Chapter 14: Finite Set and Continuous Variables - SONET Design Problem

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ECLiPSe ELearning Overview

Licence

Outline

What we want to introduce

- Finite set variables
- Continuous domains
- Optimization from below
- Advanced symmetry breaking
- SONET design problem without inter-ring flows

Outline

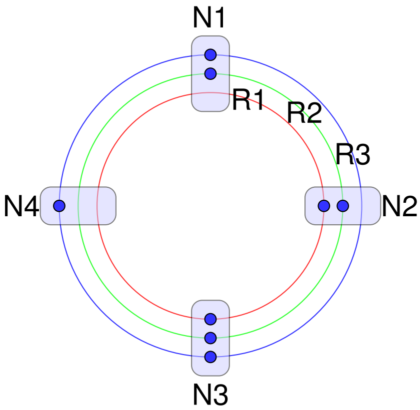
1. Problem
2. Program
3. Search
4. Conclusions

Problem Definition

SONET Design Problem

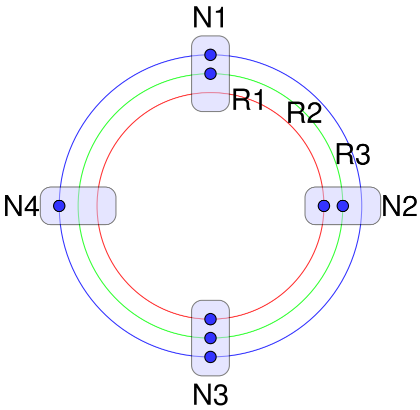
We want to design a network with multiple SONET rings, minimizing ADM equipment. Traffic can only be transported between nodes connected to the same ring, not between rings. Traffic demands between nodes are given. Decide which nodes to place on which ring(s), respecting a maximal number of ADM per ring, and capacity limits on ring traffic. If two nodes are connected on more than one ring, the traffic between them can be split arbitrarily between the rings. The objective is to minimize the overall number of ADM.

Example



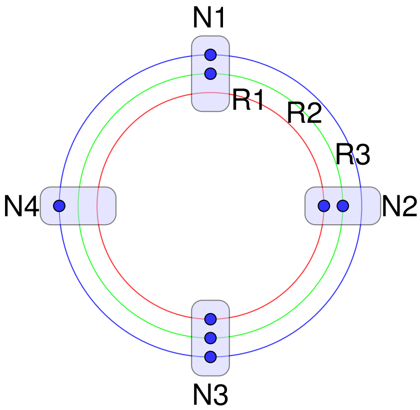
3 rings, 4 nodes, 8 ADM

Example



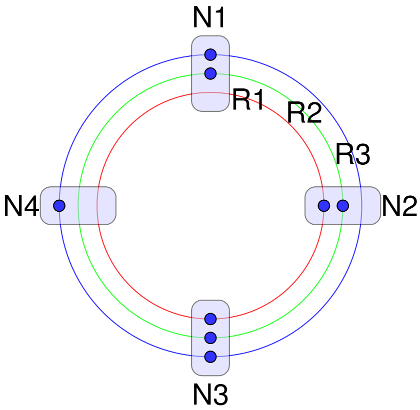
Every node connected to at least one ring

Example



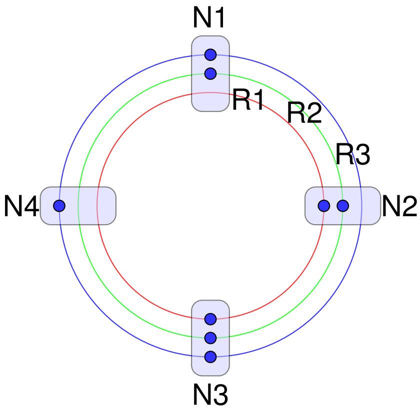
On every ring are at least two nodes

Example



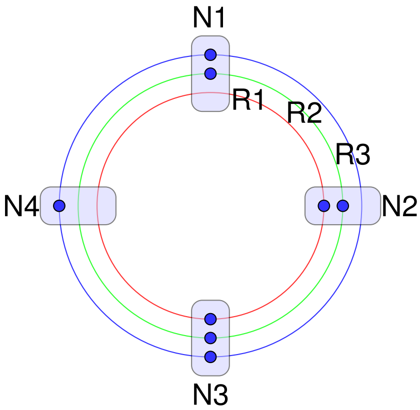
$N1$ connected to $R2$ and $R3$

Example



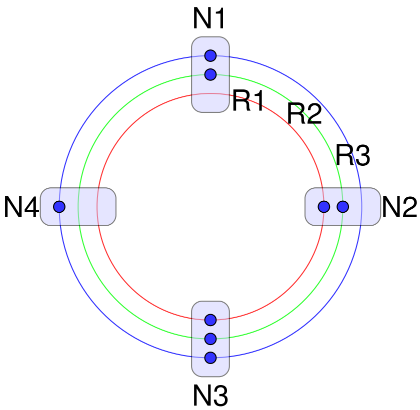
*N*4 and *N*2 can't talk to each other

Example



Traffic between *N*1 and *N*2 must use *R*2

Example



Traffic between $N2$ and $N3$ can use either $R1$ or $R2$, or both

Data

- Demands $d \in D$ between nodes f_d and t_d of size s_d
- Rings R, total of $|R| = r$ rings
- Each ring has capacity c
- Nodes N

Model

- Primary model integer 0/1 variables x_{ik}
  - Node i has a connection to ring k
  - A node can be connected to more than one ring
- Continuous [0..1] variables f_{dk}
  - Which fraction of total traffic of demand d is transported on ring k
  - A demand can use a ring only if both end-points are connected to it

Constraints

$$\min \sum_{i \in N} \sum_{k \in R} x_{ik}$$

s.t.

$$\begin{aligned}
\sum_{i \in N} x_{ik} &\leq r && (1)\\
\sum_{k \in R} f_{dk} &= 1 && (2)\\
\sum_{d \in D} s_d * f_{dk} &\leq c && (3)\\
f_{dk} &\leq x_{f_d k} && (4)\\
f_{dk} &\leq x_{t_d k} && (5)
\end{aligned}$$

Dual Models

- Introducing finite set variables
- Range over sets of integers, not just integers
- Most useful when we don't know the number of items involved
- Here: for each node, the rings on which it is placed
- Could be one, could be two, or more
- Hard to express with finite domain variables alone

Dual Model 1

- Finite set variables N_i
  - Which rings node i is connected to
- Cardinality finite domain variables n_i
  - $|N_i| = n_i$

Dual Model 2

- Finite set variables R_k
  - Which nodes ring k is connected to
- Cardinality finite domain variables r_k
  - $|R_k| = r_k$

Channeling between models

- Use the zero/one model as common ground
- $x_{ik} = 1 \Leftrightarrow k \in N_i$
- $x_{ik} = 1 \Leftrightarrow i \in R_k$

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Constraints in dual models

- For every demand, source and sink must be on (at least one) shared ring
  - $\forall d \in D: \quad |N_{f_d} \cap N_{t_d}| \geq 1$
- Every node must be on a ring
  - $n_i \geq 1$
- A ring can not have a single node connected to it
  - $r_k \neq 1$

Assignment Strategy

- Cost based decomposition
- Assign total cost first
- Then assign n_i variables
- Finally, assign x_{ik} variables
- If required, fix flow f_{dk} variables
- Might leave flows as bound-consistent continuous domains

Optimization from below

- Optimization handled by assigning cost first
- Enumerate values increasing from lower bound
- First feasible solution is optimal
- Depends on proving infeasibility rapidly
- Does not provide sub-optimal initial solutions

Redundant Constraints

- Deduce bounds in n_i variables
  - Helps with finding n_i assignment which can be extended
- Symmetry Breaking

Symmetries

- Typically no symmetries between demands
- Full permutation symmetry on rings
- Gives $r!$ permutations
- These must be handled somehow
- Further symmetries if capacity seen as discrete channels

Symmetry Breaking Choices

- As part of assignment routine
  - SBDS (symmetry breaking during search)
  - Define all symmetries as parameter
  - Search routine eliminates symmetric sub-trees
- By stating ordering constraints
  - As shown in the BIBD example
  - Ordering constraints not always compatible with search heuristic
  - Particular problem of dynamic variable ordering

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Defining finite set variables

- Library `ic_sets`
- Domain definition `X :: Low..High`
  - *Low*, *High* sets of integer values, e.g. `[1,3,4]`
- or `intsets(L,N,Min,Max)`
  - `L` is a list of `N` set variables
  - each containing all values between `Min` and `Max`

Using finite set variables

- Set Expressions: `A /\ B`, `A \/ B`
- Cardinality constraint: `#(Set,Size)`
  - *Size* integer or finite domain variable
- `membership_booleans(Set,Booleans)`
  - Channeling between set and 0/1 integer variables

Using continuous variables

- Library `ic` handles both
  - Finite domain variables
  - Continuous variables
- Use floats as domain bounds, e.g. `X :: 0.0 .. 1.0`
- Use `$=` etc for constraints instead of `#=`
- Bounds reasoning similar to finite case
- But must deal with safe rounding
- Not all constraints deal with continuous variables

Ambiguous Import

- Multiple solvers define predicates like `::`
- If we load multiple solvers in the same module, we have to tell ECLiPSe which one to use
- Compiler does not deduce this from context!
- So
  - `ic:(X :: 1..3)`
  - `ic_sets:(X :: [] .. [1,2,3])`
- Otherwise, we get loads of error messages
- Happens whenever two modules export same predicate

Top-level predicate

```
:-module(sonet).
:-export(top/0).
:-lib(ic),lib(ic_global),lib(ic_sets).

top:-
    problem(NrNodes,NrRings,Demands,
            MaxRingSize,ChannelSize),
    length(Demands,NrDemands),
    ...
```

Matrix of x_{ik} integer variables

```
...
dim(Matrix,[NrNodes,NrRings]),
ic:(Matrix[1..NrNodes,1..NrRings] :: 0..1),
...
```

Node and ring set variables

```
...
dim(Nodes,[NrNodes]),
intsets(Nodes[1..NrNodes],NrNodes,1,NrRings),
dim(NodeSizes,[NrNodes]),
ic:(NodeSizes[1..NrNodes] :: 1..NrRings),
dim(Rings,[NrRings]),
intsets(Rings[1..NrRings],NrRings,1,NrNodes),
dim(RingSizes,[NrRings]),
ic:(RingSizes[1..NrRings] :: 0..MaxRingSize),
...
```

Channeling node set variables

```
...
(for(I,1,NrNodes),
 param(Matrix,Nodes,NodeSizes,NrRings) do
     subscript(Nodes,[I],Node),
     subscript(NodeSizes,[I],NodeSize),
     #(Node,NodeSize),
     membership_booleans(Node,
                                Matrix[I,1..NrRings])
),
...
```

Channeling ring set variables

```
...
(for(J,1,NrRings),
 param(Matrix,Rings,RingSizes,NrNodes) do
     subscript(Rings,[J],Ring),
     subscript(RingSizes,[J],RingSize),
     RingSize #\= 1,
     #(Ring,RingSize),
     membership_booleans(Ring,
                               Matrix[1..NrNodes,J])
),
...
```

Demand ends must be (on atleast one) same ring

```
...
(foreach(demand(I,J,_Size),Demands),
 param(Nodes,NrRings) do
     subscript(Nodes,[I],NI),
     subscript(Nodes,[J],NJ),
     ic:(NonZero :: 1..NrRings),
     #(NI /\ NJ,NonZero)
),
...
```

Flow Variables

```
...
dim(Flow,[NrDemands,NrRings]),
ic:(Flow[1..NrDemands,1..NrRings]::0.0 .. 1.0),
(for(I,1,NrDemands),
 param(Flow,NrRings) do
    (for(J,1,NrRings),
     fromto(0.0,A,A+F,Term),
     param(Flow,I) do
    subscript(Flow,[I,J],F)
    ),
    eval(Term) $= 1.0
),
...
```

Ring Capacity Constraints

```
...
(for(I,1,NrRings),
 param(Flow,Demands,ChannelSize) do
     (foreach(demand(_,_,Size),Demands),
      count(J,1,_),
      fromto(0.0,A,A+Size*F,Term),
      param(Flow,I) do
     subscript(Flow,[J,I],F)
     ),
     eval(Term) $=< ChannelSize
),
...
```

Linking x_{ik} and f_{dk} variables

```
...
(foreach(demand(From,To,_),Demands),
 count(I,1,_),
 param(Flow,Matrix,NrRings) do
     (for(K,1,NrRings),
      param(I,From,To,Flow,Matrix) do
     subscript(Flow,[I,K],F),
     subscript(Matrix,[From,K],X1),
     subscript(Matrix,[To,K],X2),
     F $=< X1,
     F $=< X2
     )
),
...
```

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Setting up degrees

```
...
dim(Degrees,[NrNodes]),
(for(I,1,NrNodes),
 param(Degrees) do
    subscript(Degrees,[I],Degree),
    neighbors(I,Neighbors),
    length(Neighbors,Degree)
),
...
```

Defining cost and assigning values

```
...
sumlist(NodeSizesList,Cost),
assign(Cost,Handle,NrNodes,Degrees,
       NodeSizes,Matrix).
```

Assignment Routines

```
assign(Cost,Handle,NrNodes,Degrees,
       NodeSizes,Matrix):-
    indomain(Cost),
    order_sizes(NrNodes,Degrees,NodeSizes,
                OrderedSizes),
    search(OrderedSizes,1,input_order,indomain,
           complete,[]),
    order_vars(Degrees,NodeSizes,Matrix,
               VarAssign),
    search(VarAssign,0,input_order,indomain_max,
           complete,[]).
```

Order ring size variables by increasing degree

```
order_sizes(NrNodes,Degrees,NodeSizes,
            OrderedSizes):-
    (for(I,1,NrNodes),
     foreach(t(X,D),Terms),
     param(Degrees,NodeSizes) do
        subscript(Degrees,[I],D),
        subscript(NodeSizes,[I],X)
    ),
    sort(2,=<,Terms,OrderedSizes).
```

Ordering decision variables

```
order_vars(Degrees,NodeSizes,Matrix,VarAssign):-
    dim(Matrix,[NrNodes,NrRings]),
    (for(I,1,NrNodes),
     foreach(t(Size,Y,I),Terms),
     param(Degrees,NodeSizes) do
        subscript(NodeSizes,[I],Size),
        subscript(Degrees,[I],Degree),
        Y is -Degree
    ),
    sort(0,=<,Terms,Sorted),
    ...
```

Ordering decision variables

```
...
(foreach(t(_,_,I),Sorted),
 fromto(VarAssign,A1,A,[]),
 param(NrRings,Matrix) do
     (for(J,1,NrRings),
      fromto(A1,[X|AA],AA,A),
      param(I,Matrix) do
          subscript(Matrix,[I,J],X)
     )
).
```

Data (13 nodes, 7 rings, 24 demands)

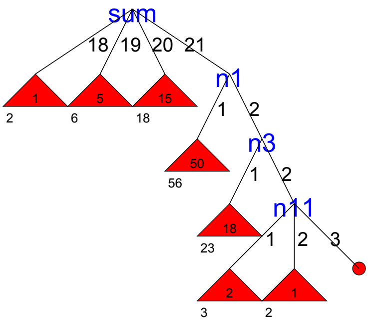
```
problem(13,7,
     [demand(1,9,8),demand(1,11,2),demand(2,3,25),
      demand(2,5,5),demand(2,9,2),demand(2,10,3),
      demand(2,13,4),demand(3,10,2),demand(4,5,4),
      demand(4,8,1),demand(4,11,5),demand(4,12,2),
      demand(5,6,5),demand(5,7,4),demand(7,9,5),
      demand(7,10,2),demand(7,12,6),demand(8,10,1),
      demand(8,12,4),demand(8,13,1),demand(9,12,5),
      demand(10,13,9),demand(11,13,3),
      demand(12,13,2)
     ],5,40).
```

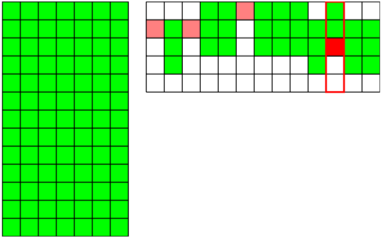
Neighbors of a node

```
neighbors(N,List):-
    problem(_,_,Demands,_,_),
    (foreach(demand(I,J,_),Demands),
     fromto([],A,A1,List),
     param(N) do
        (N = I ->
            A1 = [J|A]
        ; N = J ->
            A1 = [I|A]
        ;
            A1 = A
        )
    ).
```

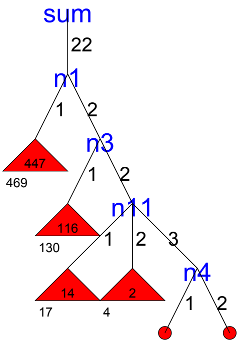
Outline

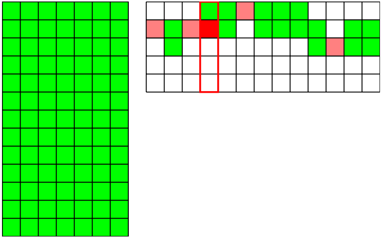
Search at Cost 18-21



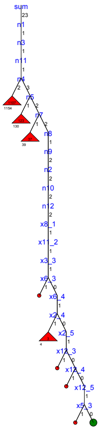


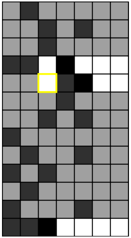
Search at Cost 22

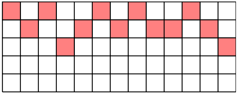




Search at Cost 23







Outline

Conclusions

- Introduced finite set and continuous domain solvers
- Finite set variables useful when values are sets of integers
- Useful when number of items assigned are unknown
- Can be linked with finite domains (cardinality) and 0/1 index variables

Continuous domain variables

- Allow to reason about non-integral values
- Bound propagation similar to bound propagation over integers
- Difficult to enumerate values
- Assignment by domain splitting

SONET Problem

- Example of optical network problems
- Competitive solution by combination of techniques
- Channeling, redundant constraints, symmetry breaking
- Decomposition by branching on objective value

More Information

Barbara M. Smith.
Symmetry and search in a network design problem.
In Roman Barták and Michela Milano, editors, *CPAIOR*, volume 3524 of *Lecture Notes in Computer Science*, pages 336–350. Springer, 2005.