to Start

▶▶ Skip Animation

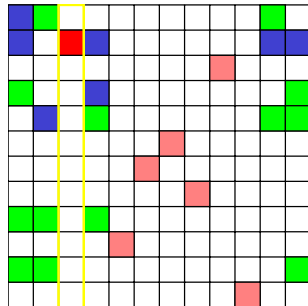
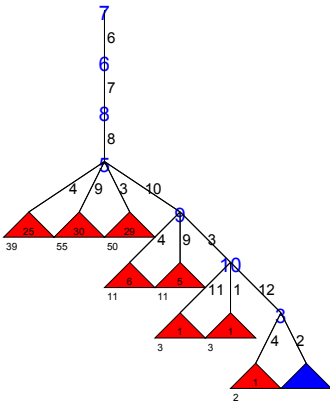
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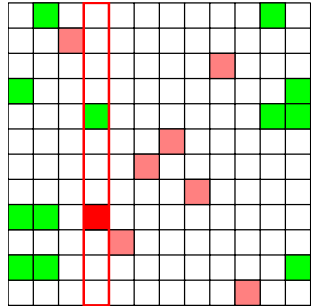
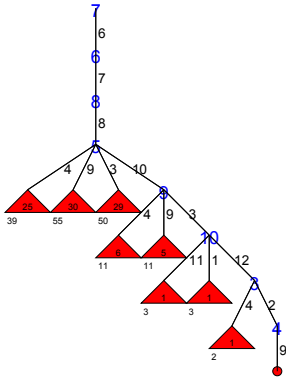
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Labeling From Middle



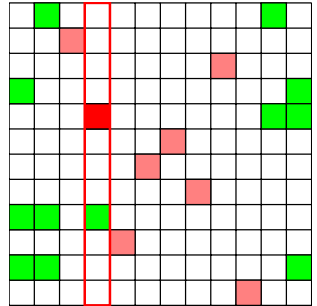
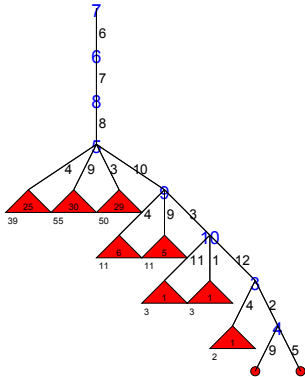
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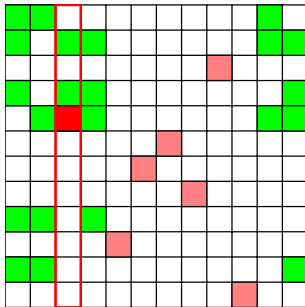
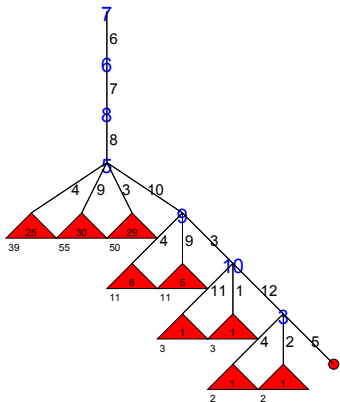
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Labeling From Middle



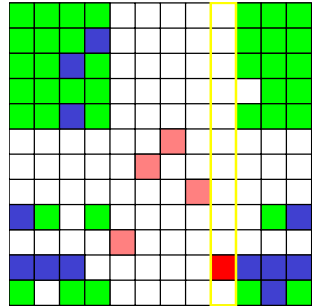
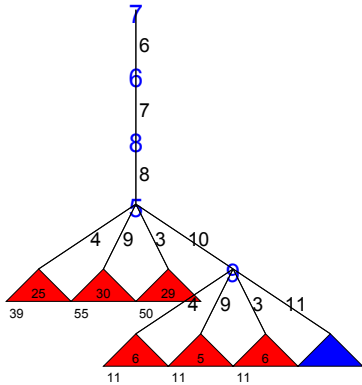
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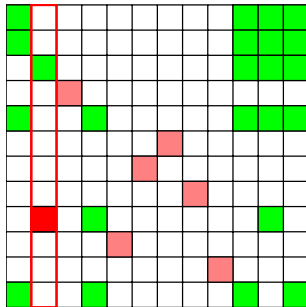
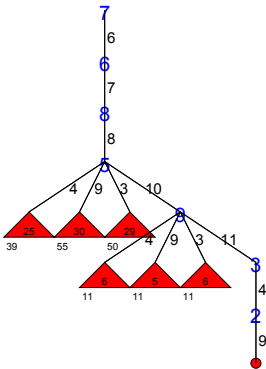
[Back to Start](#)

[Skip Animation](#)

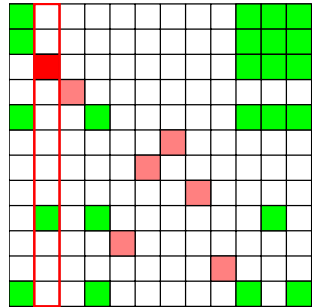
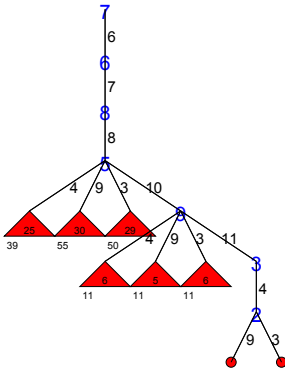
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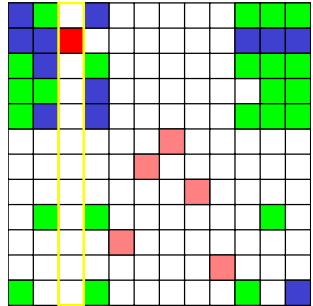
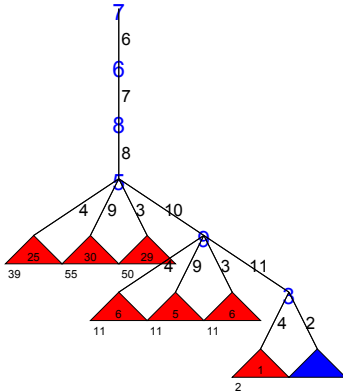
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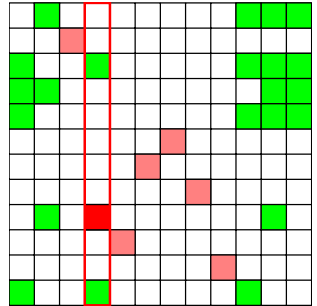
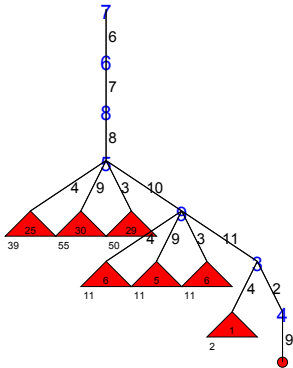
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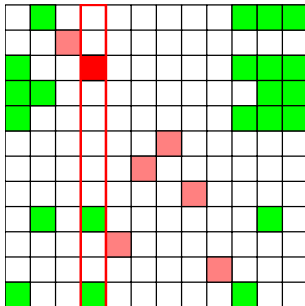
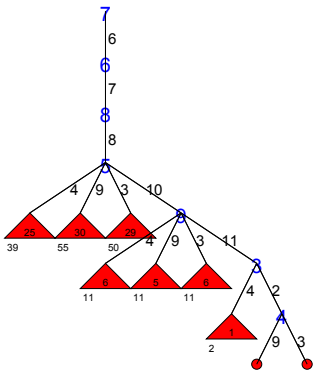
Labeling From Middle



Labeling From Middle



Labeling From Middle

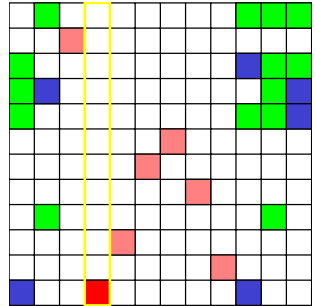
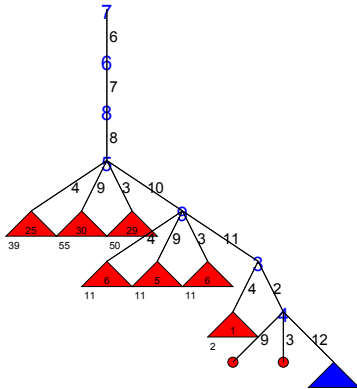


◀ Back to Start

▶▶ Skip Animation



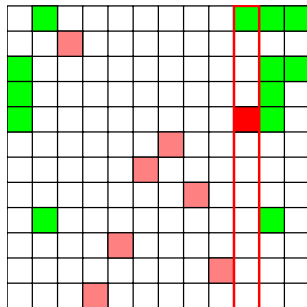
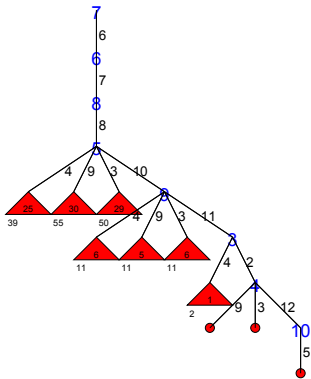
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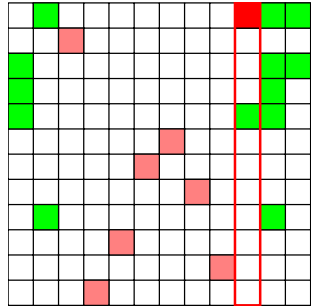
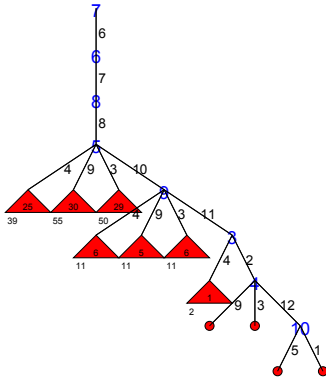
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



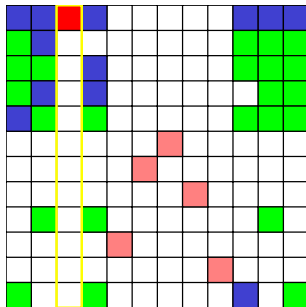
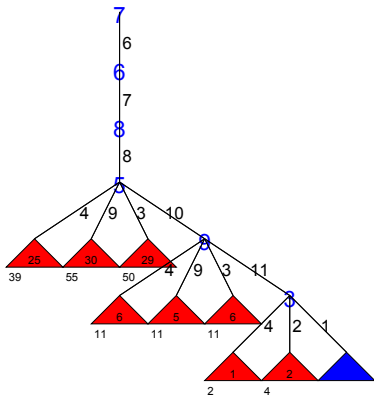
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◀ Back to Start

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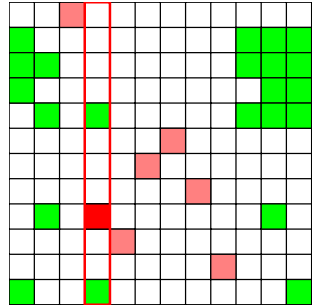
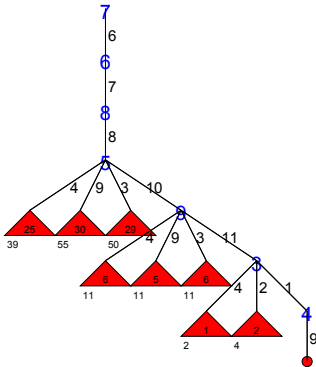
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◀ Back to Start

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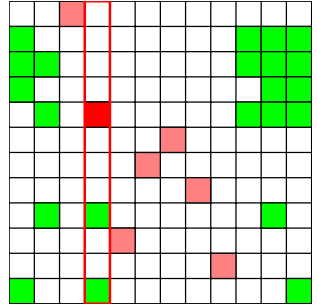
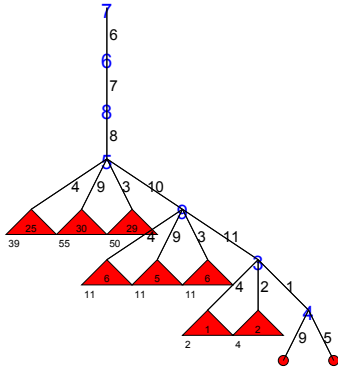
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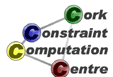
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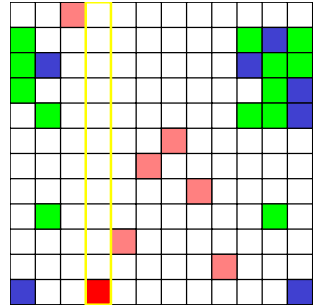
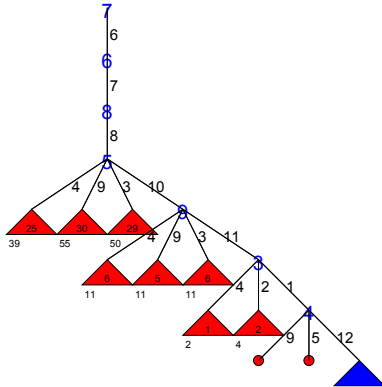
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◀ Back to Start ▶ Skip Animation



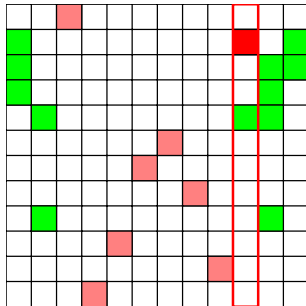
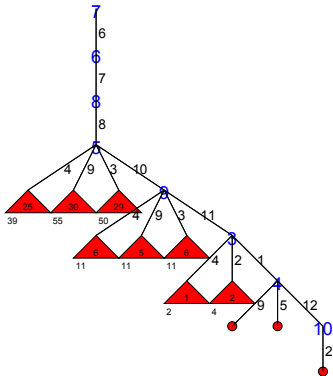
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

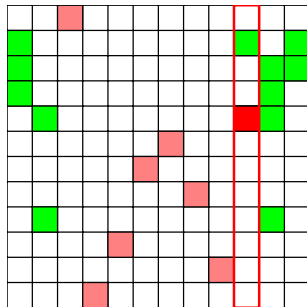
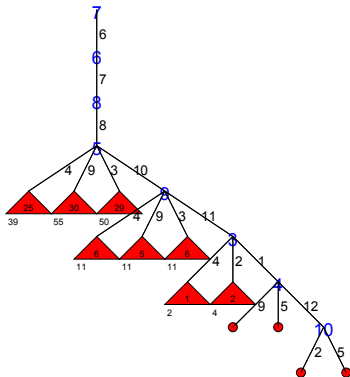
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◀ Back to Start

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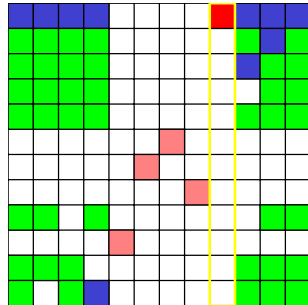
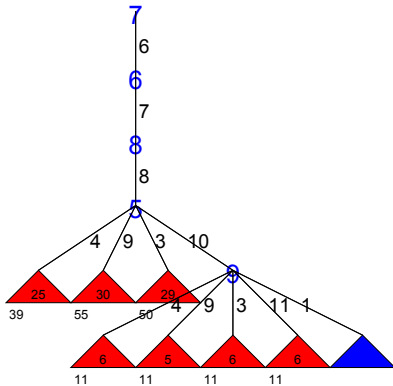
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◀ Back to Start

▶ Skip Animation

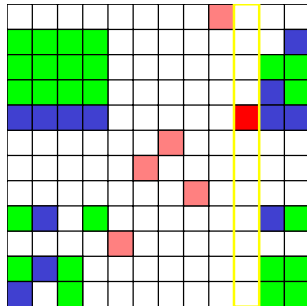
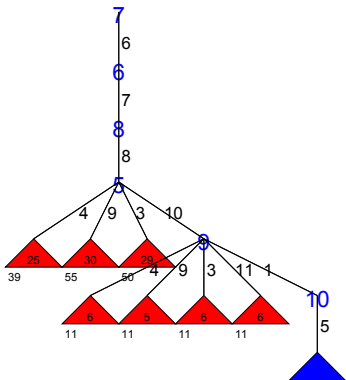
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◀ Back to Start

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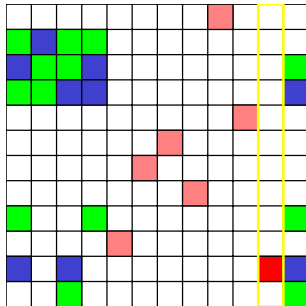
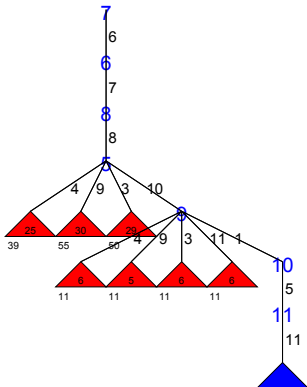
Labeling From Middle



◀ Back to Start ▶ Skip Animation



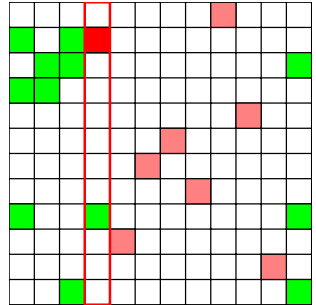
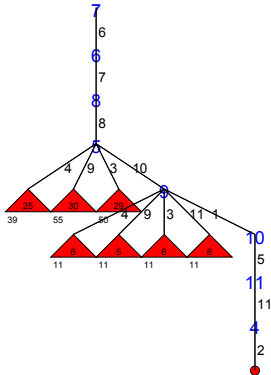
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◀ Back to Start

▶▶ Skip Animation

Labeling From Middle

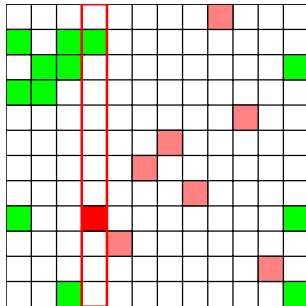
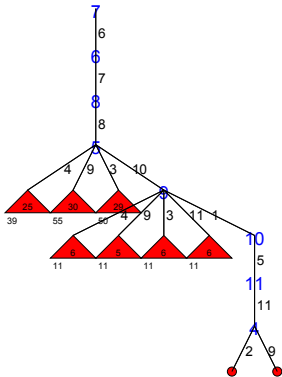


◀ Back to Start

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Labeling From Middle

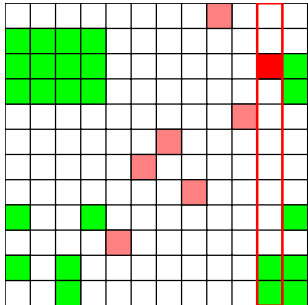
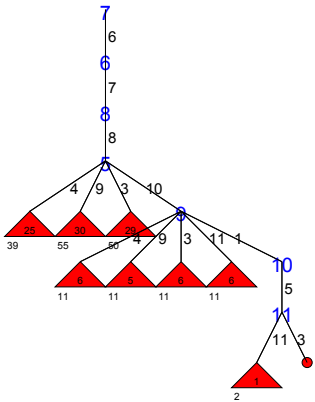


◀ Back to Start

▶ Skip Animation



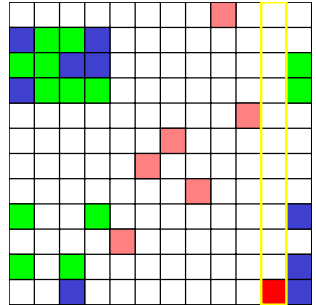
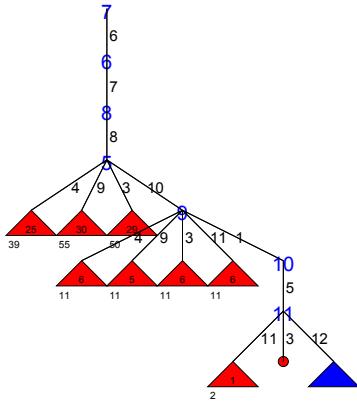
Labeling From Middle



◀ Back to Start

▶ Skip Animation

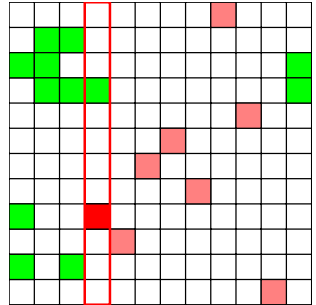
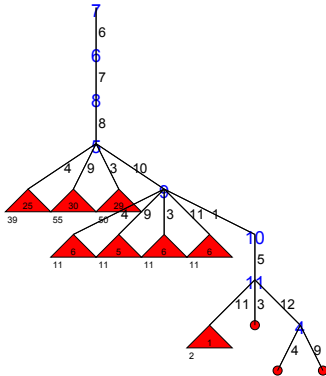
Labeling From Middle



◀ Back to Start

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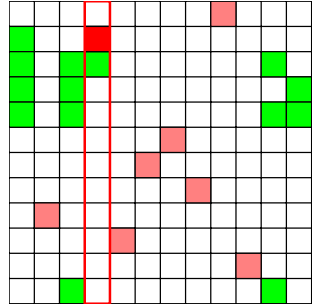
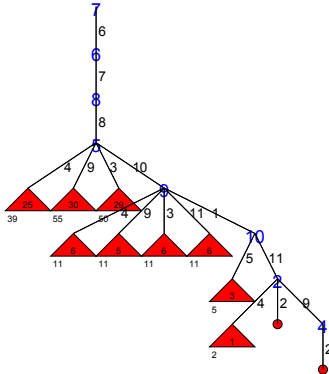
Labeling From Middle



◀ Back to Start

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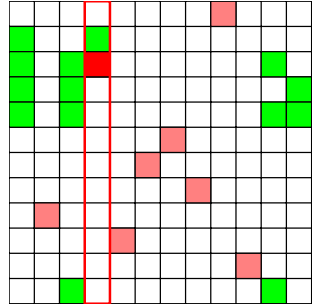
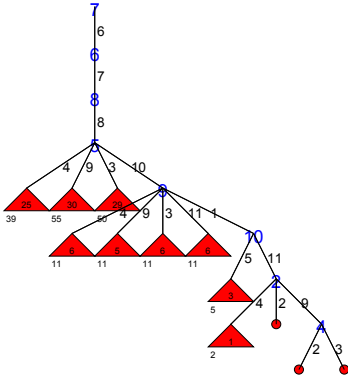
Labeling From Middle



◀ Back to Start

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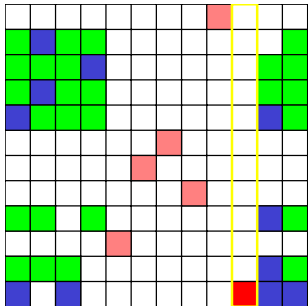
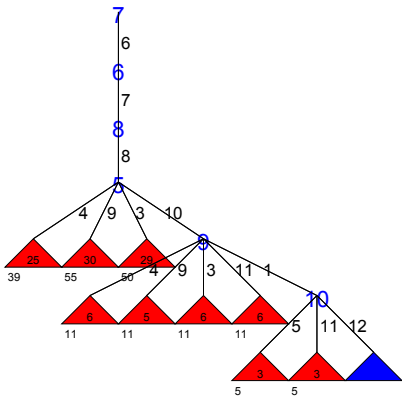
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◀ Back to Start

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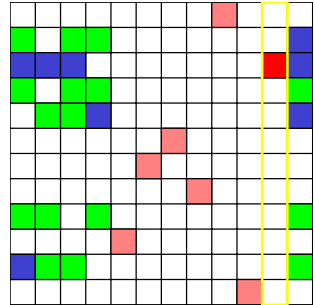
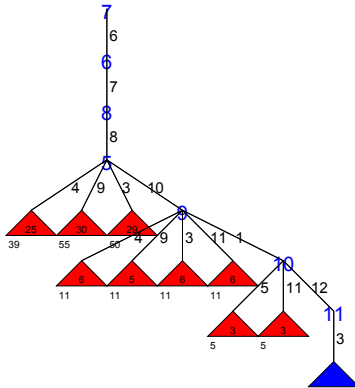
Labeling From Middle



[◀ Back to Start](#) [▶ Skip Animation](#)



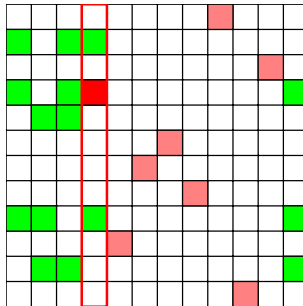
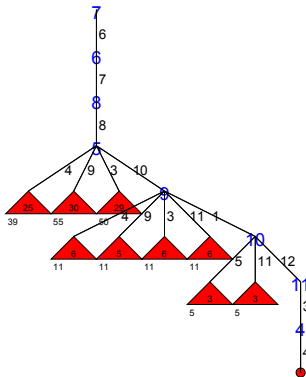
Labeling From Middle



◀ Back to Start

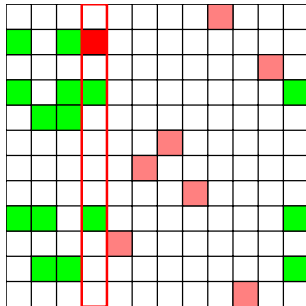
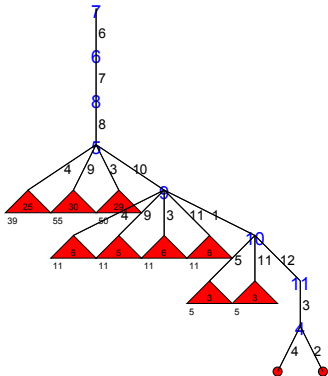
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Labeling From Middle



◀ Back to Start ▶ Skip Animation

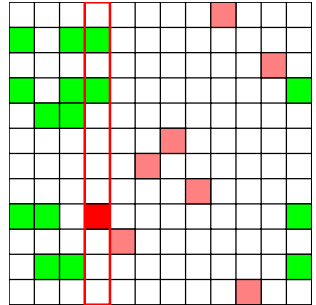
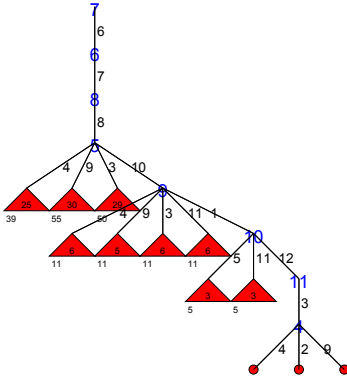
Labeling From Middle



◀ Back to Start

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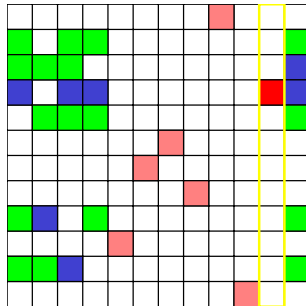
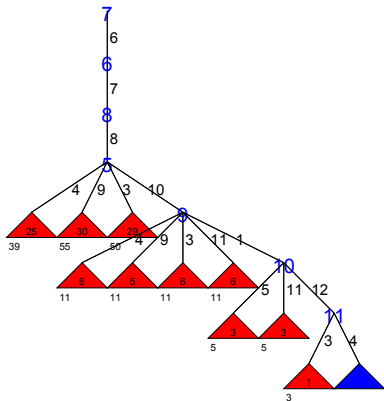
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◀ Back to Start

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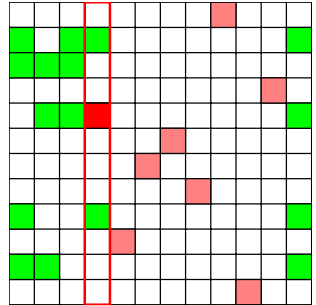
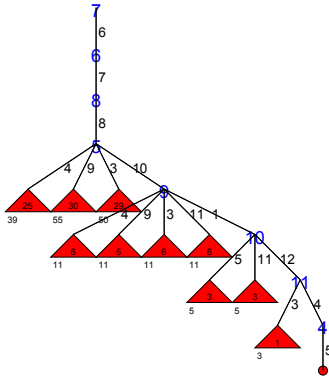
Labeling From Middle



◀ Back to Start ▶ Skip Animation



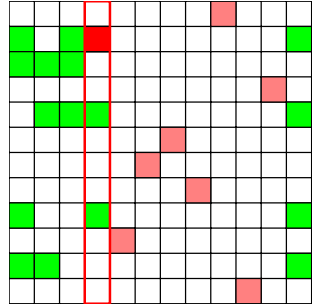
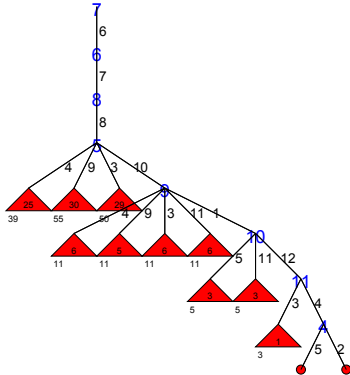
Labeling From Middle



◀ Back to Start

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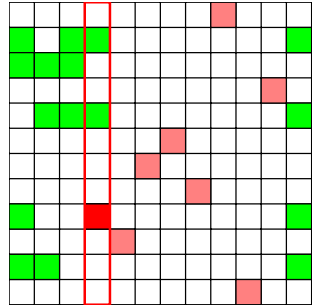
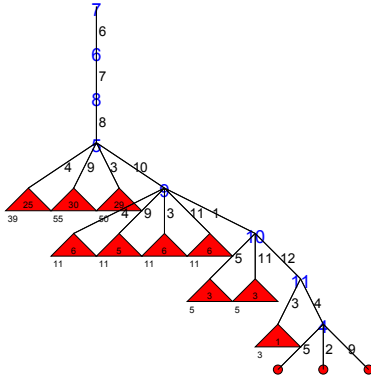
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◀ Back to Start

▶▶ Skip Animation

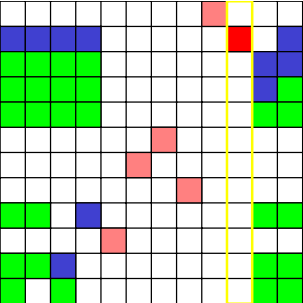
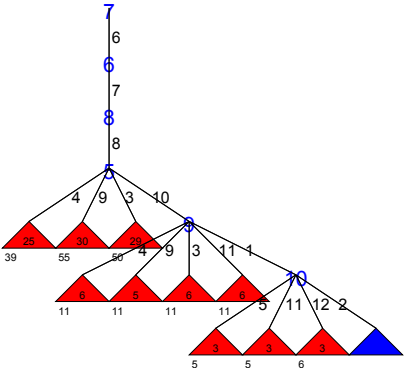
Labeling From Middle



◀ Back to Start

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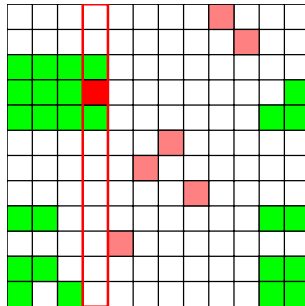
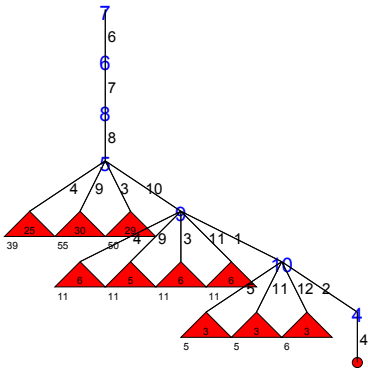
Labeling From Middle



[◀ Back to Start](#) [▶▶ Skip Animation](#)



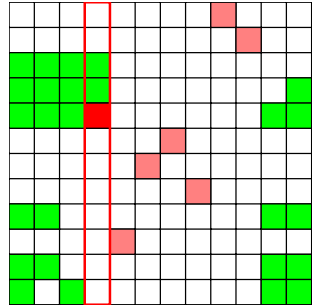
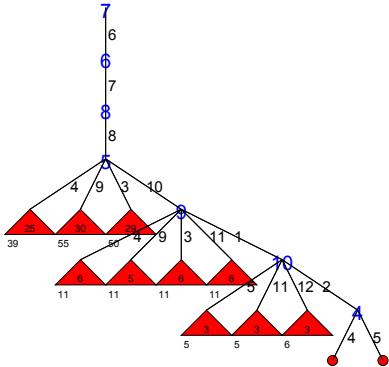
Labeling From Middle



◀ Back to Start

▶ Skip Animation

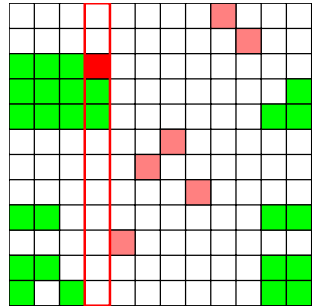
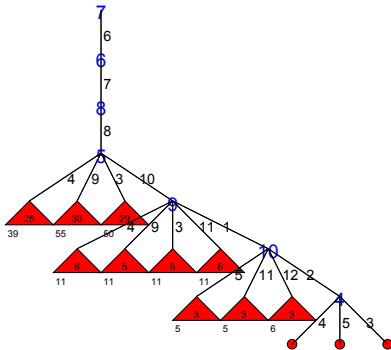
Labeling From Middle



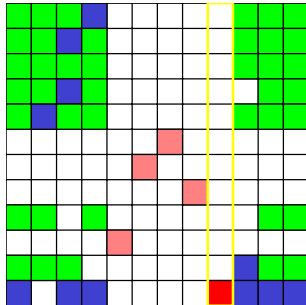
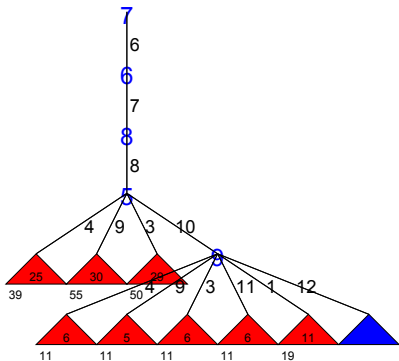
◀ Back to Start

▶▶ Skip Animation

Labeling From Middle



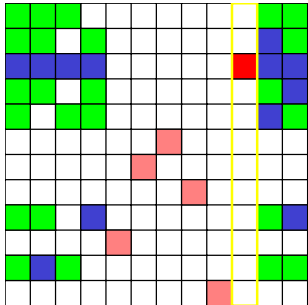
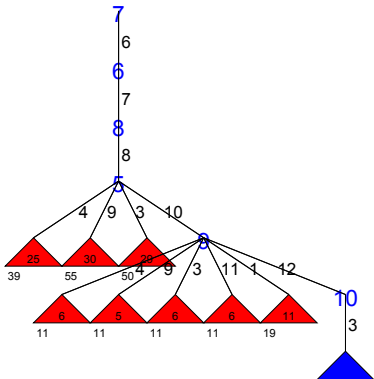
Labeling From Middle



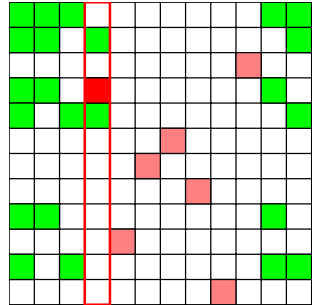
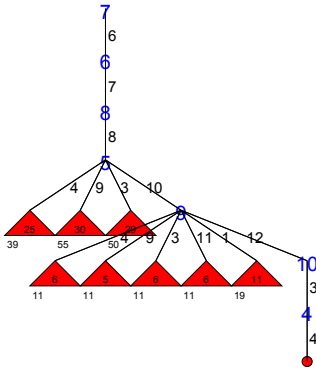
◀ Back to Start

▶ Skip Animation

Labeling From Middle



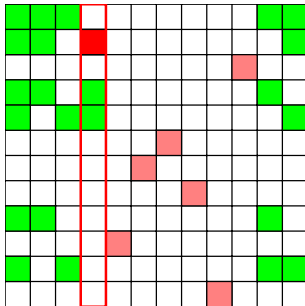
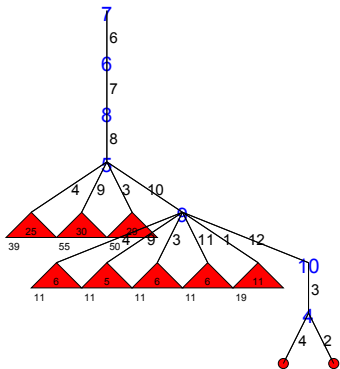
Labeling From Middle



◀ Back to Start

▶ Skip Animation

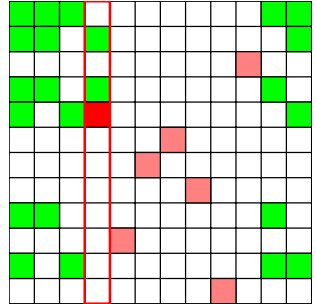
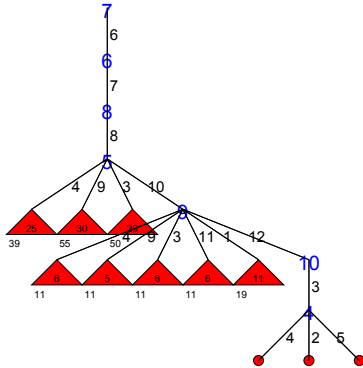
Labeling From Middle



◀ Back to Start

▶ Skip Animation

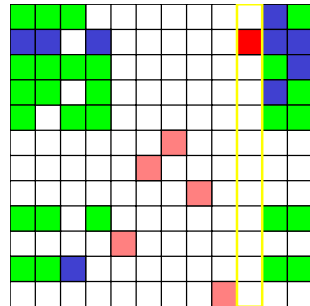
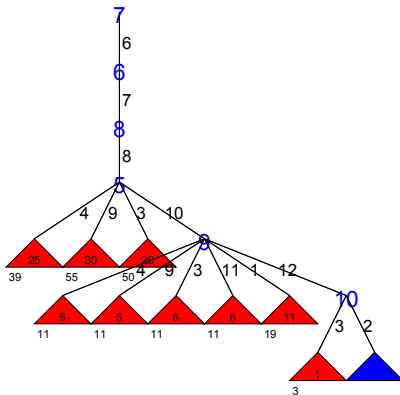
Labeling From Middle



◀ Back to Start

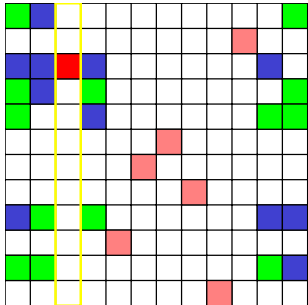
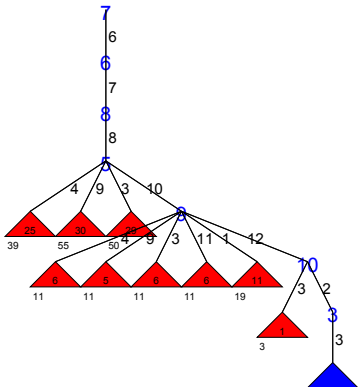
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Labeling From Middle



◀ Back to Start ▶ Skip Animation

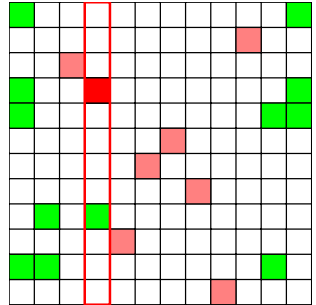
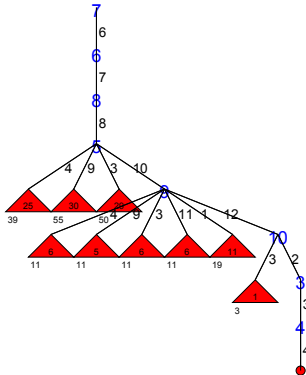
Labeling From Middle



◀ Back to Start

▶ Skip Animation

Labeling From Middle

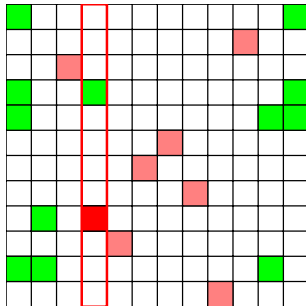
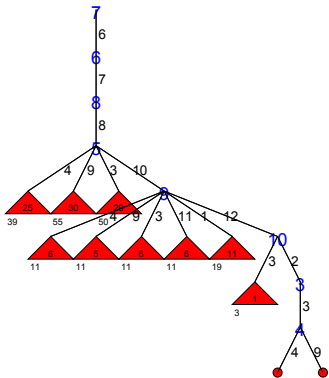


◀ Back to Start

▶ Skip Animation



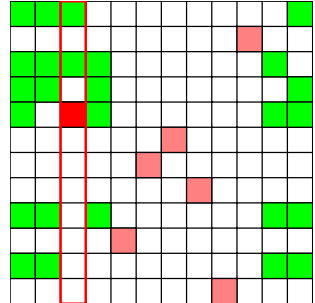
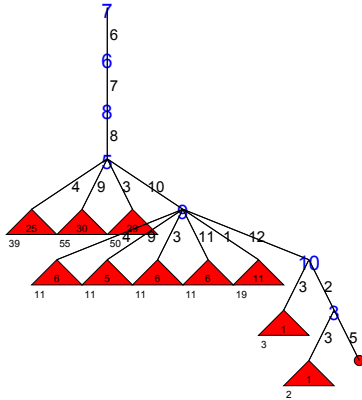
Labeling From Middle



◀ Back to Start ▶▶ Skip Animation



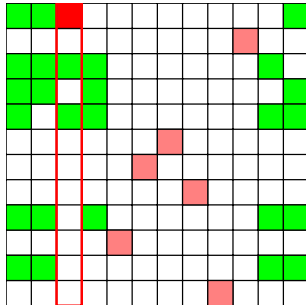
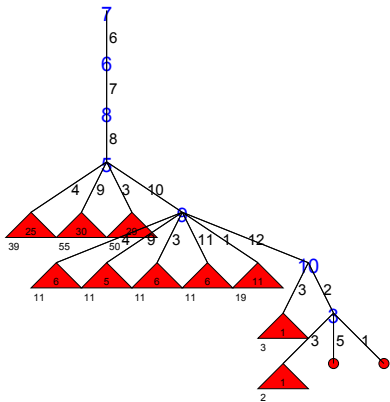
Labeling From Middle



◀ Back to Start

▶ Skip Animation

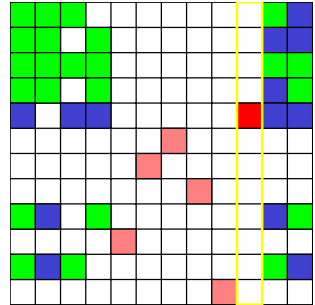
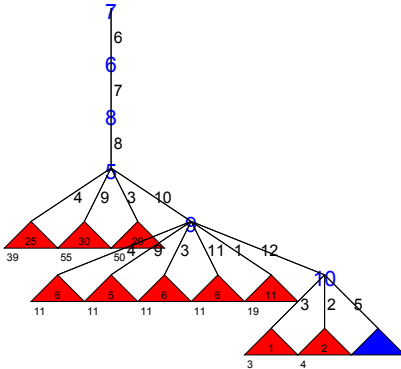
Labeling From Middle



◀ Back to Start

▶ Skip Animation

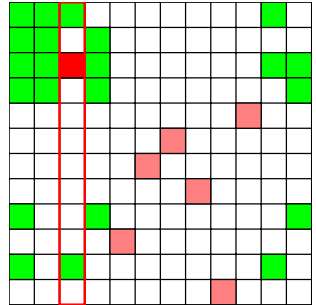
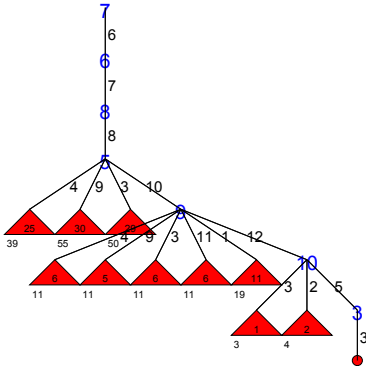
Labeling From Middle



◀ Back to Start

▶▶ Skip Animation

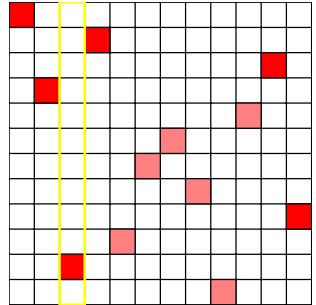
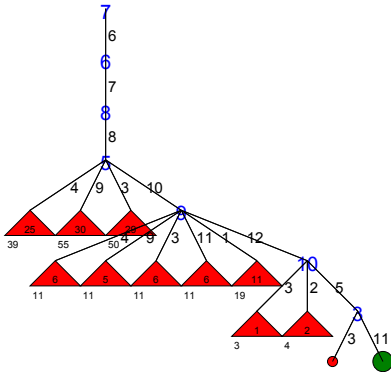
Labeling From Middle



◀ Back to Start

▶ Skip Animation

Labeling From Middle



◀ Back to Start

Other Problem Sizes

Size	Improved Model		Improved Model, Middle	
	Backtrack	Time	Backtrack	Time
10	4	0.16	1	0.01
11	77	1.44	13	0.03
12	31	0.94	72	0.26
13	216	6.22	513	1.81
14	2875	95.94	589	2.37
15	25820	1046.75	7840	34.30
16	84161	4099.52	13158	63.91
17	825590	49371.02	56390	298.16
18	55102	4530.83	19750	115.64

Observation

- Big improvement in backtracks and time
- Not for all problem sizes
- Question: Do we need improvement of model for this to work?
- Experiment: Run changes search routine on basic model



Labeling Basic Model from Middle

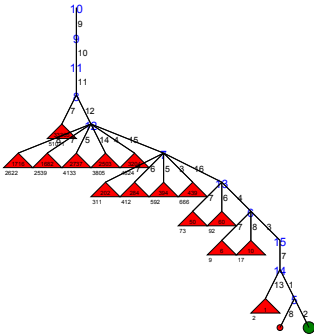
Size	Basic Model		Basic Model, Middle	
	Backtrack	Time	Backtrack	Time
10	4	0.00	1	0.00
11	118	0.08	17	0.01
12	50	0.05	97	0.09
13	335	0.36	644	0.74
14	5008	6.23	746	1.03
15	47332	68.92	10041	16.03
16	157773	271.22	17005	31.12
17	1641685	3278.19	73080	152.72
18	115745	283.97	28837	60.97

Comparison: Model Impact

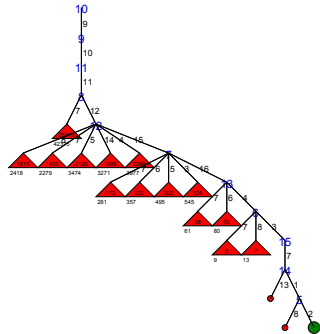
	Basic Model, Middle		Improved Model, Middle	
Size	Backtrack	Time	Backtrack	Time
10	1	0.00	1	0.01
11	17	0.01	13	0.03
12	97	0.09	72	0.26
13	644	0.74	513	1.81
14	746	1.03	589	2.37
15	10041	16.03	7840	34.30
16	17005	31.12	13158	63.91
17	73080	152.72	56390	298.16
18	28837	60.97	19750	115.64
19	1187618	3174.72	1044751	4474.56

Comparison (Search Tree, size 18)

Initial Model



Improved Model



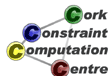
Observation

- Search strategy does not depend on model
- Variable selection is the same!
- Basic model is about two times faster
- About 50% more backtrack steps
- Again, sometimes reasoning does not pay!
- Better search strategy pays off dramatically



Outline

5 0/1 Models



A Different Model

- Model shown is not the only way to express problem



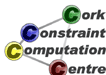
0/1 Models

- SAT (Minisat)
- Pseudo Boolean (Minisat+)
- MIP (Coin-OR)



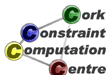
0/1 Models: Variables

- X_{iv} : Variable i takes value v
- D_{ijv} : Difference between variables i and j is v



MIP Model: Constraints

- alldifferent between variables
 - $\sum_i X_{iv} = 1$
 - $\sum_v X_{iv} = 1$
- alldifferent between differences
 - $\sum_v D_{ijv} = 1$
 - $\sum_{i-j=c} D_{ijv} \leq 1$
- link between variables and differences
 - $D_{ijv} = \sum_{v_1=v_2+v} X_{iv_1} X_{jv_2}$



More Information

- <http://www.costasarrays.org/>

