# Chapter 5: Global Constraints(Sudoku)

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ECLiPSe ELearning Overview

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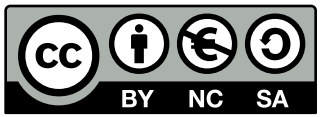

# Outline

1. Problem
2. Program
3. Initial Propagation (Forward Checking)
4. Improved Reasoning
5. Search

# What we want to introduce

- Global Constraints
  - Powerful modelling abstractions
  - Non-trivial propagation
- Consistency Levels
  - Tradeoff between speed and propagation
  - Characterisation of reasoning power
- Example: Alldifferent
  - 3 variants shown

# Methodology

- Evaluation on Sudoku puzzle
- Comparing
  - Initial setup
  - Search
  - Performance
- Explaining reasoning inside constraint
- Link to general classification of global constraints

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1 Problem
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5 Search

# Problem Definition

## Sudoku

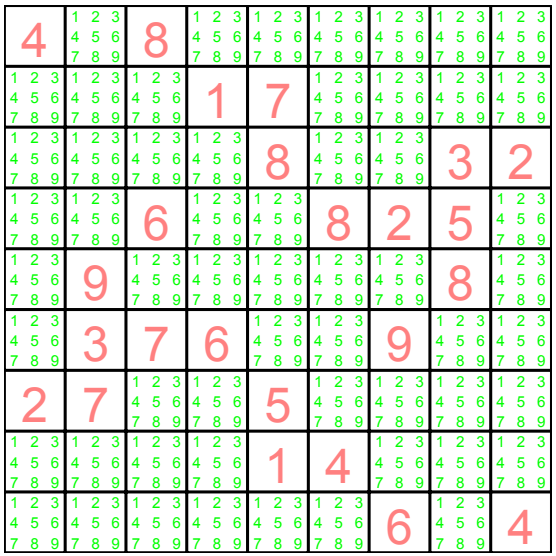
Fill in numbers from 1 to 9 so that each row, column and block contain each number exactly once

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | | 8 | | | | | | |
| | | | 1 | 7 | | | | |
| | | | | 8 | | | 3 | 2 |
| | | 6 | | | 8 | 2 | 5 | |
| | 9 | | | | | | 8 | |
| | 3 | 7 | 6 | | | 9 | | |
| 2 | 7 | | | 5 | | | | |
| | | | | 1 | 4 | | | |
| | | | | | | 6 | | 4 |

# Problem Definition

## Sudoku

Fill in numbers from 1 to 9 so that each row, column and block contain each number exactly once



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 | 9 |
| 3 | 5 | 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 | 9 | 8 | 2 | 5 | 7 |
| 5 | 9 | 2 | 7 | 4 | 1 | 3 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 9 | 5 | 6 | 8 | 1 | 3 |
| 6 | 8 | 3 | 2 | 1 | 4 | 7 | 9 | 5 |
| 9 | 1 | 5 | 8 | 3 | 7 | 6 | 2 | 4 |

# Model

- A variable for each cell, ranging from 1 to 9
- A 9x9 matrix of variables describing the problem
- Preassigned integers for the given hints
- `alldifferent` constraints for each row, column and 3x3 block

# Reminder: `alldifferent`

- Argument: list of variables
- Meaning: variables are pairwise different
- Reasoning: Forward Checking (FC)
  - When variable is assigned to value, remove the value from all other variables
  - If a variable has only one possible value, then it is assigned
  - If a variable has no possible values, then the constraint fails
  - Constraint is checked whenever one of its variables is assigned
  - Equivalent to decomposition into binary disequality constraints

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# Declarations

```
:-module(sudoku).
:-export(top/0).
:-lib(ic).

top:-
    problem(Matrix),
    model(Matrix),
    writeln(Matrix).
```

# Data

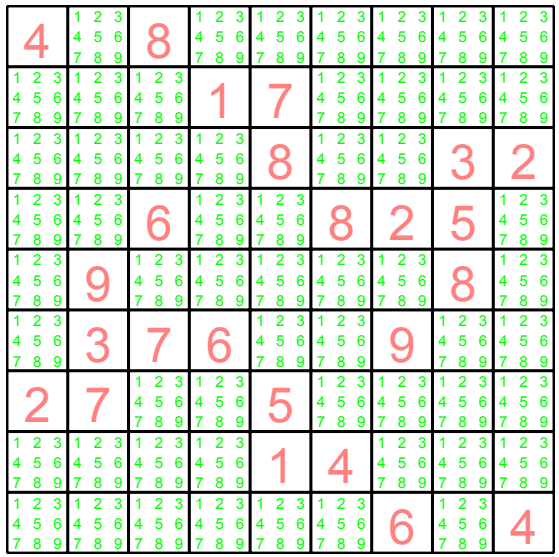
```
problem([](4, _, 8, _, _, _, _, _, _),
          [](_, _, _, 1, 7, _, _, _, _),
          [](_, _, _, _, 8, _, _, 3, 2),
          [](_, _, 6, _, _, 8, 2, 5, _),
          [](_, 9, _, _, _, _, _, 8, _),
          [](_, 3, 7, 6, _, _, 9, _, _),
          [](2, 7, _, _, 5, _, _, _, _),
          [](_, _, _, _, 1, 4, _, _, _),
          [](_, _, _, _, _, _, 6, _, 4))).
```

# Main Program

```
model(Matrix):-
    Matrix[1..9,1..9] :: 1..9,
    (for(I,1,9),
     param(Matrix) do
         alldifferent(Matrix[I,1..9]),
         alldifferent(Matrix[1..9,I])
    ),
    (multifor([I,J],[1,1],[7,7],[3,3]),
     param(Matrix) do
         alldifferent(flatten(Matrix[I..I+2,J..J+2]))
    ),
    flatten_array(Matrix,List),
    labeling(List).
```

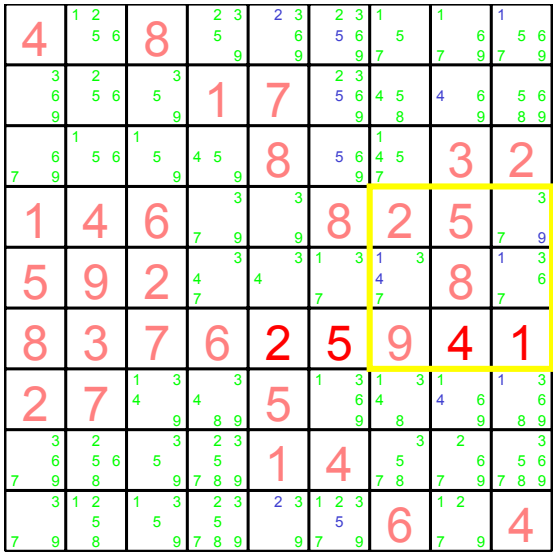
# Domain Visualizer

- Problem shown as matrix
- Each cell corresponds to a variable
- Instantiated: Shows integer value (large)
- Uninstantiated: Shows values in domain



# Constraint Visualizer

- Problem shown as matrix
- Currently active constraint highlighted
- Values removed at this step shown in blue
- Values assigned at this step shown in red



# Outline

1 Problem

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5 Search

# Initial State (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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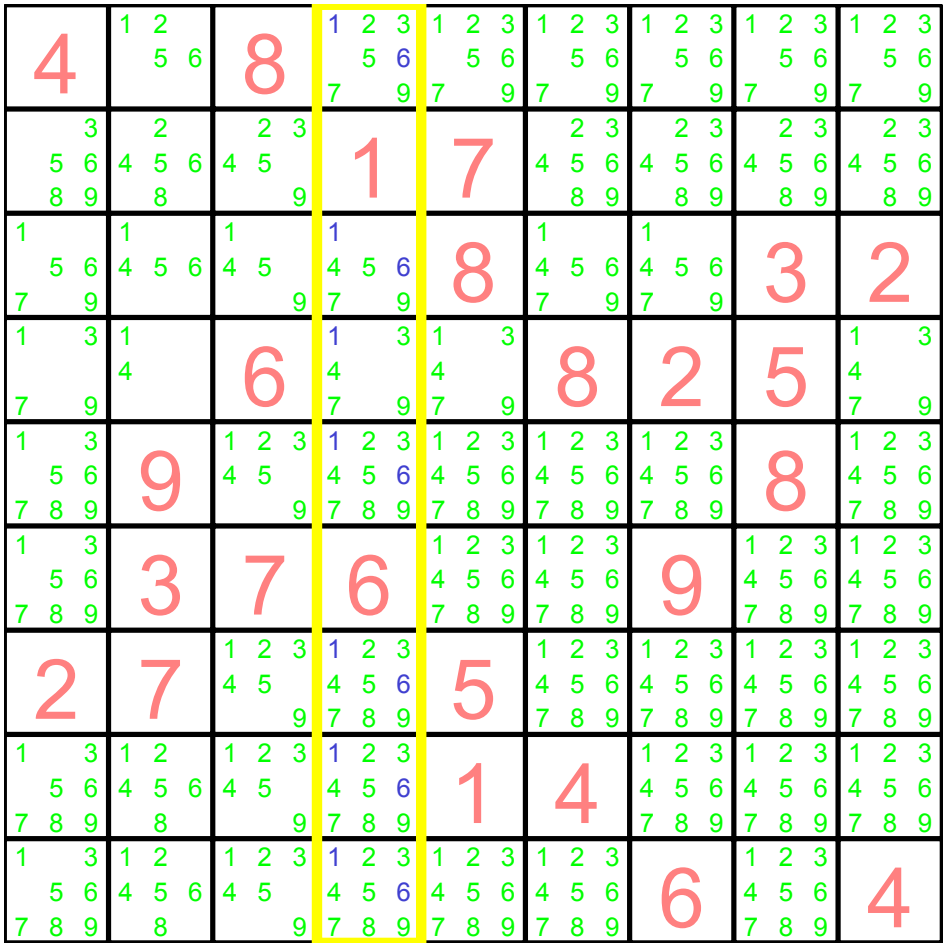
# Propagation Steps (Forward Checking)

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
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| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
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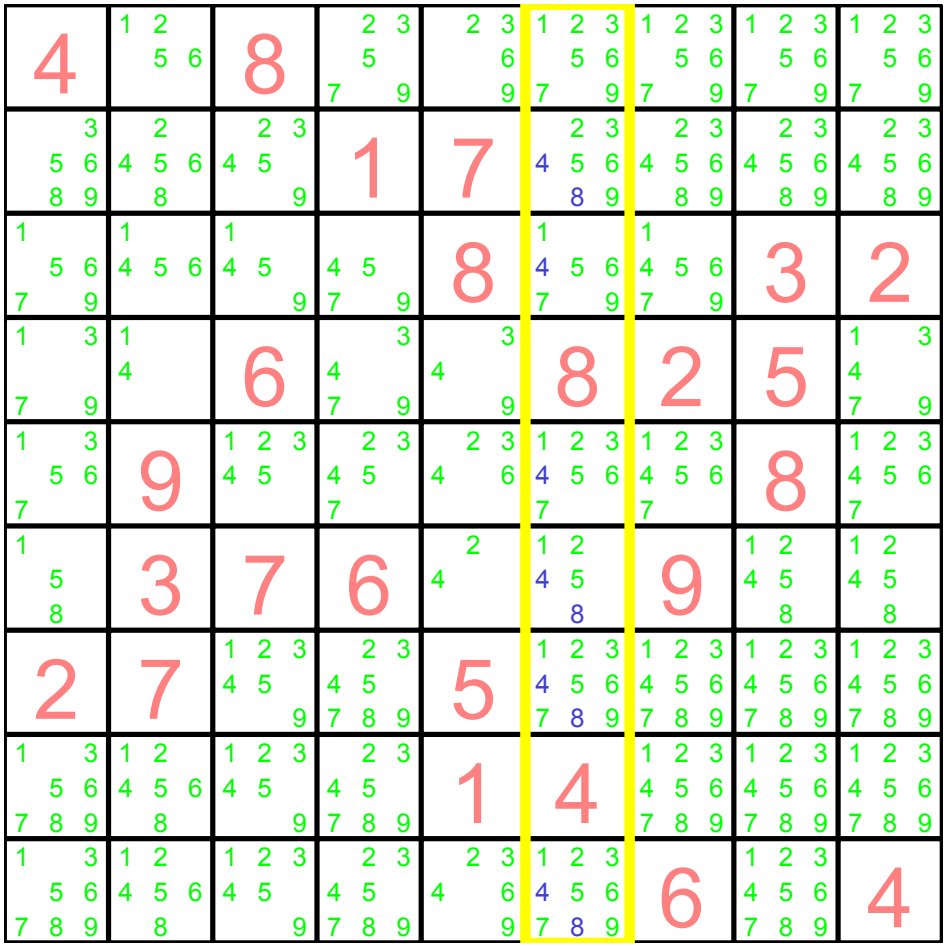
# Propagation Steps (Forward Checking)

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
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| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
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| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
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| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
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| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 3 4 5 7 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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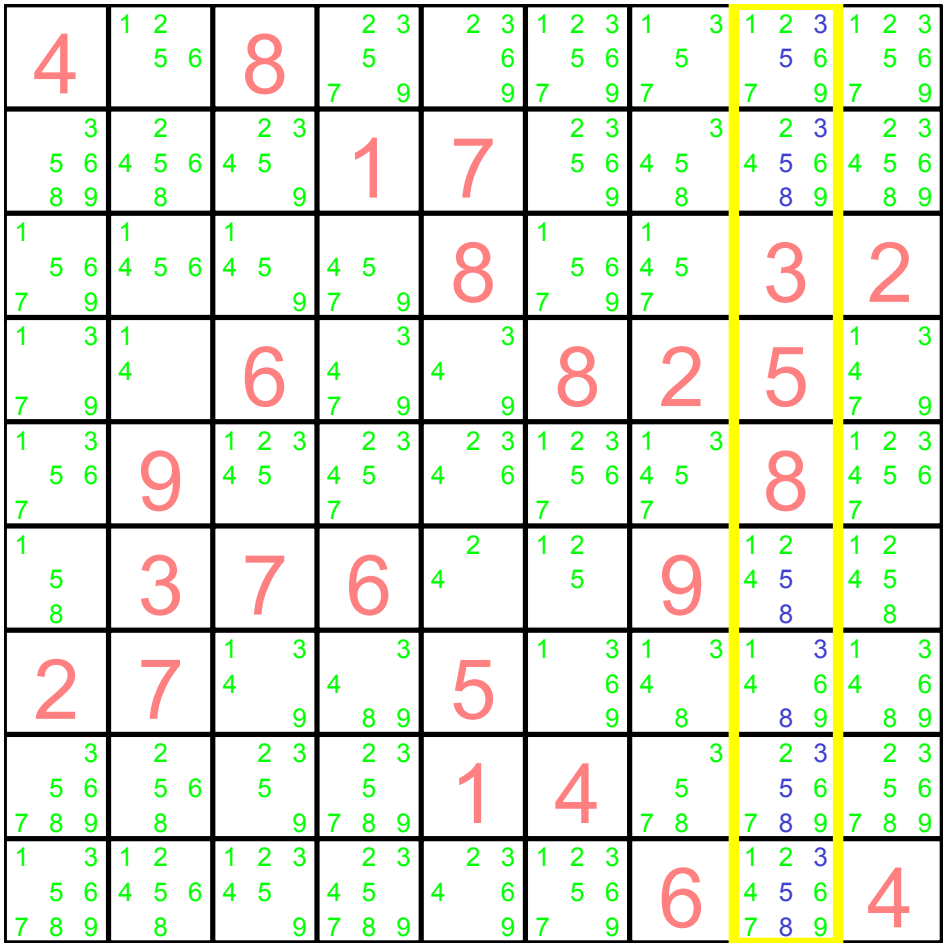
# Propagation Steps (Forward Checking)




Back to Start  Skip Animation

# Propagation Steps (Forward Checking)

| | | | | | | | | |
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| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 2 3 5 6 7 9 |
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| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 4 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 4 6 7 9 | 4 |

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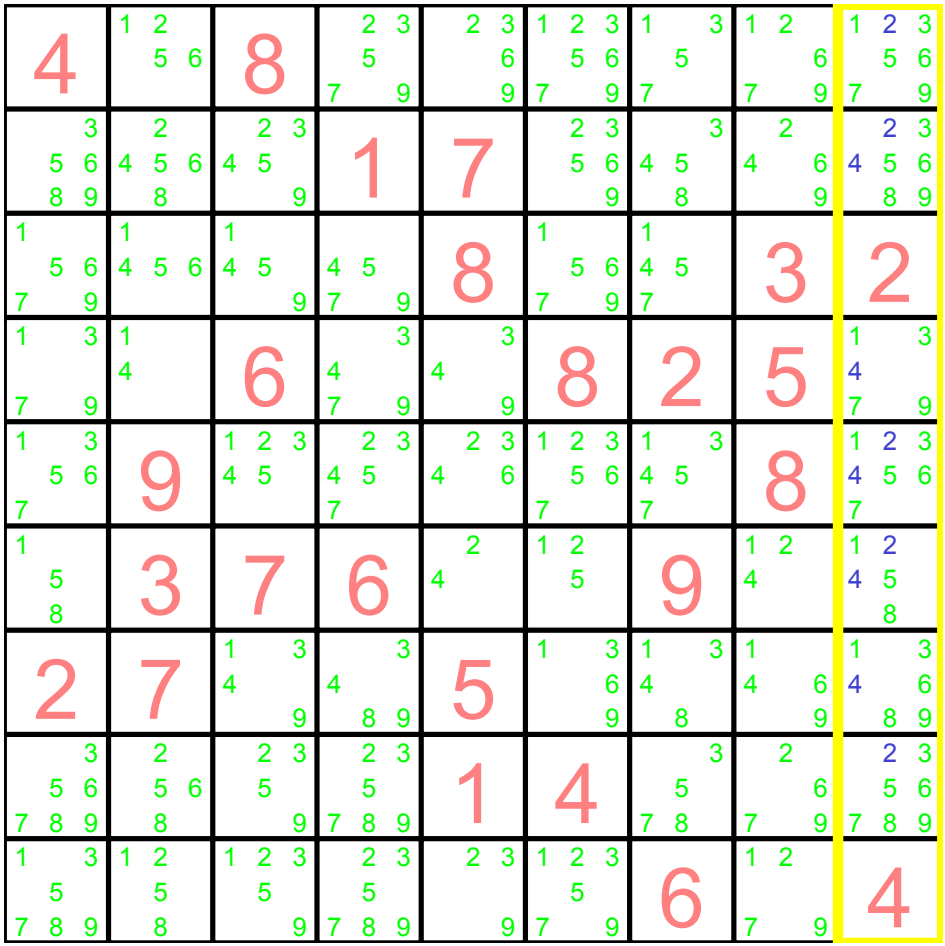
# Propagation Steps (Forward Checking)

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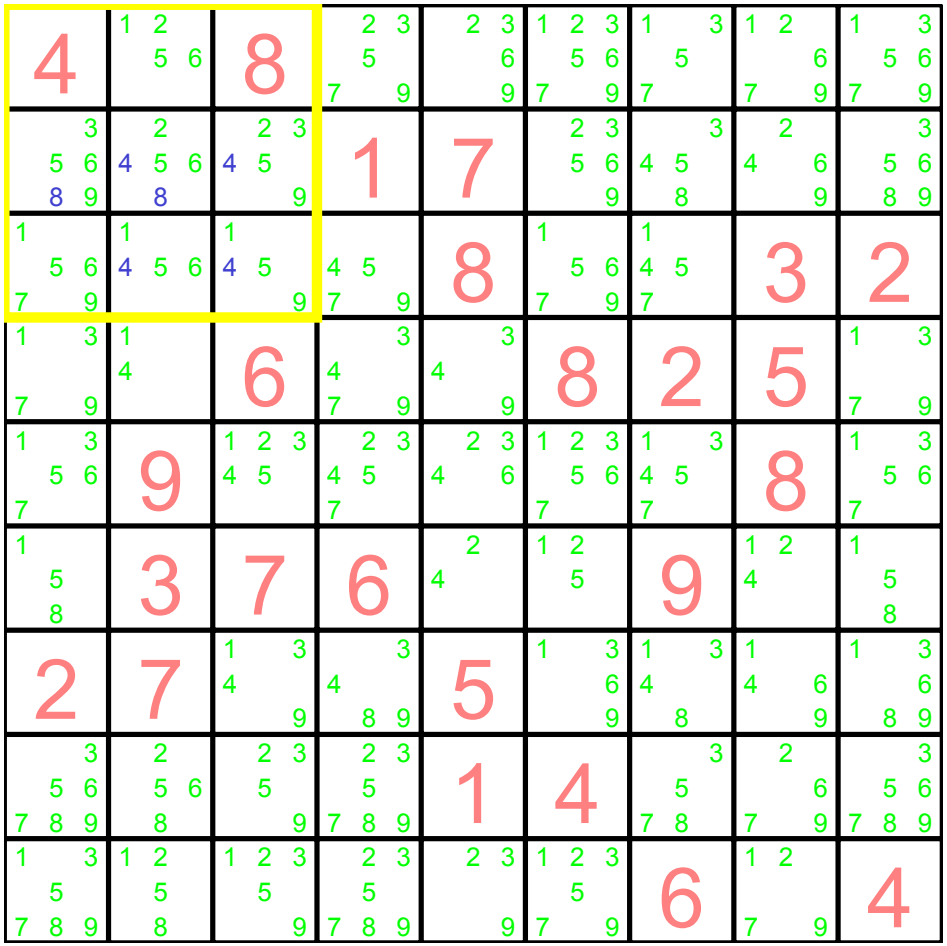
# Propagation Steps (Forward Checking)

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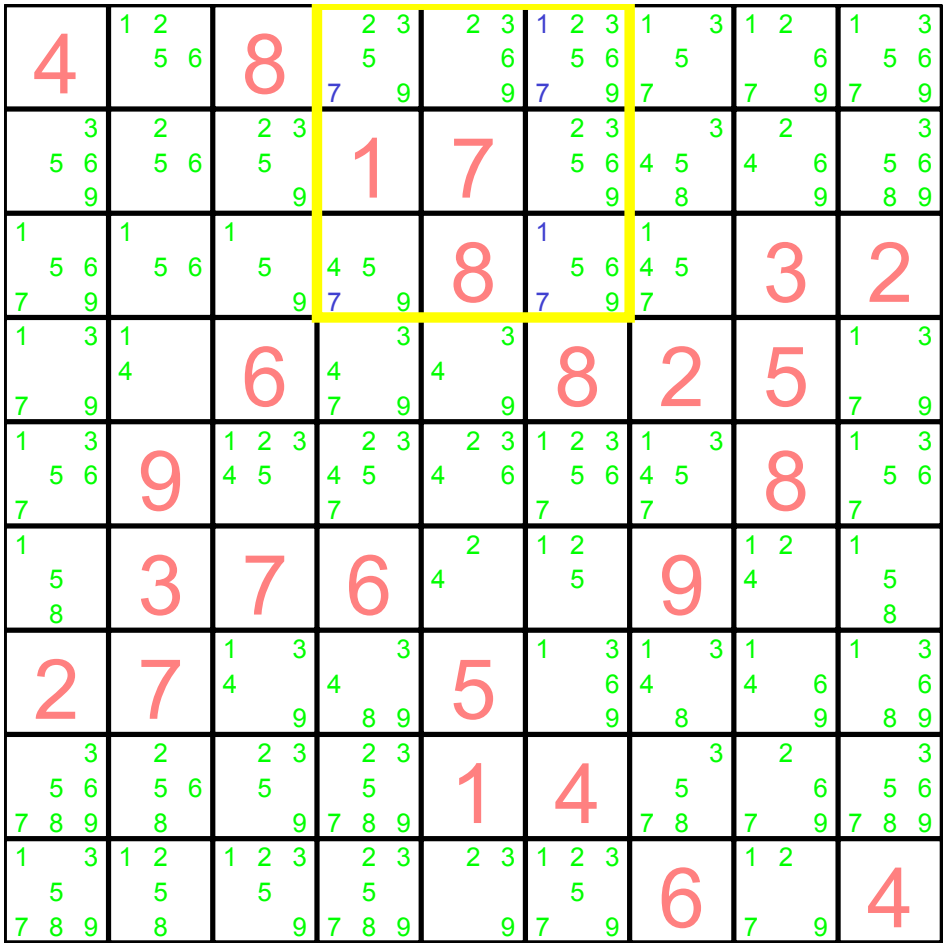
# Propagation Steps (Forward Checking)

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 2 3 6 9 | 2 3 5 6 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 1 5 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 5 6 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 7 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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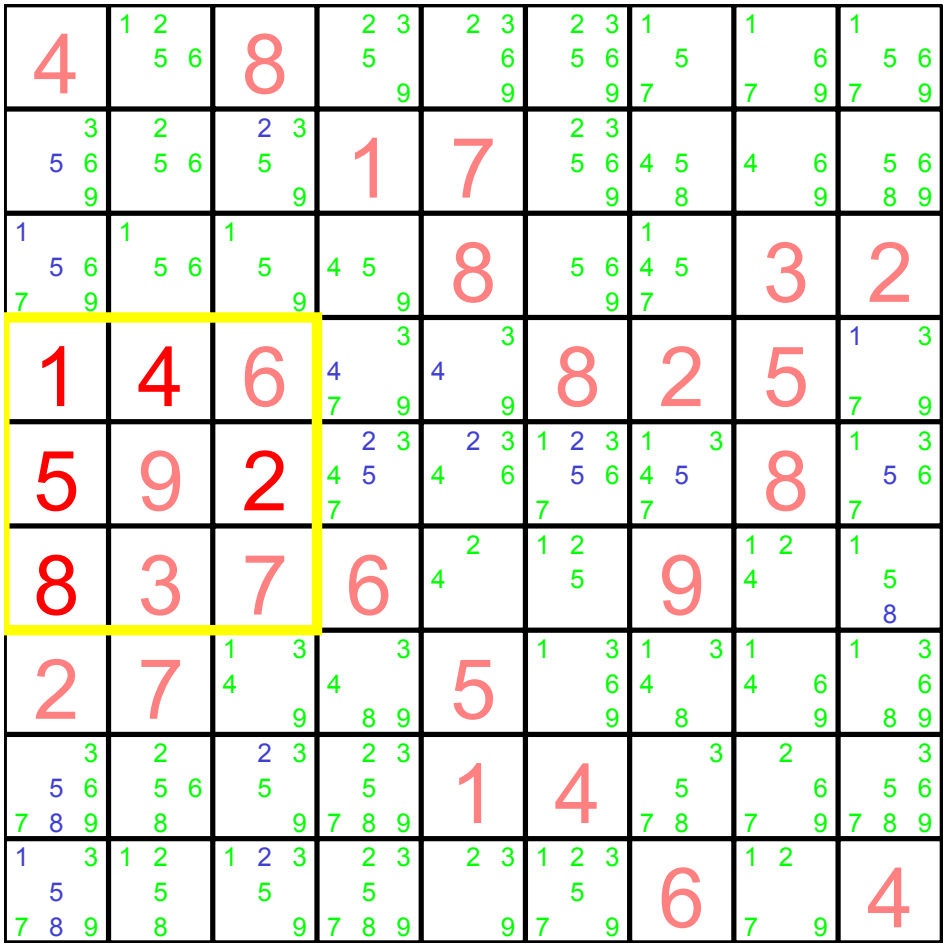
# Propagation Steps (Forward Checking)

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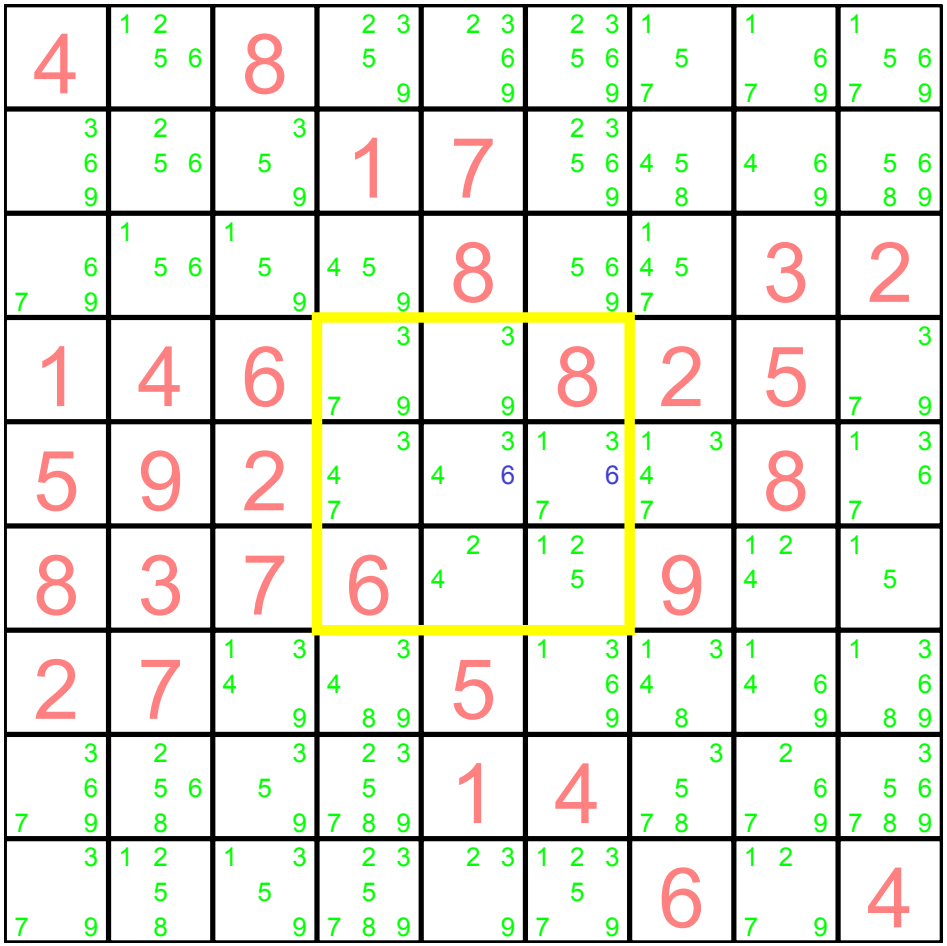
# Propagation Steps (Forward Checking)

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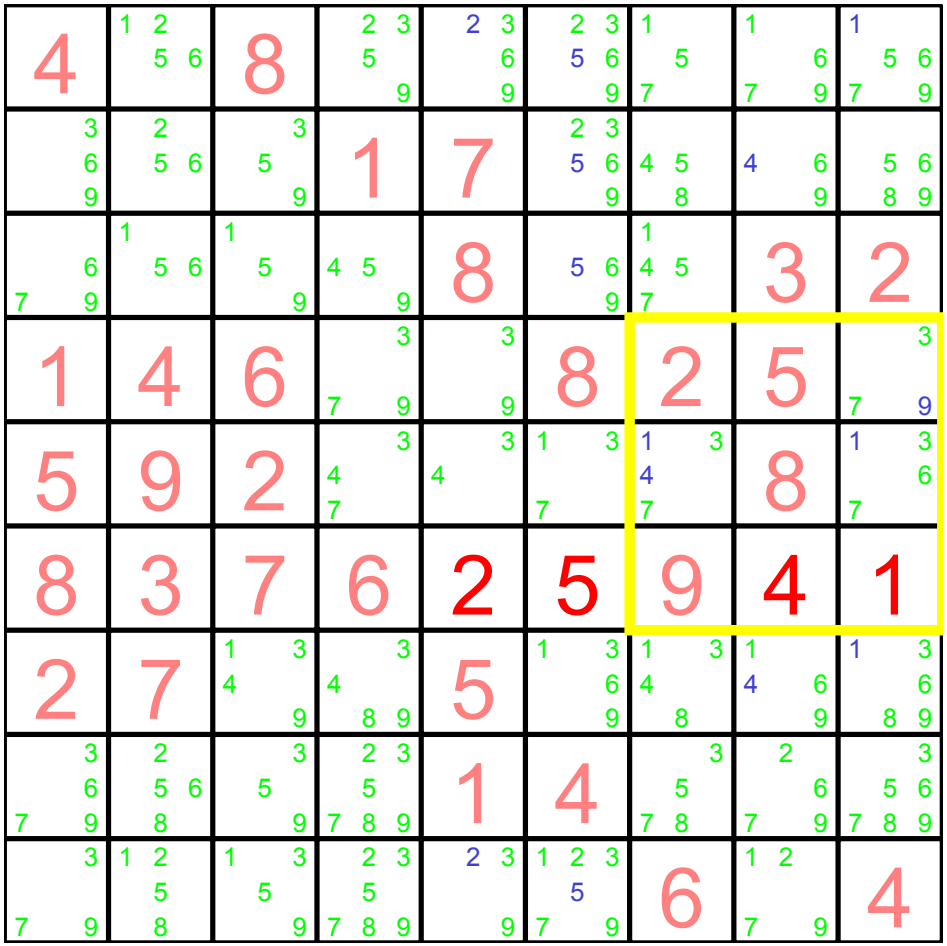
# Propagation Steps (Forward Checking)

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
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| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 6 9 | 3 6 8 9 |
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| 3 7 9 | 1 2 5 8 | 1 3 5 9 | 2 3 5 7 8 9 | 3 9 | 1 2 3 7 9 | 6 | 1 2 7 9 | 4 |

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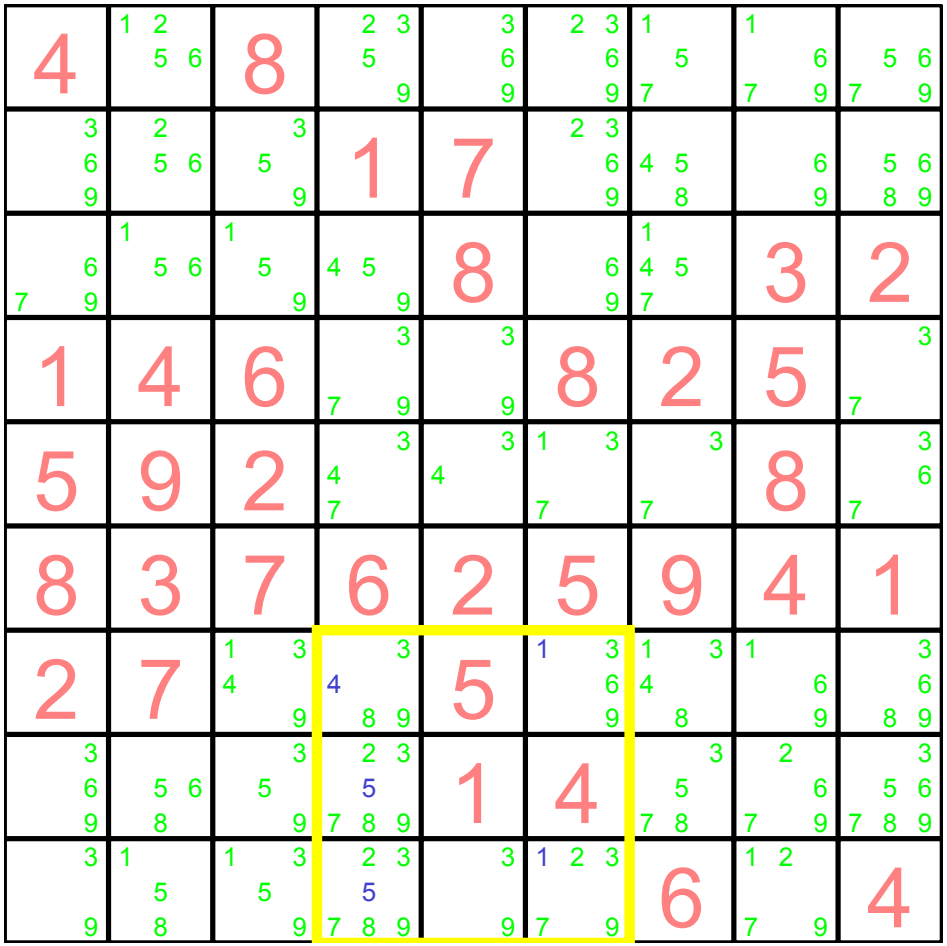
# Propagation Steps (Forward Checking)

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 4 8 | 1 6 9 | 3 6 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
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| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

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# After Setup (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

# Outline

1 Problem

2 Program

3 Initial Propagation (Forward Checking)

4 Improved Reasoning
- Bounds Consistency
- Domain Consistency
- Comparison

5 Search

# Can we do better?

- The alldifferent constraint is missing propagation
  - How can we do more propagation?
  - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables

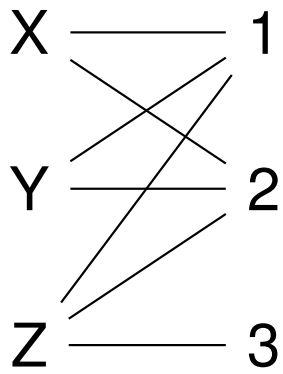
# A Simpler Example

```
:-lib(ic).

top:-
    X :: 1..2,
    Y :: 1..2,
    Z :: 1..3,
    alldifferent([X,Y,Z]),
    writeln([X,Y,Z]).
```

# Using Forward Checking

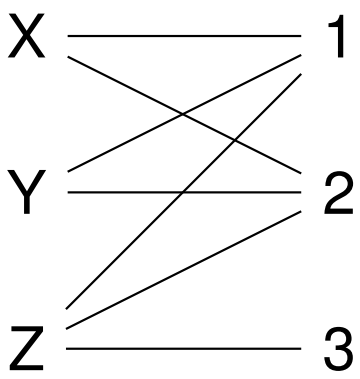
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

# Visualization of alldifferent as Graph




- Show problem as graph with two types of nodes
  - Variables on the left
  - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph

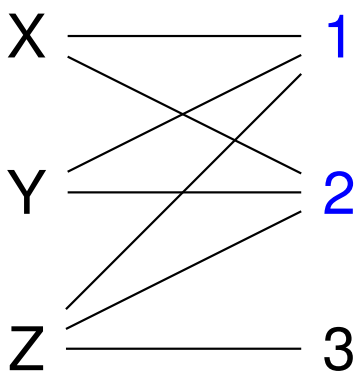
# A Simpler Example




Value Graph for

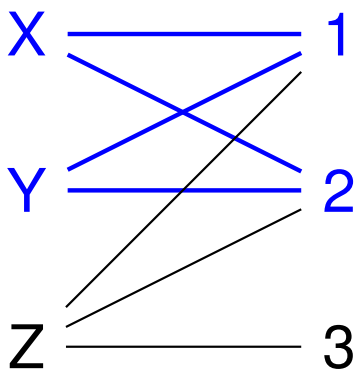
```
X :: 1..2,

Y :: 1..2,

Z :: 1..3
```

# A Simpler Example



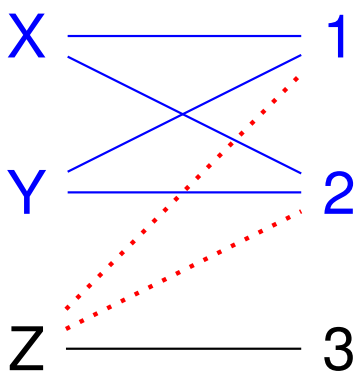

Check interval [1,2]

# A Simpler Example



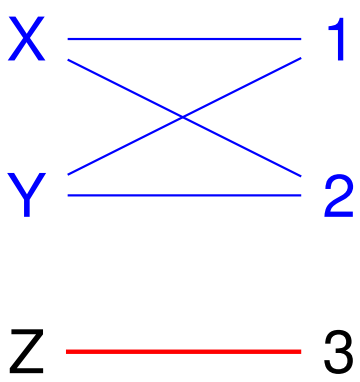

- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

# A Simpler Example




No other variable can use that interval

# A Simpler Example




Only one value left in domain of Z, this can be assigned

# Idea (Hall Intervals)

- Take each interval of possible values, say size $N$
- Find all $K$ variables whose domain is completely contained in interval
- If $K > N$ then the constraint is infeasible
- If $K = N$ then no other variable can use that interval
- Remove values from such variables if their bounds change
- If $K < N$ do nothing
- Re-check whenever domain bounds change

# Implementation

- Problem: Too many intervals ($O(n^2)$) to consider
- Solution:
  - Check only those intervals which update bounds
  - Enumerate intervals incrementally
  - Starting from lowest(highest) value
  - Using sorted list of variables
- Complexity: $O(n \log(n))$ in standard implementations
- Important: Only looks at min/max bounds of variables

# Bounds Consistency

## Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

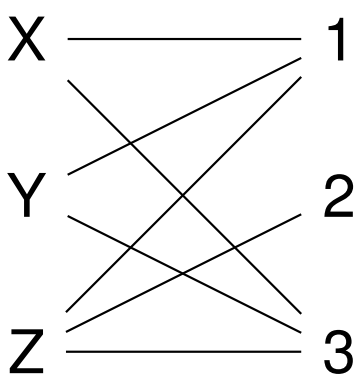
# Can we do better?

- Bounds consistency only considers min/max bounds
- Ignores “holes” in domain
- Sometimes we can improve propagation looking at those holes

# Another Simple Example

```
:-lib(ic).

top:-
    X :: [1,3],
    Y :: [1,3],
    Z :: 1..3,
    alldifferent([X,Y,Z]),
    writeln([X,Y,Z]).
```

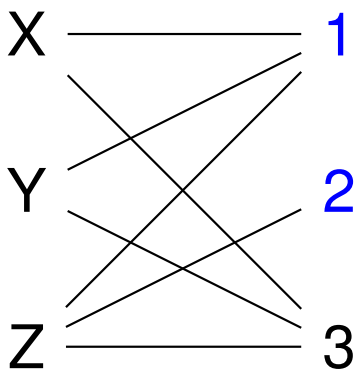
# Another Simple Example




Value Graph for

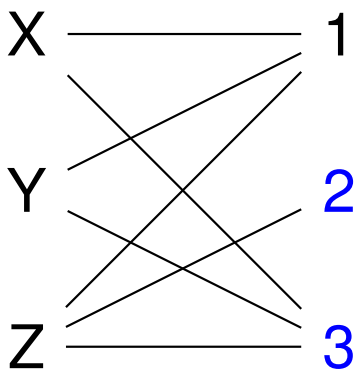
```
X :: [1,3],

Y :: [1,3],

Z :: 1..3
```

# Another Simple Example



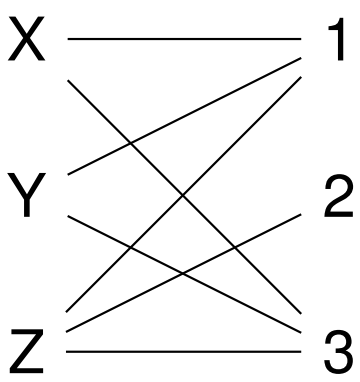

- Check interval [1,2]
- No domain of a variable completely contained in interval
- No propagation

# Another Simple Example



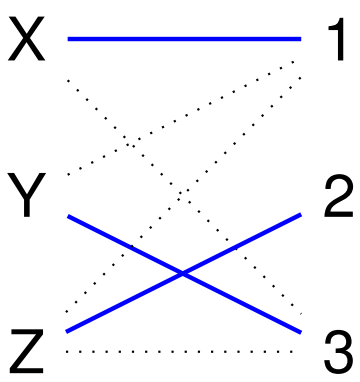

- Check interval [2,3]
- No domain of a variable completely contained in interval
- No propagation

# Another Simple Example



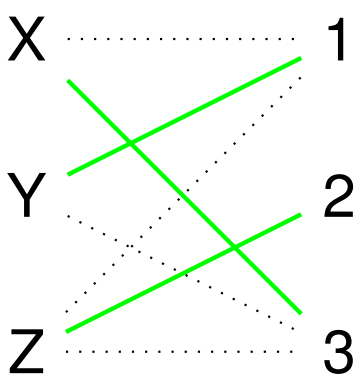

But, more propagation is possible, there are only two solutions

# Another Simple Example



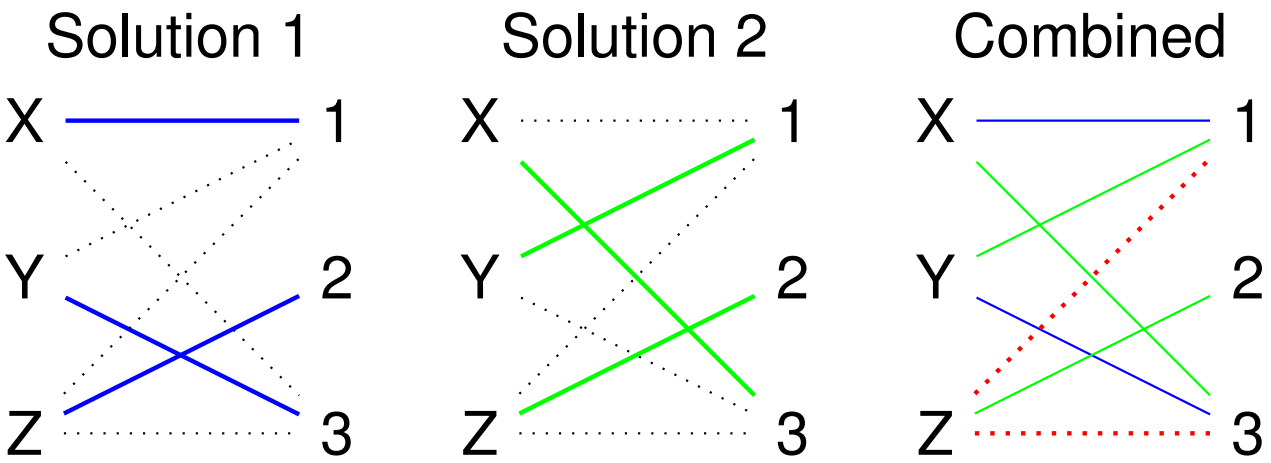

Solution 1: assignment in blue

# Another Simple Example




Solution 2: assignment in green

# Another Simple Example




Combining solutions shows that Z=1 and Z=3 are not possible.

# Another Simple Example

Can we deduce this without enumerating solutions?

# Solutions and maximal matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

# Implementation

- Possible to compute all links which belong to some matching
  - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity: $O(n^{1.5}d)$

# Domain Consistency

**Definition**

A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

# Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

# Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
  - Overkill for simple problems
  - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
  - We have to live with more restricted propagation

# Improved Propagation in ECLiPSe

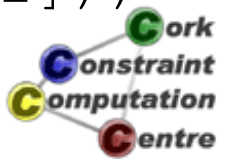
- `ic_global` library bounds consistent version
- `ic_global_gac` library domain consistent version
- Choose which version to use by using module annotation
- Choice can be passed as parameter

# Declarations

```
:-module(sudoku).
:-export(top/0).
:-lib(ic).
:-lib(ic_global).
:-lib(ic_global_gac).

top:-
    problem(Matrix),
    model(ic_global,Matrix),
    writeln(Matrix).
```

# Main Program

```
model(Method,Matrix):-
    Matrix[1..9,1..9] :: 1..9,
    (for(I,1,9),
     param(Method,Matrix) do
        Method:alldifferent(Matrix[I,1..9]),
        Method:alldifferent(Matrix[1..9,I])
    ),
    (multifor([I,J],[1,1],[7,7],[3,3]),
     param(Method,Matrix) do
        Method:alldifferent(flatten(Matrix[I..I+2,
                                           J..J+2]))
    ),
    flatten_array(Matrix,List),labeling(List).
```



# Initial State (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
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| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
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| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
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| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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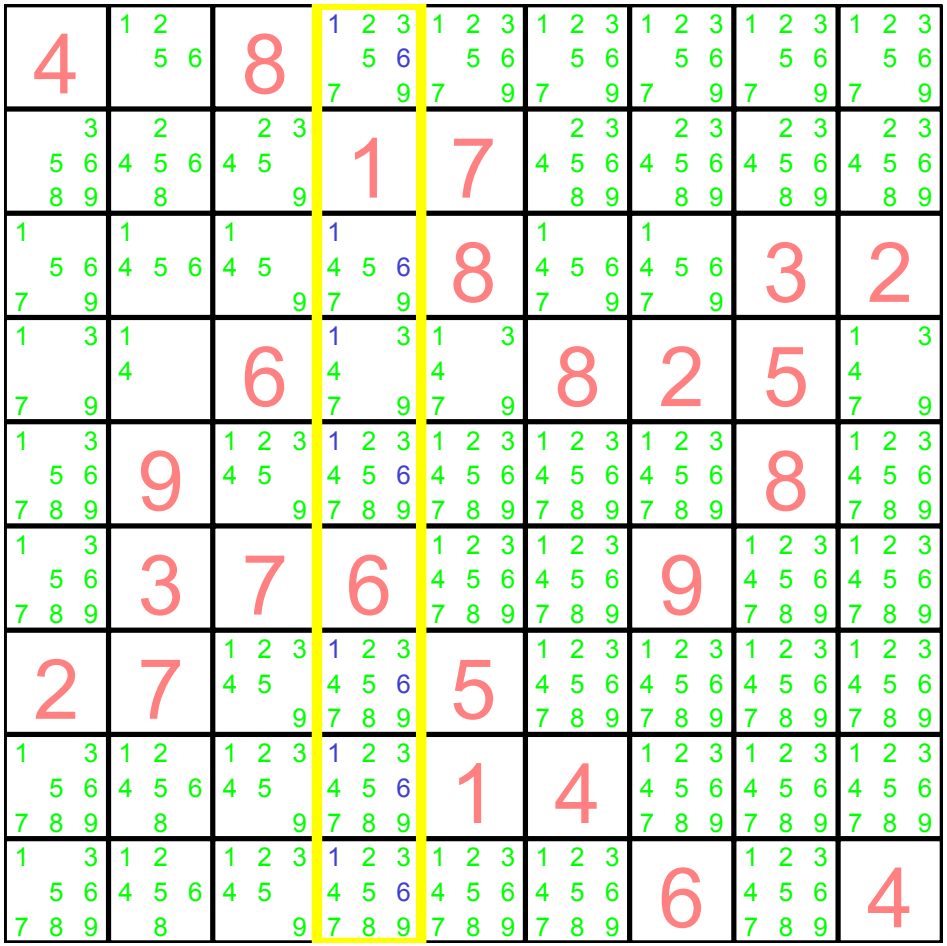
# Propagation Steps (Bounds Consistency)

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 4 5 8 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 5 6 7 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 3 4 5 7 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 3 5 6 7 8 9 | 2 3 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 4 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 2 3 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 4 6 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 4 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 2 3 5 6 7 8 9 |
| 1 3 5 7 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 7 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 7 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 7 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 1 5 6 9 | 1 4 5 | 3 | 2 |
| 1 3 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 2 3 6 9 | 2 3 5 6 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 7 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 5 6 9 | 1 4 5 | 3 | 2 |
| 1 3 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 2 3 6 9 | 2 3 5 6 9 | 1 5 7 | 1 6 7 9 | 1 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 4 5 8 | 4 6 9 | 5 6 8 9 |
| 7 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 5 6 9 | 1 4 5 | 3 | 2 |
| 1 | 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 5 | 9 | 2 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 5 | 6 | 2 3 5 6 9 | 1 5 7 | 1 6 7 9 | 1 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 5 6 9 | 4 5 8 | 4 6 9 | 5 6 8 9 |
| 7 | 6 | 1 5 9 | 4 | 8 | 9 | 1 4 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 9 |
| 5 | 9 | 2 | 3 4 7 | 3 4 6 | 1 | 1 3 4 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 4 8 9 | 5 | 6 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 6 | 2 5 6 8 | 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 3 9 | 1 2 5 8 | 1 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 7 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 9 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 4 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 2 5 8 | 3 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 2 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 2 5 8 | 3 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 2 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

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# Propagation Steps (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

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# After Setup (Bounds Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

# Initial State (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 3 5 6 7 9 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 6 8 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 2 | 5 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 1 4 5 6 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 1 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
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| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 8 9 | 9 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 8 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
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| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 1 3 4 7 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 3 5 6 7 8 9 | 3 | 7 | 6 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 4 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 4 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 4 5 8 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 4 5 6 7 8 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 5 | 1 2 3 5 6 7 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 6 7 9 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 2 3 4 5 6 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 1 | 4 | 1 3 4 5 7 8 | 1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 3 5 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 3 4 5 6 8 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 5 8 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 3 4 6 8 9 | 1 3 4 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 3 5 6 7 8 9 | 2 3 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 3 4 5 6 7 8 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 4 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 2 3 5 6 7 8 9 |
| 1 3 5 6 7 8 9 | 1 2 4 5 6 8 | 1 2 3 4 5 9 | 2 3 4 5 7 8 9 | 2 3 4 6 9 | 1 2 3 5 6 7 9 | 6 | 1 2 4 6 7 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 2 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 2 3 4 5 6 8 9 |
| 1 5 6 7 9 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 1 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 4 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 2 3 4 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 2 4 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 4 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 2 3 5 6 7 8 9 |
| 1 3 5 7 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 7 9 | 2 3 6 9 | 1 2 3 5 6 7 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 8 9 | 2 4 5 6 8 | 2 3 4 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 7 | 1 4 5 6 | 1 4 5 9 | 4 5 7 9 | 8 | 1 5 6 7 9 | 1 4 5 7 | 3 | 2 |
| 1 3 7 9 | 4 | 6 | 3 4 7 9 | 3 4 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 7 | 9 | 1 2 3 4 5 | 2 3 4 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 4 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 7 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 7 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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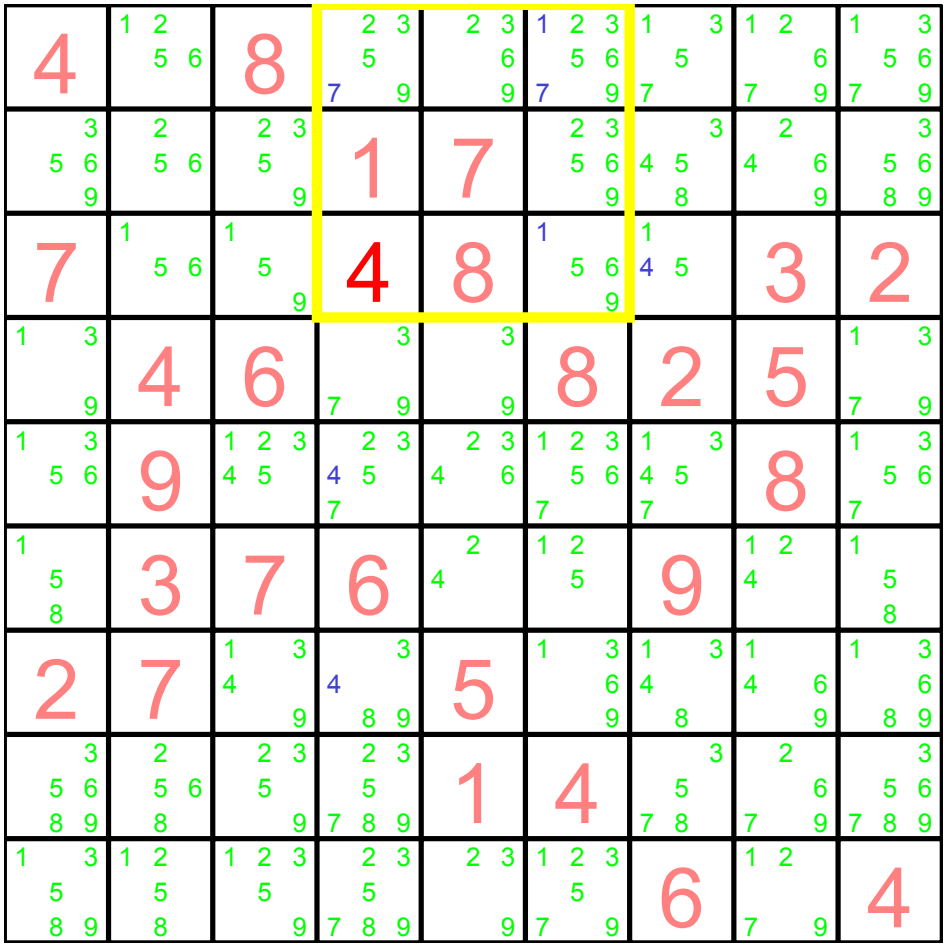
# Propagation Steps (Domain Consistency)

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 2 3 6 9 | 2 3 5 6 9 | 1 3 5 7 | 1 2 6 7 9 | 1 3 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 3 4 5 8 | 2 4 6 9 | 3 5 6 8 9 |
| 7 | 1 5 6 | 1 5 9 | 4 | 8 | 5 6 9 | 1 5 | 3 | 2 |
| 1 3 9 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 1 3 7 9 |
| 1 3 5 6 | 9 | 1 2 3 4 5 | 2 3 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 1 5 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 2 3 6 9 | 2 3 5 6 9 | 1 5 7 | 1 6 7 9 | 1 5 6 7 9 |
| 3 5 6 9 | 2 5 6 | 2 3 5 9 | 1 | 7 | 2 3 5 6 9 | 4 5 8 | 4 6 9 | 5 6 8 9 |
| 7 | 1 5 6 | 1 5 9 | 4 | 8 | 5 6 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 1 3 7 9 |
| 5 | 9 | 2 | 2 3 5 7 | 2 3 4 6 | 1 2 3 5 6 7 | 1 3 4 5 7 | 8 | 1 3 5 6 7 |
| 8 | 3 | 7 | 6 | 2 4 | 1 2 5 | 9 | 1 2 4 | 1 5 8 |
| 2 | 7 | 4 | 3 8 9 | 5 | 1 3 6 9 | 1 3 4 8 | 1 4 6 9 | 1 3 6 8 9 |
| 3 5 6 8 9 | 2 5 6 8 | 2 3 5 9 | 2 3 5 7 8 9 | 1 | 4 | 3 5 7 8 | 2 6 7 9 | 3 5 6 7 8 9 |
| 1 3 5 8 9 | 1 2 5 8 | 1 2 3 5 9 | 2 3 5 7 8 9 | 2 3 9 | 1 2 3 5 7 9 | 6 | 1 2 7 9 | 4 |

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# Propagation Steps (Domain Consistency)

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 9 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

Back to Start  Skip Animation

# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

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# Propagation Steps (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

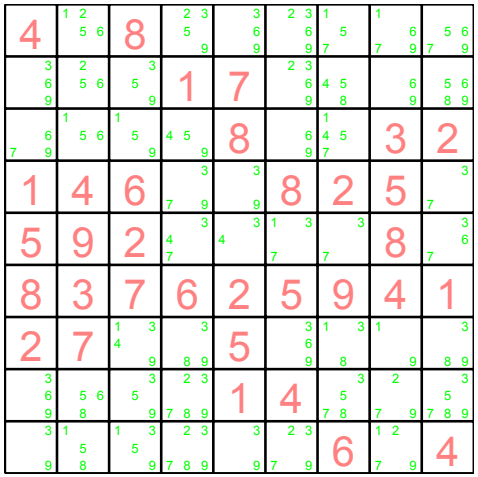
Back to Start

# After Setup (Domain Consistency)

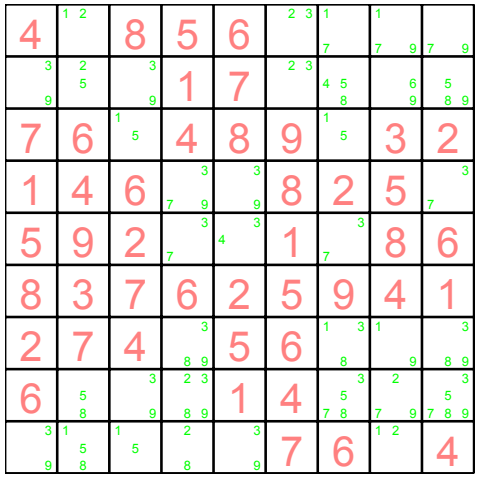
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

# Comparison

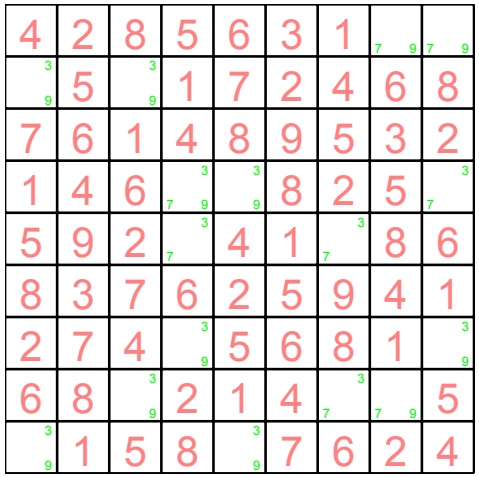
## Forward Checking



## Bounds Consistency



## Domain Consistency



# Typical?

- This does not always happen
- Sometimes, two methods produce same amount of propagation
- Possible to predict in certain special cases
- In general, tradeoff between speed and propagation
- Not always fastest to remove inconsistent values early
- But often required to find a solution at all

# Outline

1 Problem

2 Program

3 Initial Propagation (Forward Checking)

4 Improved Reasoning

5 Search
- Solution

# Simple search routine

- Enumerate variables in given order
- Try values starting from smallest one in domain
- Complete, chronological backtracking

# Search Tree (Forward Checking)

2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 5 6 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
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| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

# Search Tree (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
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| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

# Search Tree (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 8 | 2 | 3 6 9 | 2 3 6 9 | 5 7 | 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 5 6 | 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 5 8 | 1 3 5 9 | 2 3 7 8 9 | 3 9 | 2 3 7 9 | 6 | 1 2 7 9 | 4 |

# Search Tree (Forward Checking)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 8 | 2 | 3 | 3 6 9 | 3 6 9 | 5 7 | 6 7 9 | 5 6 7 9 |
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| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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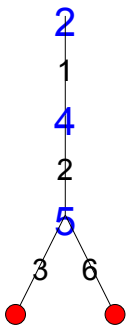
# Search Tree (Forward Checking)

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| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 | |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 | |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 | |
| 3 6 9 | 5 6 8 | 3 5 9 | 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 | |
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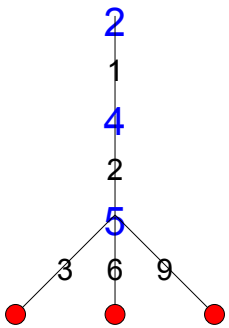
# Search Tree (Forward Checking)

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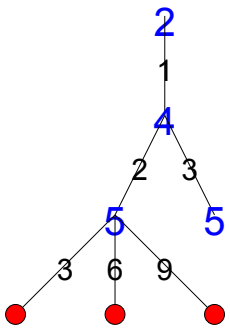
# Search Tree (Forward Checking)

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| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
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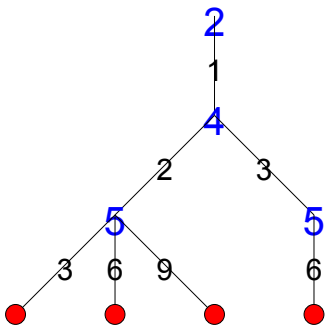
# Search Tree (Forward Checking)

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| 2 | 7 | 1 3 4 9 | 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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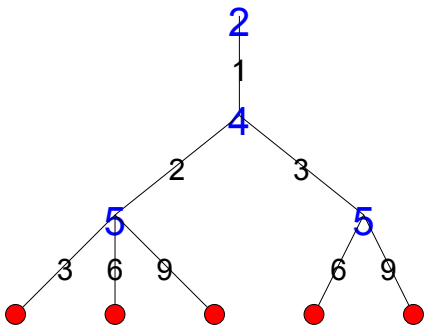
# Search Tree (Forward Checking)

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| 5 | 9 | 2 | 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
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| 2 | 7 | 1 3 4 9 | 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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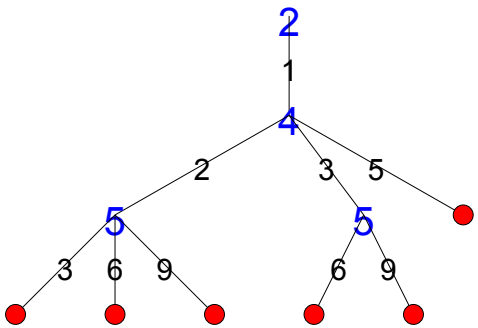
# Search Tree (Forward Checking)

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| 6 7 9 | 5 6 | 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
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| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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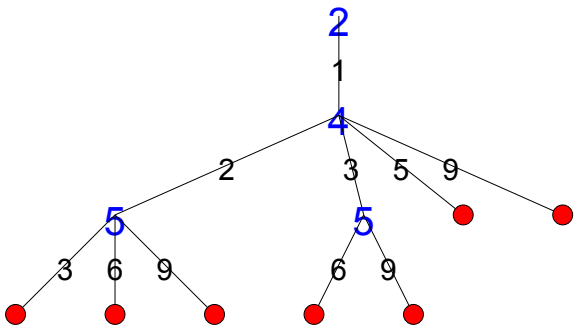
# Search Tree (Forward Checking)

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| 6 7 9 | 5 6 | 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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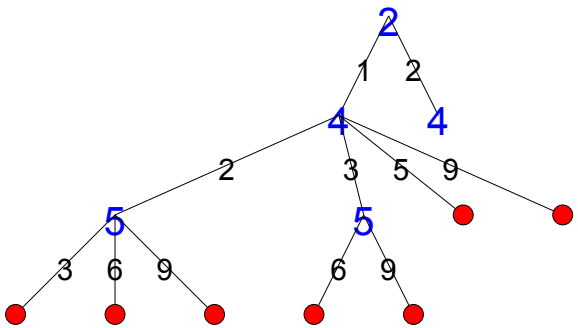
# Search Tree (Forward Checking)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 2 3 5 9 | 3 6 9 | 2 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 2 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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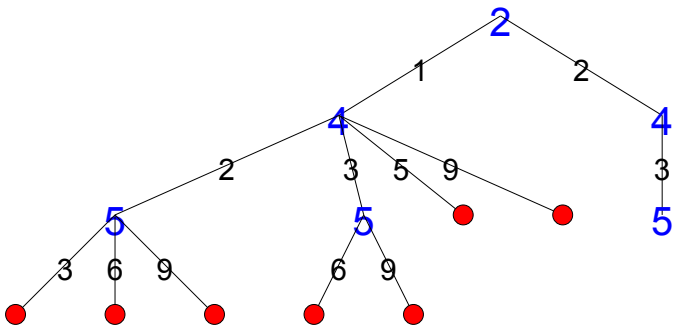
# Search Tree (Forward Checking)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 3 | 3 6 9 | 3 6 9 | 1 5 7 | 1 6 7 9 | 5 6 7 9 |
| 3 6 9 | 5 6 | 3 5 9 | 1 | 7 | 2 3 6 9 | 4 5 8 | 6 9 | 5 6 8 9 |
| 6 7 9 | 1 5 6 | 1 5 9 | 4 5 9 | 8 | 6 9 | 1 4 5 7 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 3 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
| 3 6 9 | 5 6 8 | 3 5 9 | 2 3 7 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
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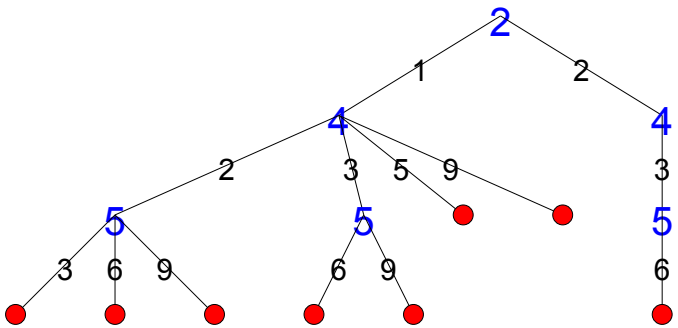
# Search Tree (Forward Checking)

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| 1 | 4 | 6 | 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 4 7 | 3 4 | 1 3 7 | 3 7 | 8 | 3 6 7 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 1 3 4 9 | 8 9 | 5 | 3 6 9 | 1 3 8 | 1 9 | 3 8 9 |
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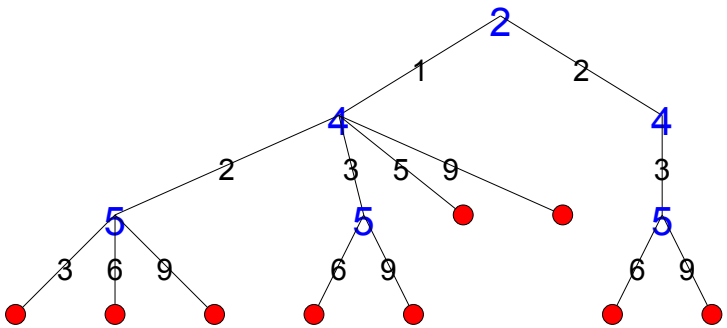
# Search Tree (Forward Checking)

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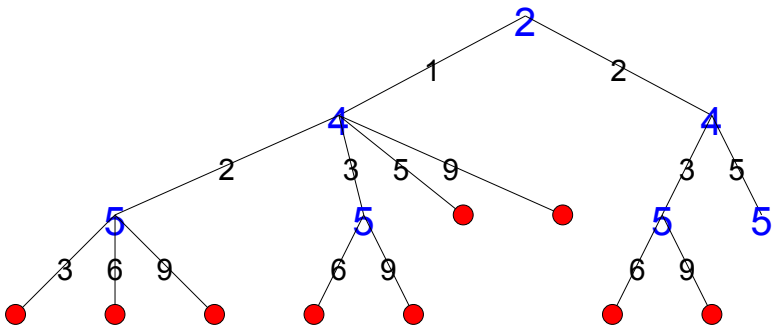
# Search Tree (Forward Checking)

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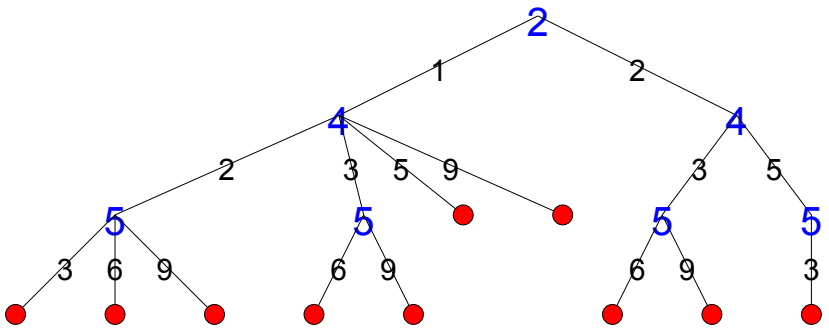
# Search Tree (Forward Checking)

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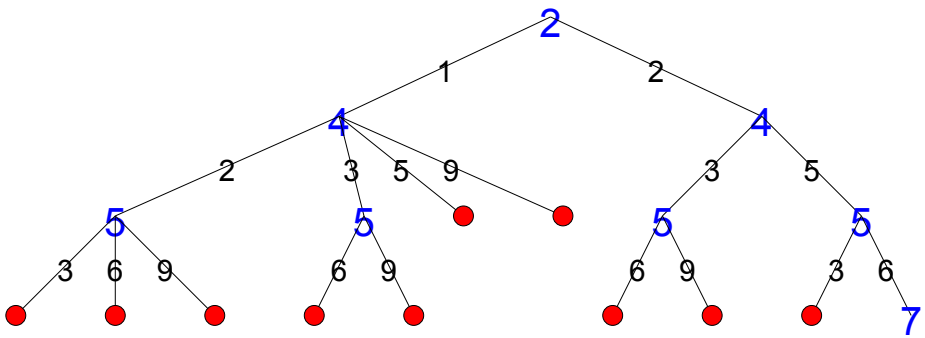
# Search Tree (Forward Checking)

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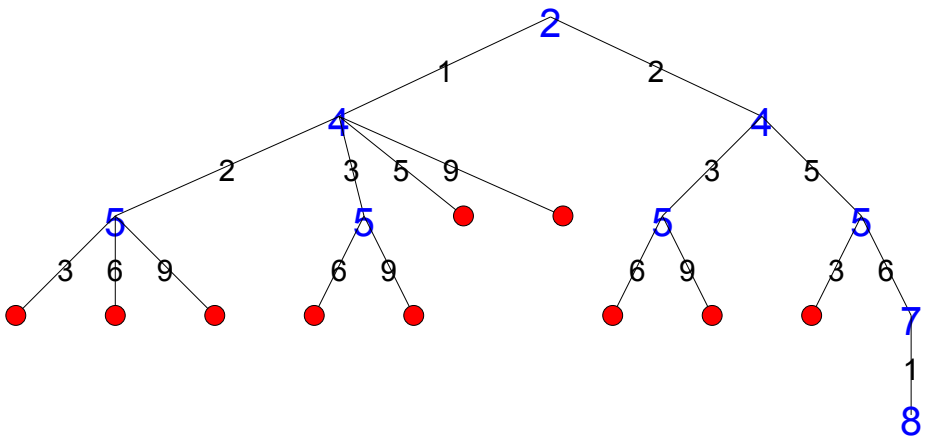
# Search Tree (Forward Checking)

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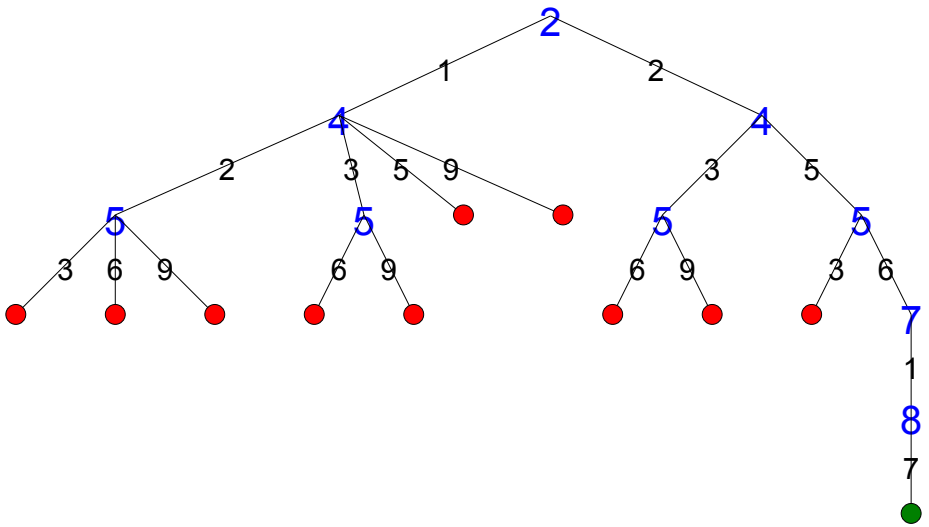
| 3 9 | 1 5 8 | 1 3 5 9 | 2 3 8 9 | 3 9 | 7 | 6 | 1 2 9 | 4 |

# Search Tree (Forward Checking)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 | 9 |
| 3 | 5 | 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 | 9 | 8 | 2 | 5 | 7 |
| 5 | 9 | 2 | 7 | 4 | 1 | 3 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 9 | 5 | 6 | 8 | 1 | 3 |
| 6 | 8 | 3 | 2 | 1 | 4 | 7 | 9 | 5 |
| 9 | 1 | 5 | 8 | 3 | 7 | 6 | 2 | 4 |

# Search Tree (Bounds Consistency)

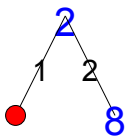
2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 2 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

# Search Tree (Bounds Consistency)

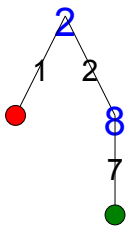
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 8 | 5 | 6 | 2 3 | 1 7 | 1 7 9 | 7 9 |
| 3 9 | 2 5 | 3 9 | 1 | 7 | 2 3 | 4 5 8 | 6 9 | 5 8 9 |
| 7 | 6 | 1 5 | 4 | 8 | 9 | 1 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 1 3 8 | 1 9 | 3 8 9 |
| 6 | 5 8 | 3 9 | 2 3 8 9 | 1 | 4 | 3 5 7 8 | 2 7 9 | 3 5 7 8 9 |
| 3 9 | 1 5 8 | 1 5 | 2 8 | 3 9 | 7 | 6 | 1 2 | 4 |

# Search Tree (Bounds Consistency)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 1 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 5 8 9 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 3 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 8 9 | 5 | 6 | 8 | 1 | 3 8 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 5 7 8 | 2 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

# Search Tree (Bounds Consistency)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 | 9 |
| 3 | 5 | 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 | 9 | 8 | 2 | 5 | 7 |
| 5 | 9 | 2 | 7 | 4 | 1 | 3 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 9 | 5 | 6 | 8 | 1 | 3 |
| 6 | 8 | 3 | 2 | 1 | 4 | 7 | 9 | 5 |
| 9 | 1 | 5 | 8 | 3 | 7 | 6 | 2 | 4 |

# Search Tree (Domain Consistency)

8

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 9 | 7 9 |
| 3 9 | 5 | 3 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 7 9 | 3 9 | 8 | 2 | 5 | 3 7 |
| 5 | 9 | 2 | 3 7 | 4 | 1 | 3 7 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 3 9 | 5 | 6 | 8 | 1 | 3 9 |
| 6 | 8 | 3 9 | 2 | 1 | 4 | 3 7 | 7 9 | 5 |
| 3 9 | 1 | 5 | 8 | 3 9 | 7 | 6 | 2 | 4 |

# Search Tree (Domain Consistency)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 | 9 |
| 3 | 5 | 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 | 9 | 8 | 2 | 5 | 7 |
| 5 | 9 | 2 | 7 | 4 | 1 | 3 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 9 | 5 | 6 | 8 | 1 | 3 |
| 6 | 8 | 3 | 2 | 1 | 4 | 7 | 9 | 5 |
| 9 | 1 | 5 | 8 | 3 | 7 | 6 | 2 | 4 |

# Observations

- Search tree much smaller for bounds/domain consistency
- Does not always happen like this
- Smaller tree = Less execution time
- Less reasoning = Less execution time
- Problem: Finding best balance
- For Sudoku: not good enough, should not require any search!

# Solution

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 8 | 5 | 6 | 3 | 1 | 7 | 9 |
| 3 | 5 | 9 | 1 | 7 | 2 | 4 | 6 | 8 |
| 7 | 6 | 1 | 4 | 8 | 9 | 5 | 3 | 2 |
| 1 | 4 | 6 | 3 | 9 | 8 | 2 | 5 | 7 |
| 5 | 9 | 2 | 7 | 4 | 1 | 3 | 8 | 6 |
| 8 | 3 | 7 | 6 | 2 | 5 | 9 | 4 | 1 |
| 2 | 7 | 4 | 9 | 5 | 6 | 8 | 1 | 3 |
| 6 | 8 | 3 | 2 | 1 | 4 | 7 | 9 | 5 |
| 9 | 1 | 5 | 8 | 3 | 7 | 6 | 2 | 4 |

# Outline

6 Conclusions

7 Complete Example: Domain Consistent Alldifferent

8 Generic Model

9 Exercises

# Global Constraints

- Powerful modelling abstractions
- Efficient reasoning

# Consistency Levels

- Defined levels of propagation
- Tradeoff speed/reasoning
- Characterisation of power of constraint

# Alldifferent Variants

- Forward Checking
  - Only reacts when variables are assigned
  - Equivalent to decomposition into binary constraints
- Bounds Consistency
  - Typical best compomise speed/reasoning
  - Works well if no holes in domain
- Domain Consistency
  - Extracts all information from single constraint
  - Cost only justified for very hard problems

# Outline

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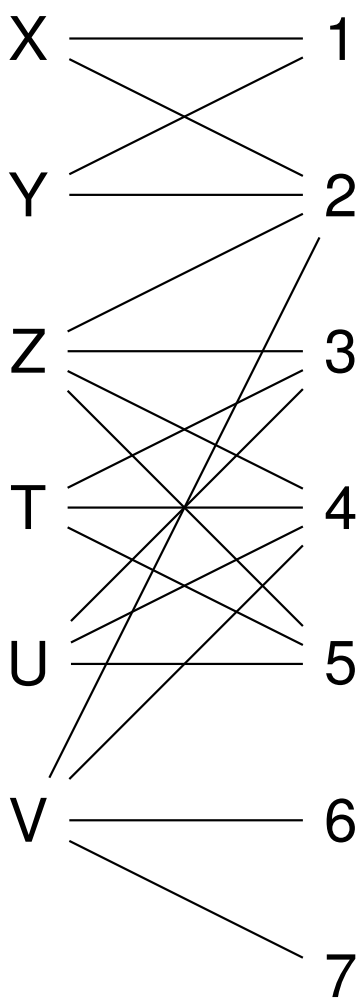
# Bigger Example

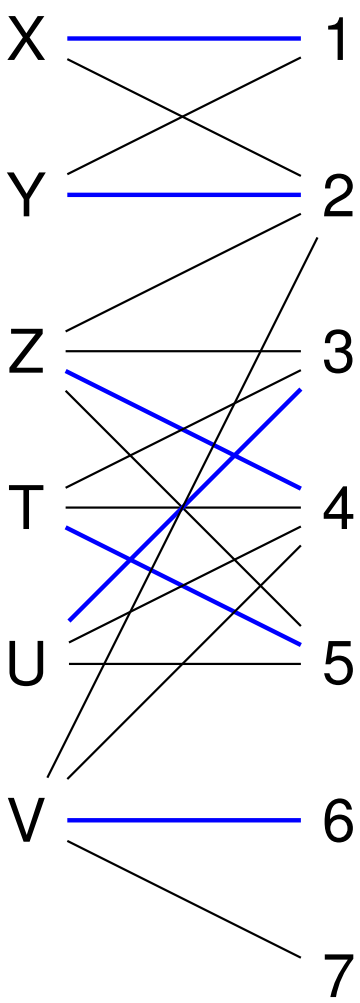
```
:-lib(ic).
:-lib(ic_global_gac).

top:-
    [X,Y] :: 1..2,
    Z :: 2..5,
    [T,U] :: 3..5,
    V :: [2,4,6,7],
    ic_global_gac:alldifferent([X,Y,Z,T,U,V]).
```

# Making constraint domain consistent



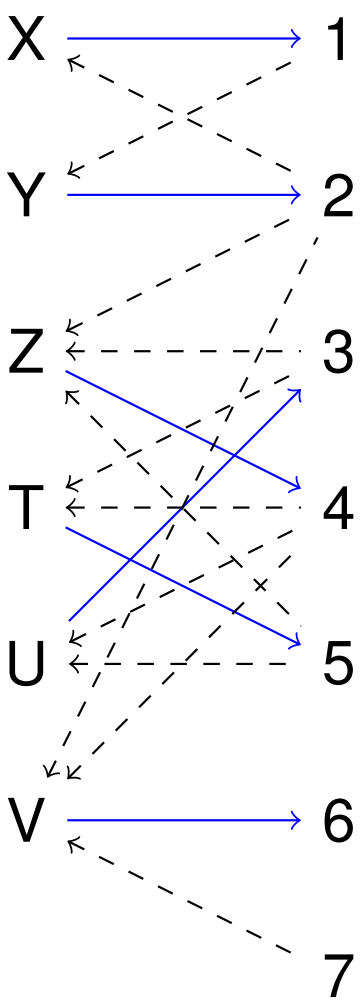

Problem shown as bipartite graph

# Making constraint domain consistent



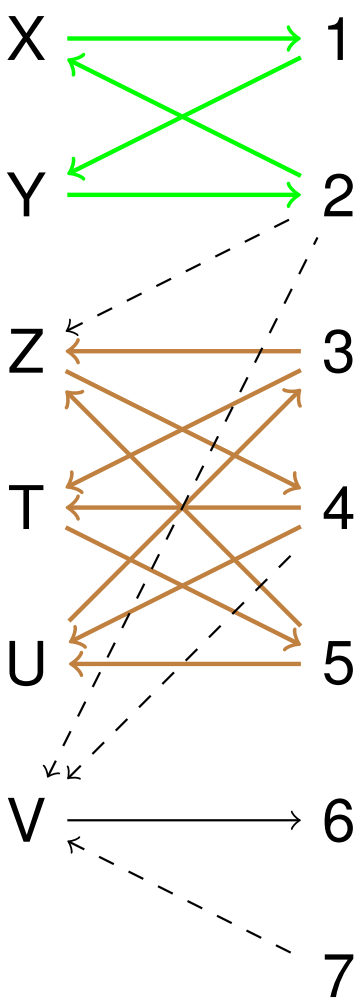

Find maximal matching (in blue)

# Making constraint domain consistent



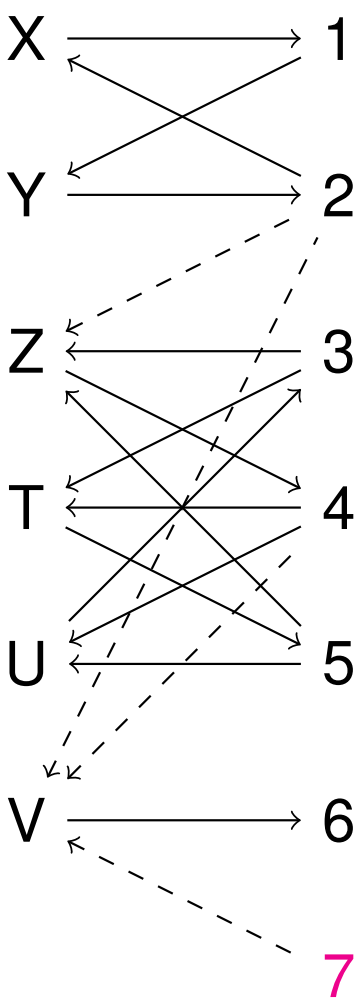

Orient graph (edges in matching from variables to values, all others from values to variables), mark edges in matching

# Making constraint domain consistent



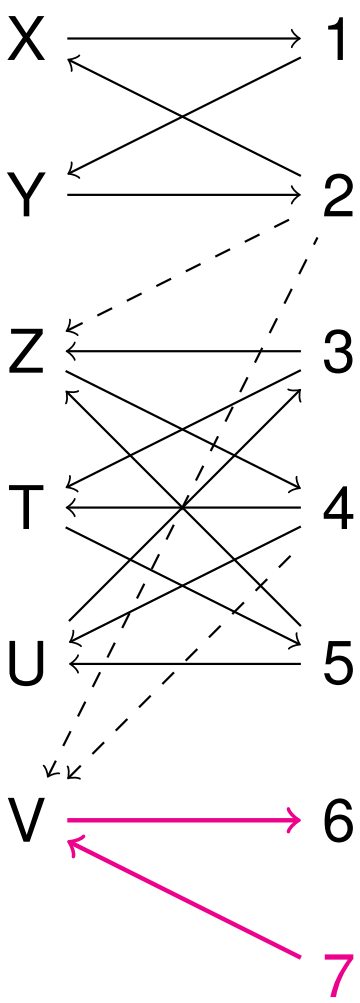

Find strongly connected components (green and brown), mark their edges

# Making constraint domain consistent



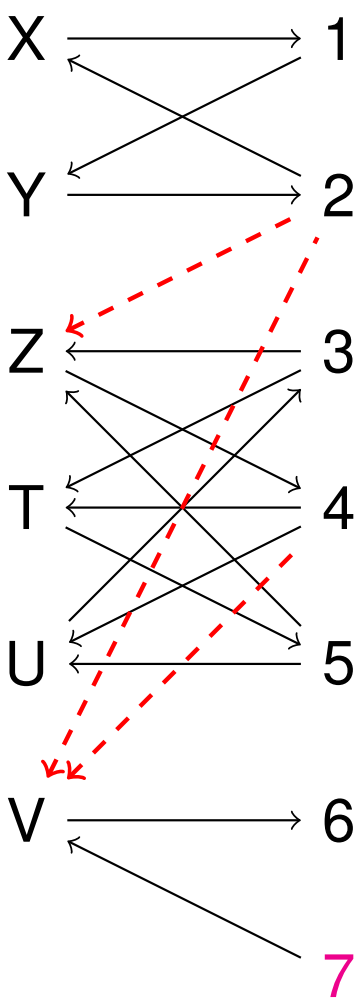

Find unmatched value nodes (here node 7, magenta)

# Making constraint domain consistent



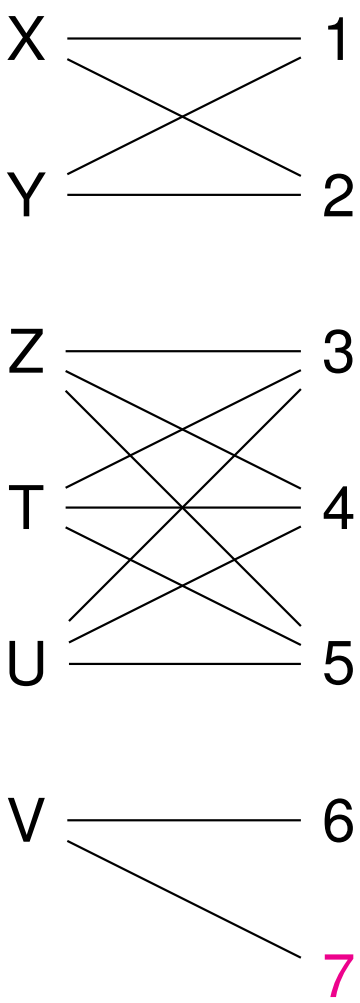

Find alternating paths from such nodes (in magenta), mark their edges

# Making constraint domain consistent




All unmarked edges can be removed

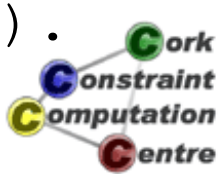
# Making constraint domain consistent



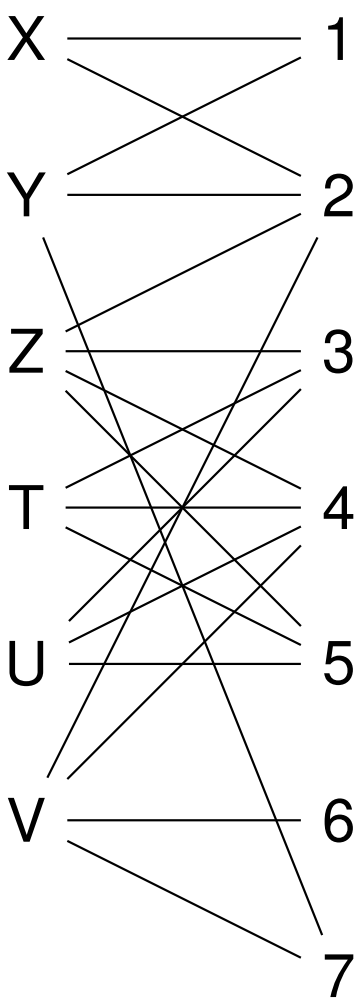

Resulting graph, constraint is domain consistent

# Extended Example

```
:-lib(ic).
:-lib(ic_global_gac).

top:-
    X :: 1..2,
    Y :: [1,2,7],
    Z :: 2..5,
    [T,U] :: 3..5,
    V :: [2,4,6,7],
    ic_global_gac:alldifferent([X,Y,Z,T,U,V]).
```

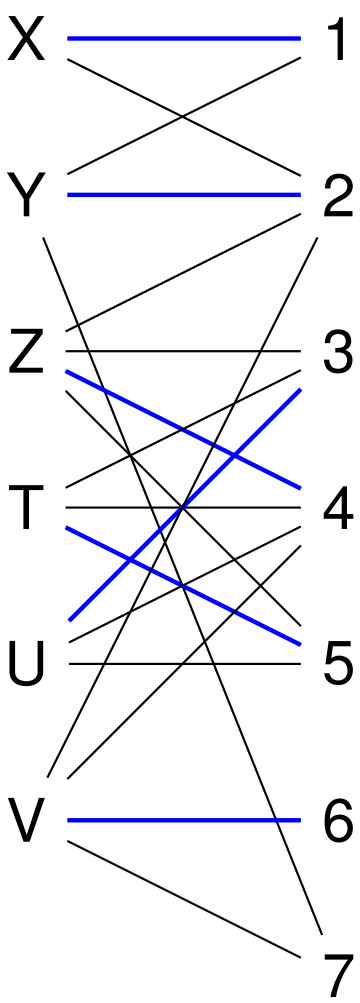


# No propagation in expanded example



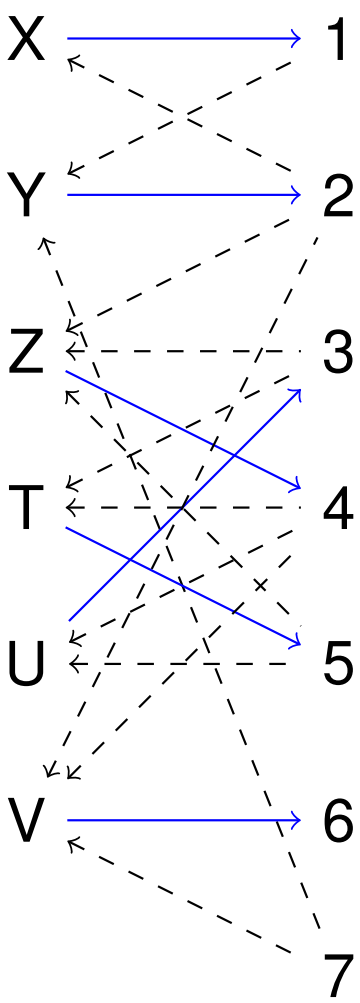

Problem shown as bipartite graph

# No propagation in expanded example



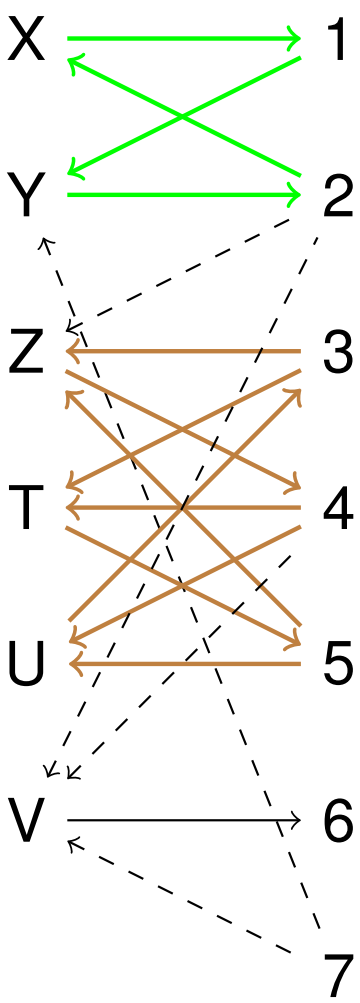

Find maximal matching (in blue)

# No propagation in expanded example



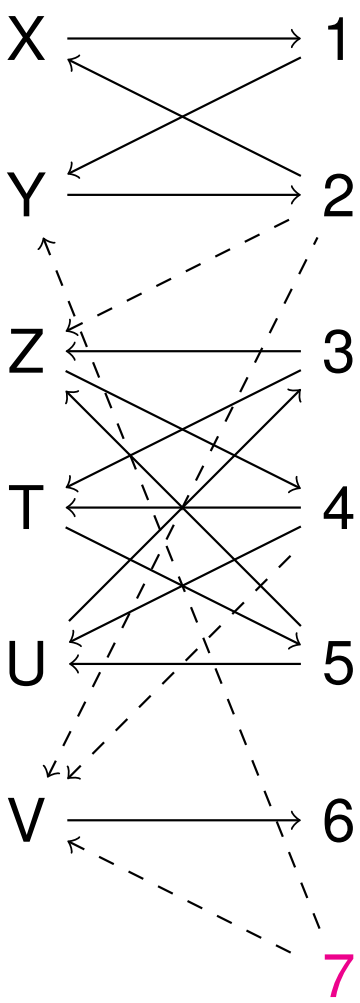

Orient graph (edges in matching from variables to values, all others from values to variables), mark edges in matching

# No propagation in expanded example



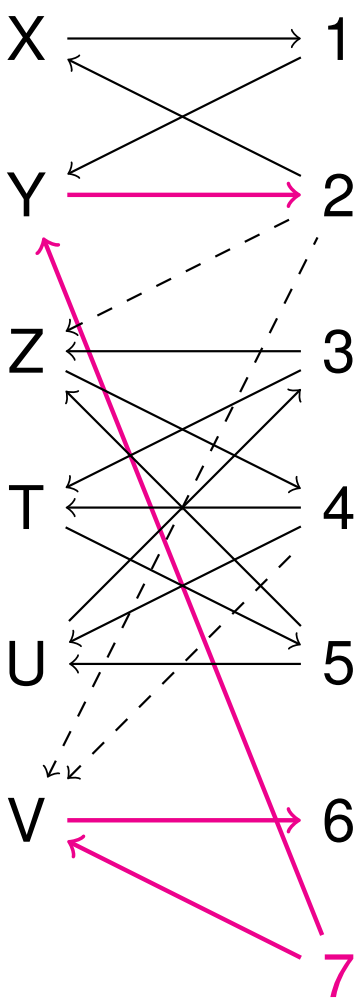

Find strongly connected components (green and brown), mark their edges

# No propagation in expanded example



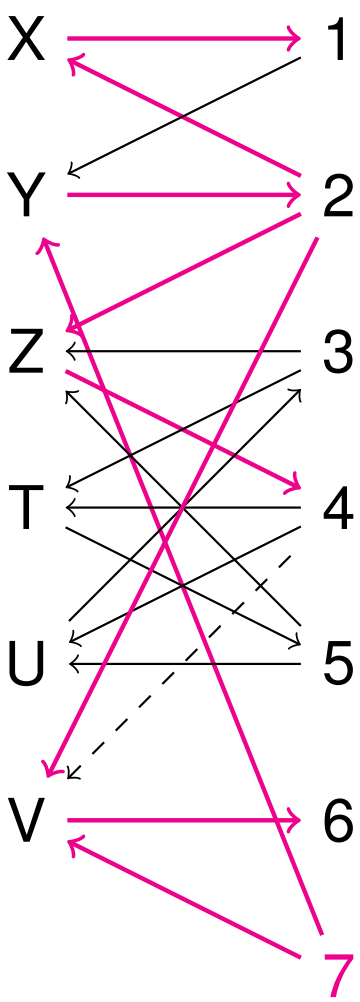

Find unmatched value nodes (here node 7, magenta)

# No propagation in expanded example



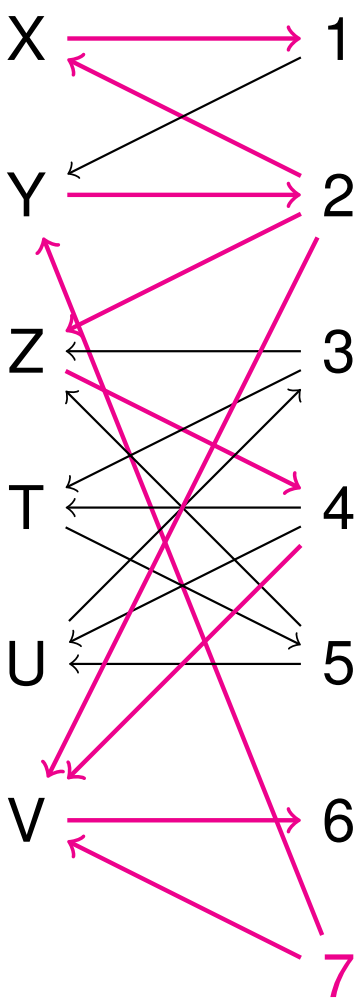

Find alternating paths from such nodes (in magenta), mark their edges

# No propagation in expanded example




Continue with alternating paths

# No propagation in expanded example




Continue with alternating paths, all edges marked, no propagation, constraint is domain consistent

# Observation

- A lot of effort for no propagation
- Problem: Slows down search without any upside
- Constraint is woken every time any domain is changed
- How often does the constraint do actual pruning?

# Outline

6 Conclusions

7 Complete Example: Domain Consistent Alldifferent

8 Generic Model

9 Exercises

# Generalize Program for different sizes

- How to generalize program for different sizes (4,9,16,25,36...)
- Add parameter R (Order, number of blocks in a row/column)
- Size N is square of R
- Remove explicit integer bounds by expressions
- Useful to do this change as rewriting of working program

# Main Program

```
model(R,M,Matrix):-
    N is R*R,Matrix[1..N,1..N] :: 1..N,
    (for(I,1,N),
     param(N,M,Matrix) do
        M:alldifferent(Matrix[I,1..N]),
        M:alldifferent(Matrix[1..N,I])
    ),
    (multifor([I,J],[1,1],[N-R+1,N-R+1],[R,R]),
     param(R,M,Matrix) do
        M:alldifferent(flatten(Matrix[I..I+R-1,
                                      J..J+R-1]))
    ),
    flatten_array(Matrix,List),labeling(List).
```

# More Information

Krzysztof R. Apt and Mark Wallace.
*Constraint Logic Programming using ECLiPSe*.
Cambridge University Press, New York, NY, USA, 2007.

Willem Jan van Hoeve.
The alldifferent constraint: A survey.
*CoRR*, cs.PL/0105015, 2001.

Claude-Guy Quimper.
*Efficient Propagators for Global Constraints*.
PhD thesis, University of Waterloo, 2006.

# More Information

H. Simonis.
Sudoku as a constraint problem.
In B. Hnich, P. Prosser, and B. Smith, editors, *Proceedings of the 4th International Workshop on Modelling and Reformulating Constraint Satisfaction Problems*, pages 13–27, September 2005.

I. Lynce and J. Ouaknine.
Sudoku as a SAT problem.
In *9th International Symposium on Artificial Intelligence and Mathematics*, January 2006.

# More Information

H. Simonis.
Kakuro as a constraint problem.
In P. Flener, H. Simonis, editors, *Proceedings of the 8th International Workshop on Modelling and Reformulating Constraint Satisfaction Problems*, September 2008.

# Outline

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# Exercises

1